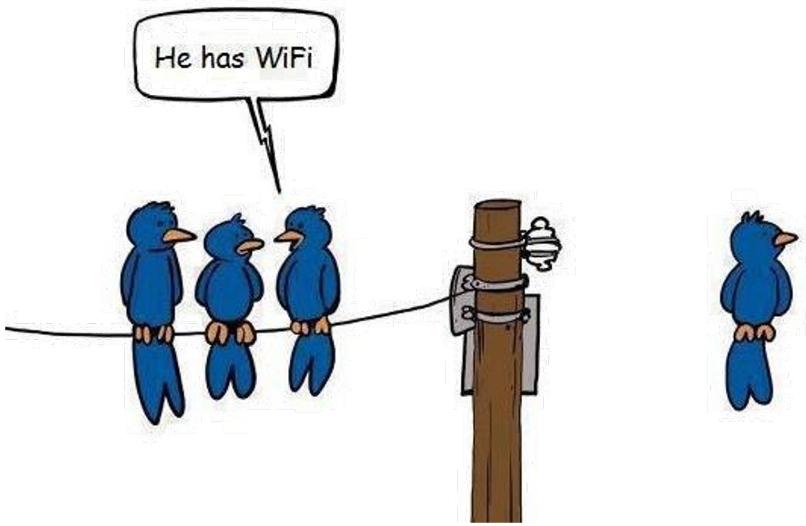
COM-405: Mobile Networks

Lecture 2.0: Wireless Channel

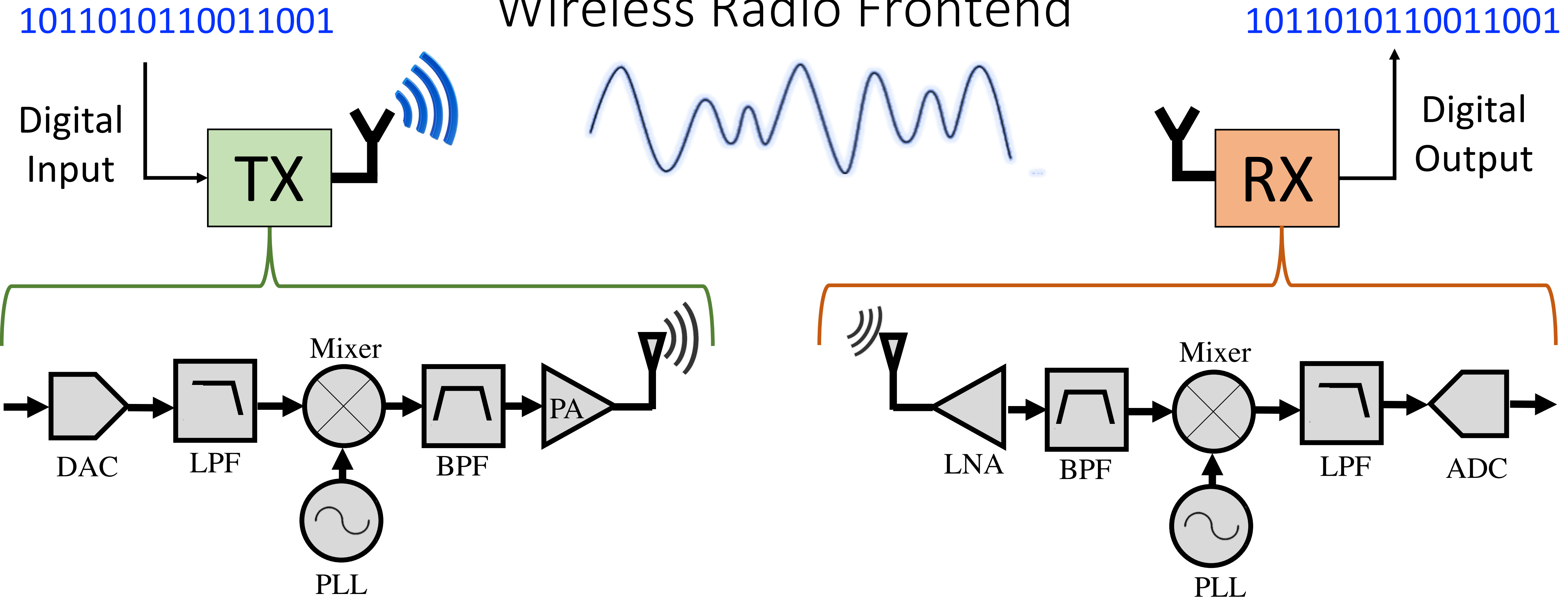
Haitham Hassanieh



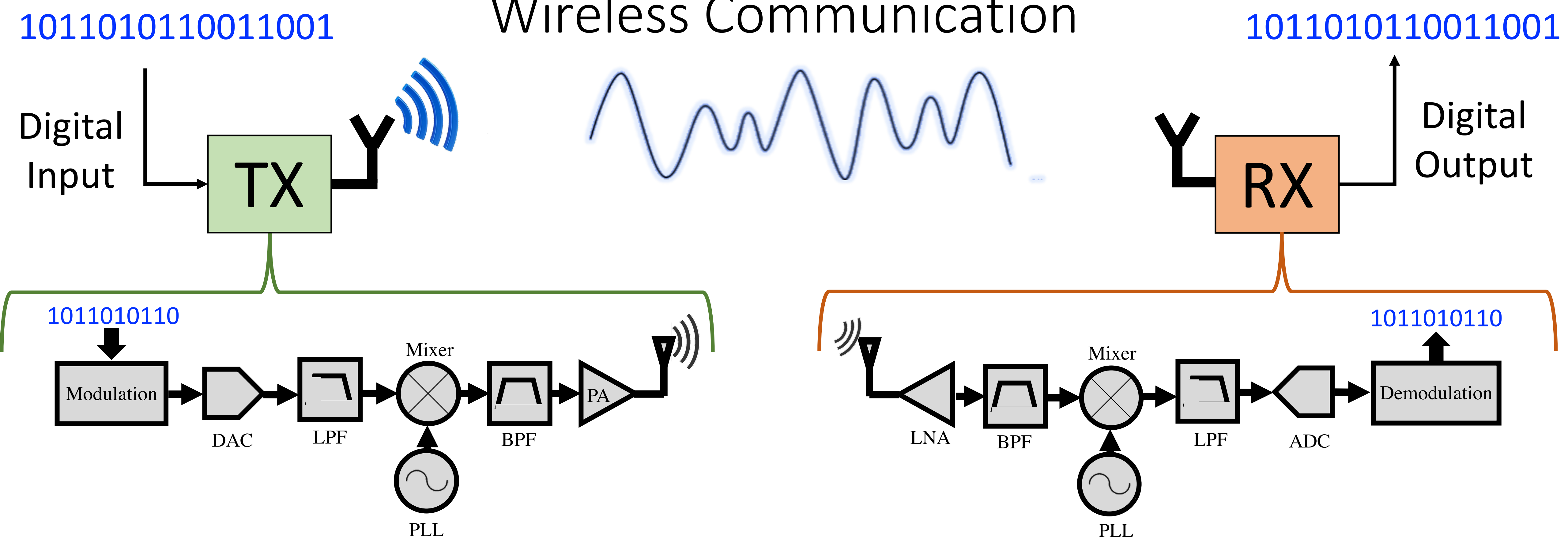
EPFL

Laboratory of SEnsing
& Networking Systems

Wireless Radio Frontend

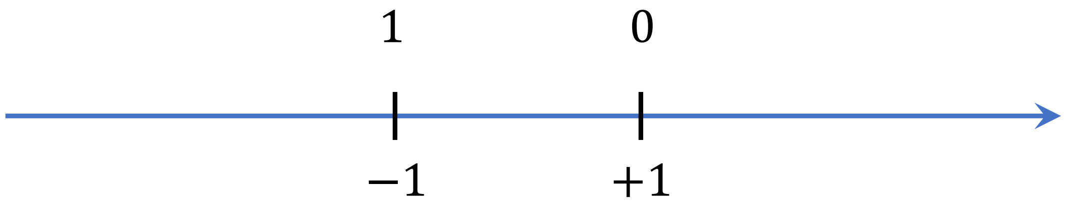


Wireless Communication

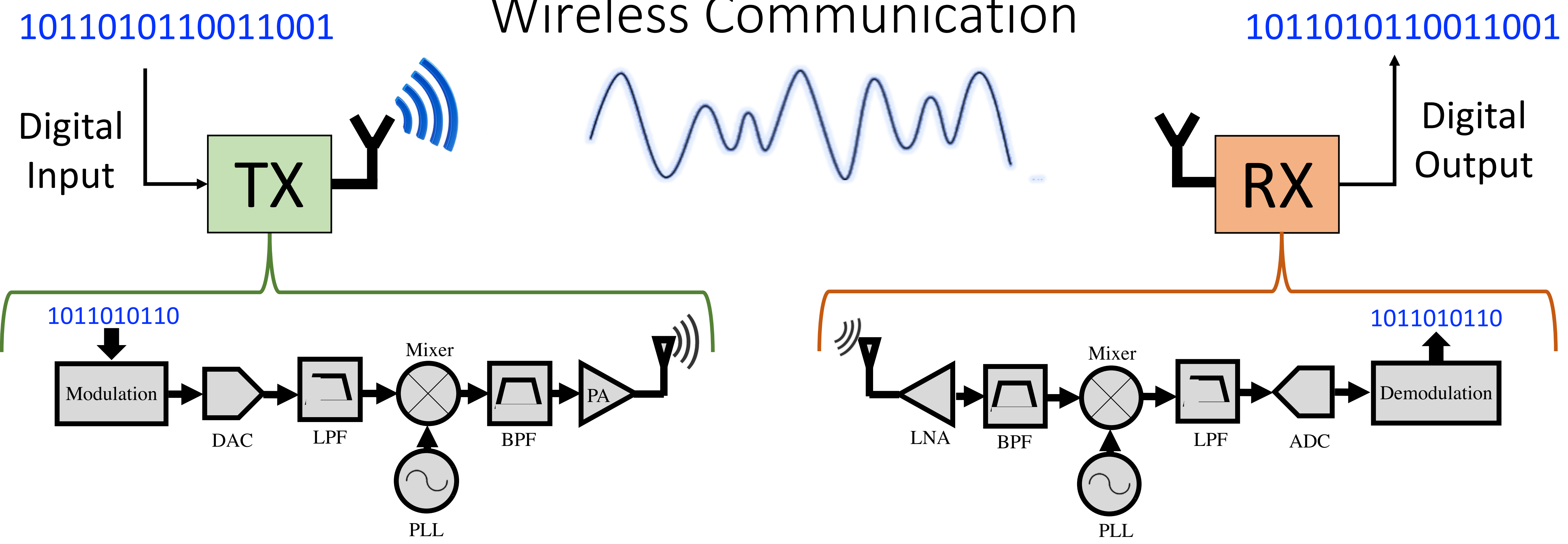


Modulation:
Mapping of Bits to Symbols

BPSK: 1 bits/symbol

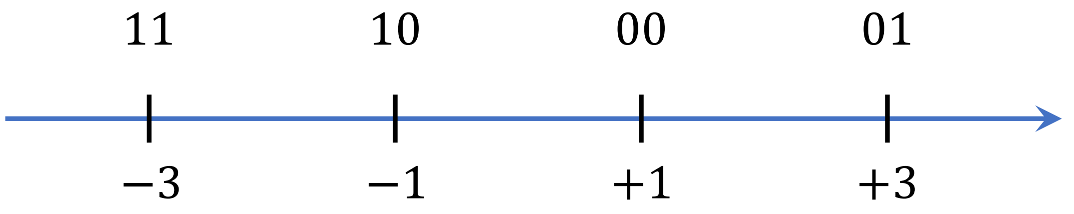


Wireless Communication

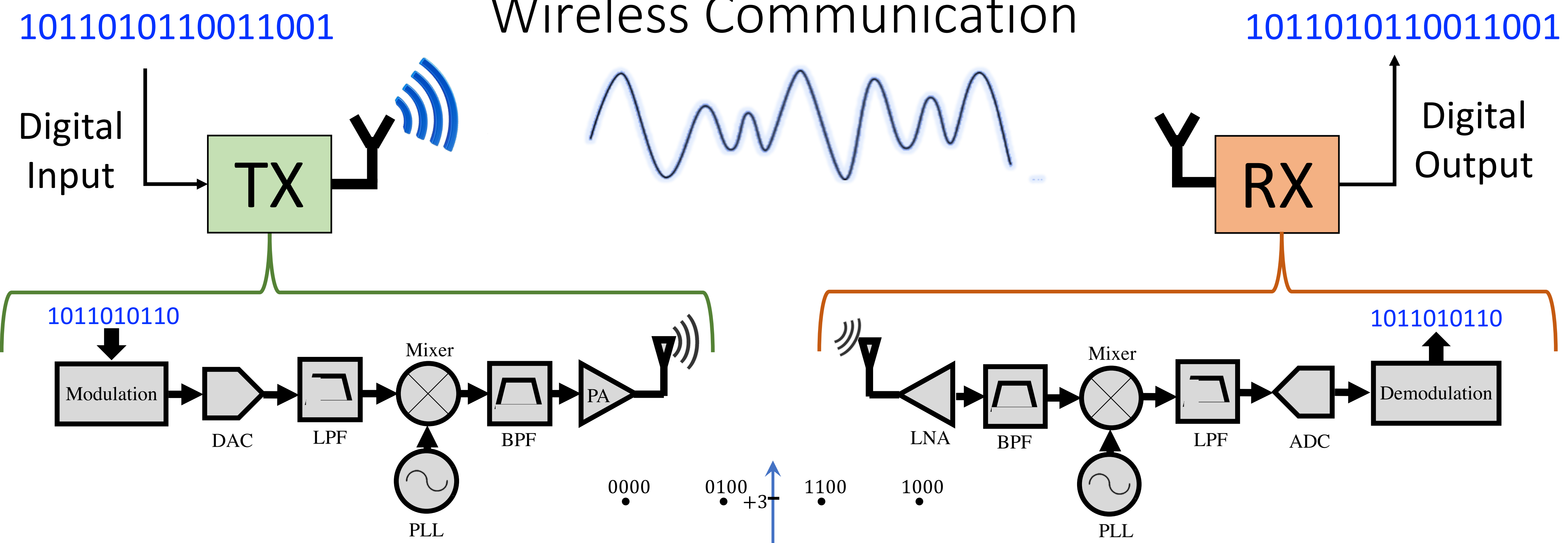


PAM: 2 bits/symbol

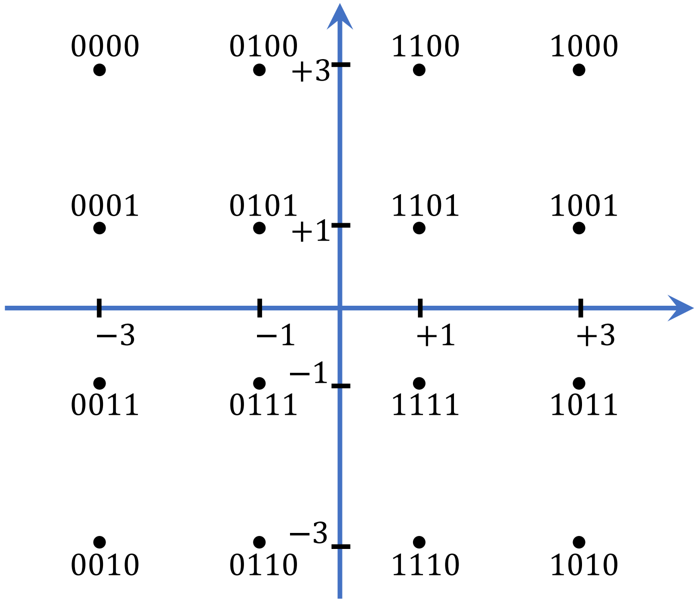
Modulation:
Mapping of Bits to Symbols



Wireless Communication



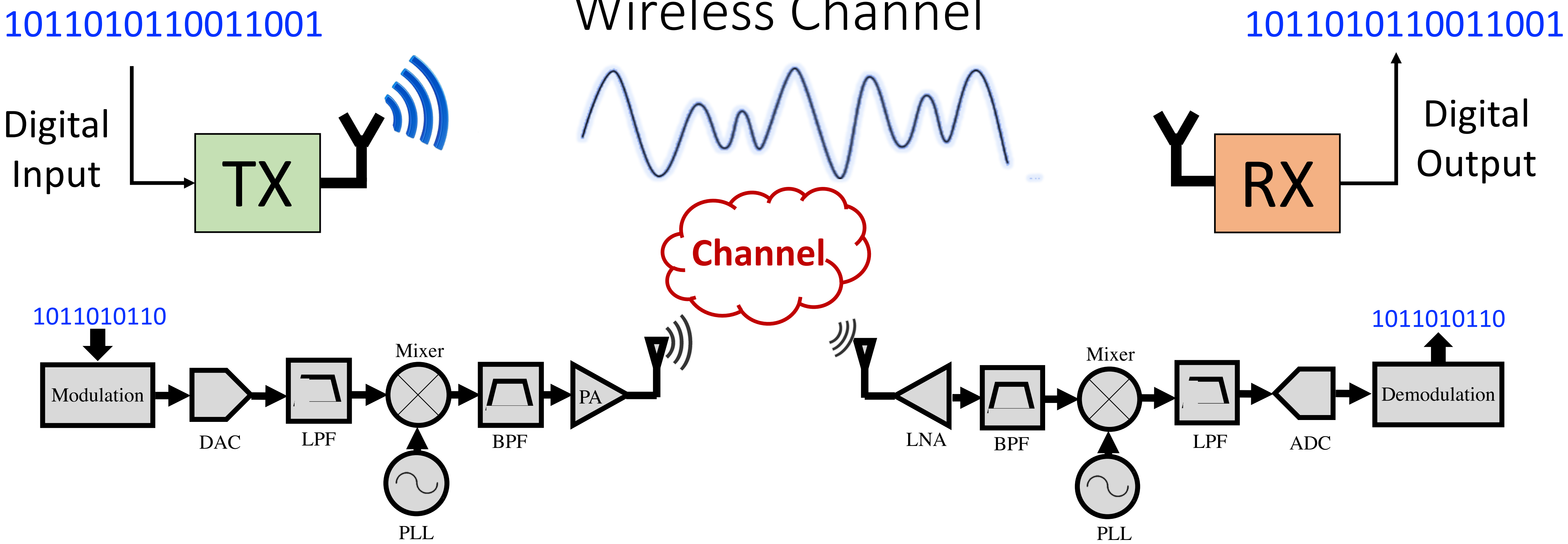
Modulation:
Mapping of Bits to Symbols



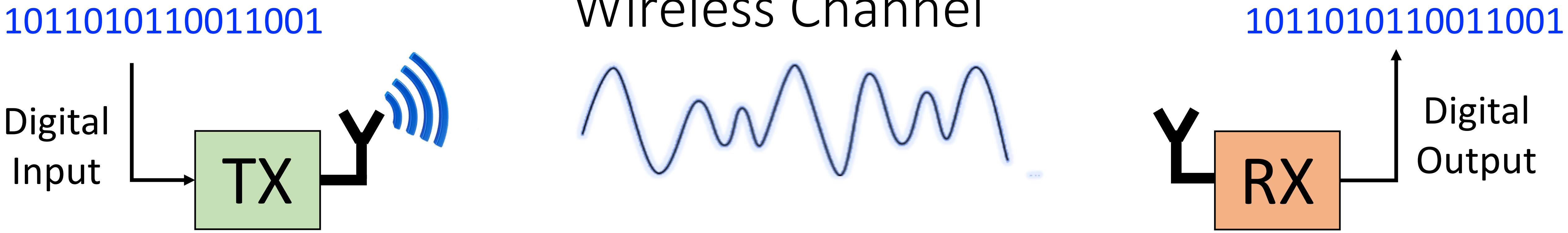
16-QAM:
4 bits/symbol

Complex Symbols!

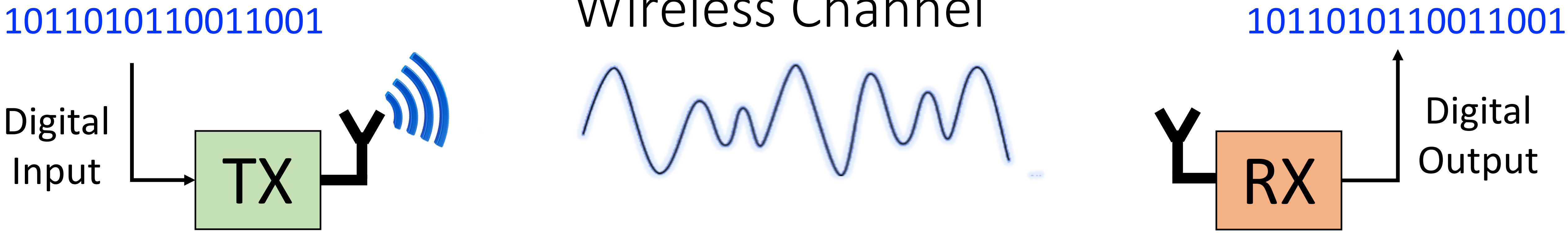
Wireless Channel



Wireless Channel



Wireless Channel



$$y(t) = x(t) + v(t)$$

Channel adds noise (AWGN)!

$$v(t) \sim N(0, \sigma)$$

Noise

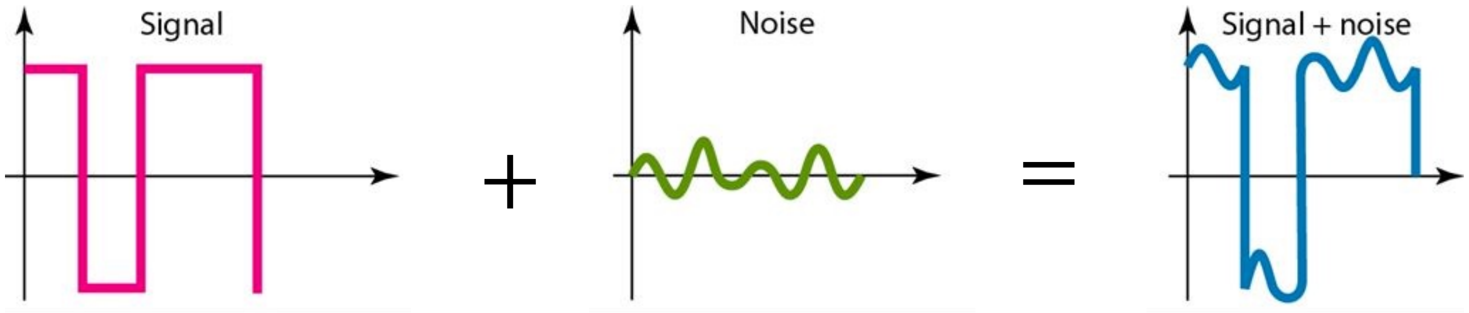
Circuit Components and wireless channel add noise to the wireless signal.

Transmit		Receive
$x(t)$	➡	$y(t) = x(t) + v(t)$

$v(t)$ is Additive White Gaussian Noise (AWGN)

Low noise is good

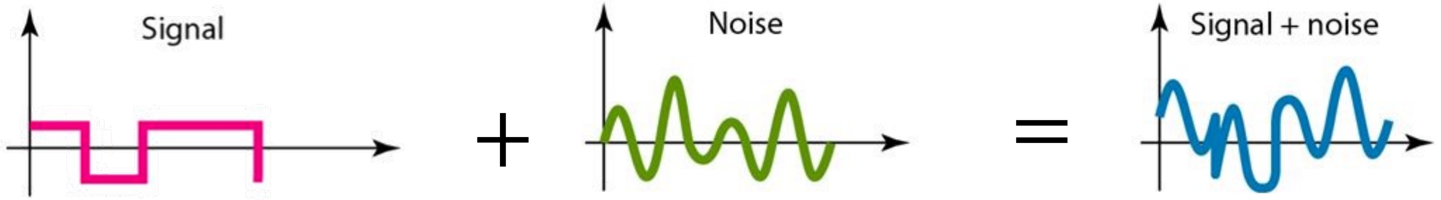
→ Can decode Bits correctly



Bits: 0 1 0 0

High noise is bad

→ Cannot decode Bits correctly



Bits: 0 1 0 0

Noise

Circuit Components and wireless channel add noise to the wireless signal.

Transmit		Receive
$x(t)$	➡	$y(t) = x(t) + v(t)$

$v(t)$ is Additive White Gaussian Noise (AWGN)

Low noise is good → Can decode Bits correctly

High noise is bad → Cannot decode Bits correctly

Signal-to-Noise Ratio (SNR)

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E[|x(t)|^2]}{E[|v(t)|^2]} = \frac{E_s}{N_0}$$

Noise

Circuit Components and wireless channel add noise to the wireless signal.

Transmit		Receive
$x(t)$	➡	$y(t) = x(t) + v(t)$

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Signal-to-Noise Ratio (SNR)

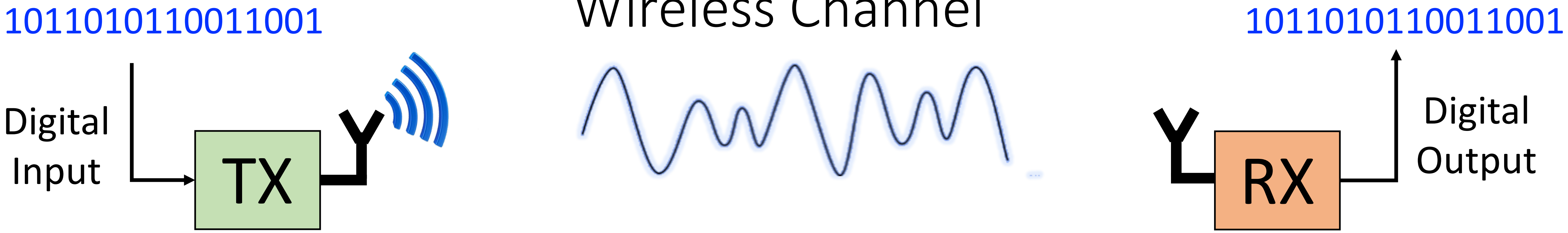
$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E[|x(t)|^2]}{E[|v(t)|^2]} = \frac{E_s}{N_0}$$

We typically measure and report SNR in dB scale: $SNR_{dB} = 10 \log_{10} SNR$

e.g., if $SNR = 100$, then $SNR_{dB} = 20\ dB$,

if $SNR = 2$, then $SNR_{dB} \approx 3dB$.

Wireless Channel

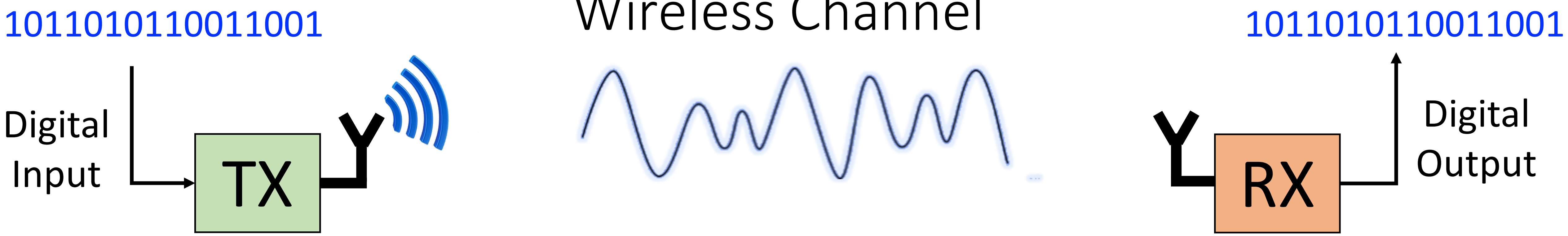


$$y(t) = x(t-\tau) + v(t)$$

Channel delays the signal!

$$\tau = \frac{d}{c}$$

Wireless Channel



$$y(t) = \boldsymbol{h}\, x(t - \tau) + v(t)$$

Channel attenuates the signal (Pathloss)

Given a distance d and frequency of operation f, the received signal power is proportional to:

A. $\propto d$, $\propto f$

B. $\propto 1/d, \propto 1/f$

C. $\propto 1/d^2, \propto 1/f^2$

D. $\propto 1/d^2, \propto f^2$

Given a distance d and frequency of operation f, the received signal power is proportional to:

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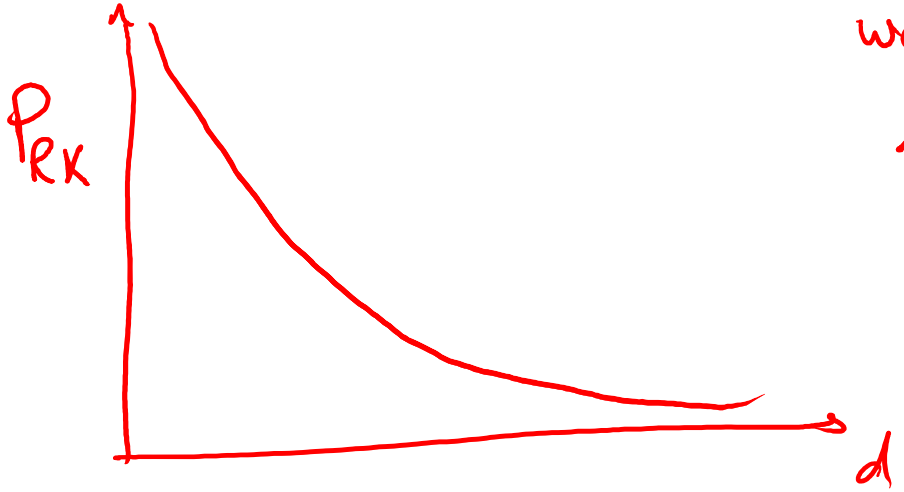


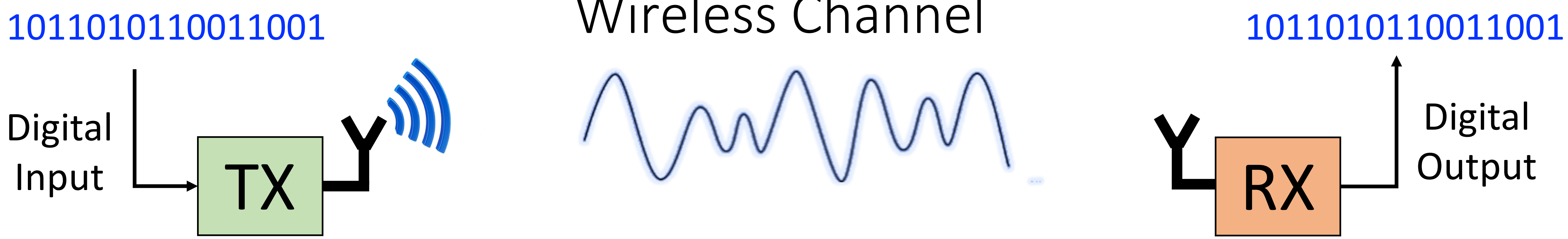
wavelength

$\lambda = \frac{c}{f}$

$$P_{RX} = G_{RX} G_{TX} \frac{\lambda^2}{(4\pi d)^2} P_{TX}$$

Antenna
Gain
Typically 1

Wireless Channel

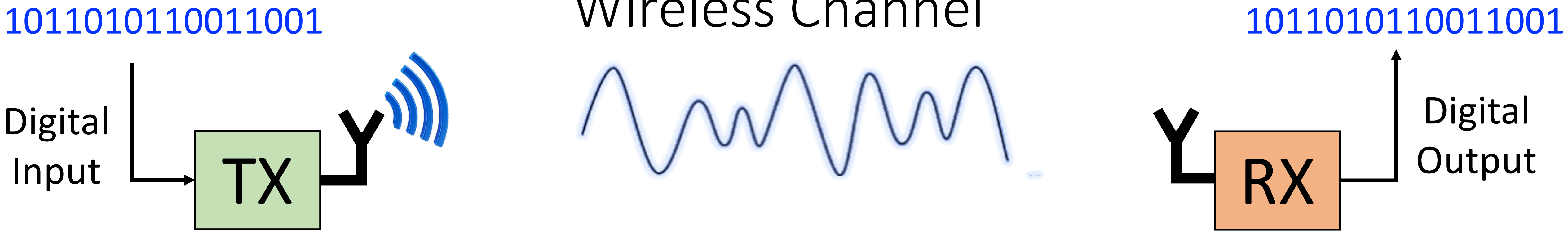


$$y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$$

Channel attenuates the signal (Pathloss)

$$P_{RX} = G_{TX} G_{RX} \frac{\lambda^2}{(4\pi d)^2} P_{TX} \quad \Rightarrow \quad |h| \propto \frac{\lambda}{d}$$

Wireless Channel

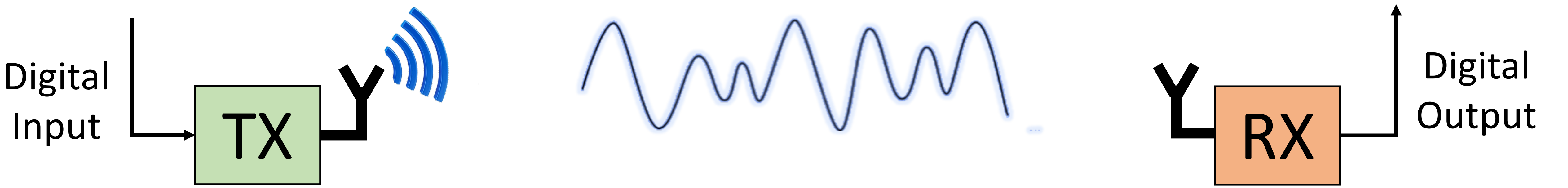


$$y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$$

Channel rotates the signal (Adds Phase)

$$h \propto \frac{\lambda}{d} e^{j\phi} \quad \rightarrow \phi = 2\pi \frac{d}{\lambda}$$

Wireless Channel

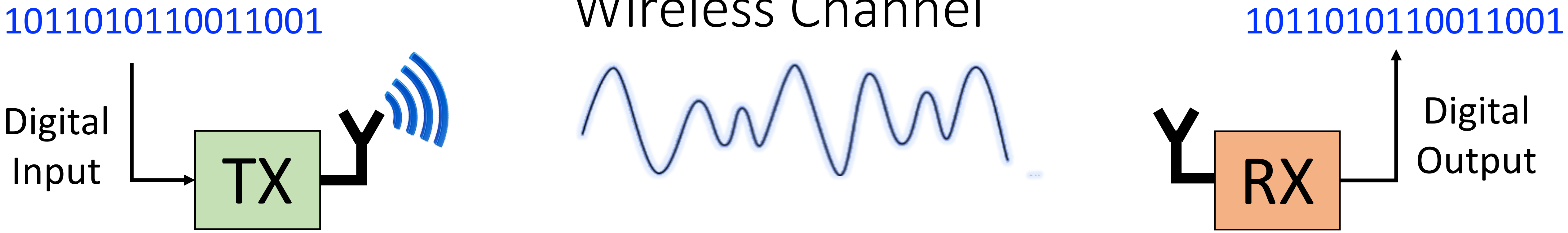


$$y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$$

$$x(t) \times e^{-j2\pi f_c t} \Rightarrow |h| x(t-\tau) e^{-j2\pi f_c (t-\tau)} \Rightarrow \times e^{j2\pi f_c t} \Rightarrow |h| x(t-\tau) e^{j2\pi f_c \tau}$$

$$h \propto \frac{\lambda}{d} e^{j\phi} \rightarrow \phi = 2\pi f_c \tau = 2\pi \frac{c}{\lambda} \frac{d}{c} = 2\pi \frac{d}{\lambda} \rightarrow h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Wireless Channel



$$y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$$

Channel:

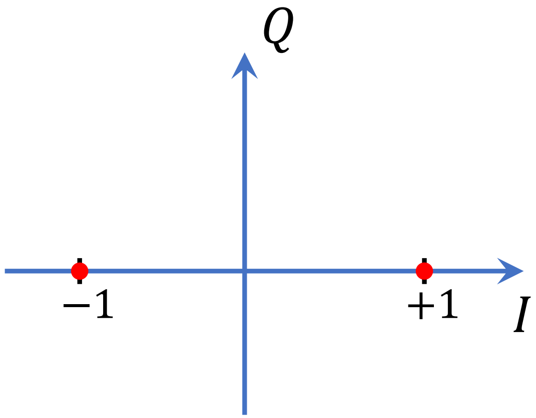
- Adds Noise
- Delays the Signal
- Attenuates the Signal
- Rotates the Phase of the Signal

$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

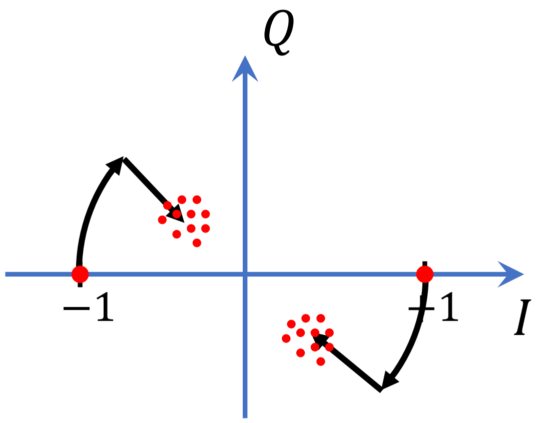
The Channel

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

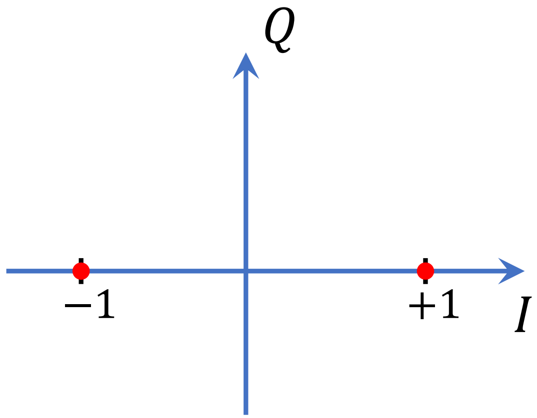


$h \times x(t) + v(t)$

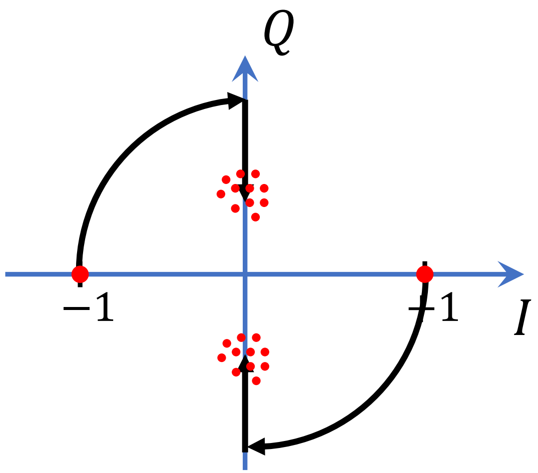
The Channel

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



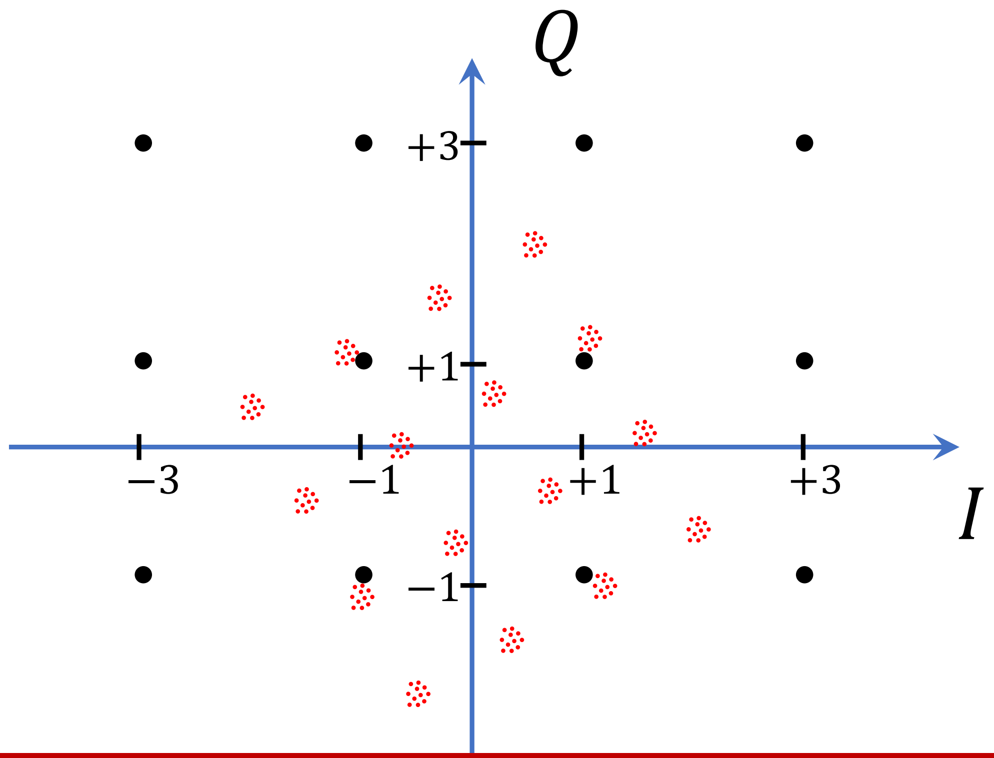
$x(t)$



$h \times x(t) + v(t)$

The Channel

Consider QAM Modulation

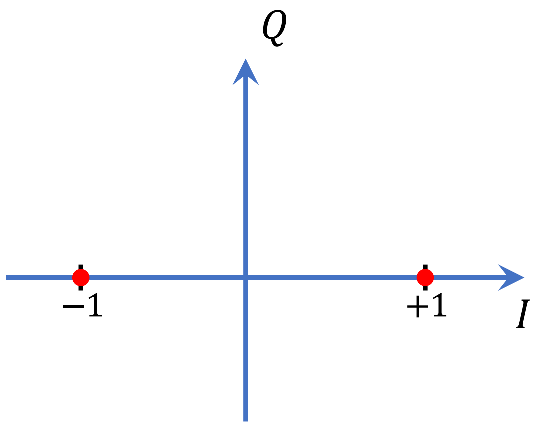


Demodulating correctly requires COHERENCE!
i.e., Need to estimate & correct for the channel h

Channel Estimation & Correction

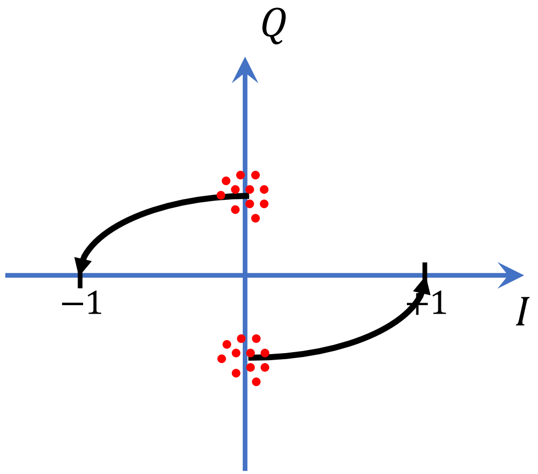
Channel Estimation & Correction

Consider BPSK Modulation.



$x(t)$

$$0 \to -1$$
$$1 \to +1$$



$h \times x(t) + v(t)$

Send Training Sequence (Preamble Bits): Known Bits

$x(0) = 1 \longrightarrow y(0) = h + v(0)$

$x(1) = 1 \longrightarrow y(1) = h + v(1)$

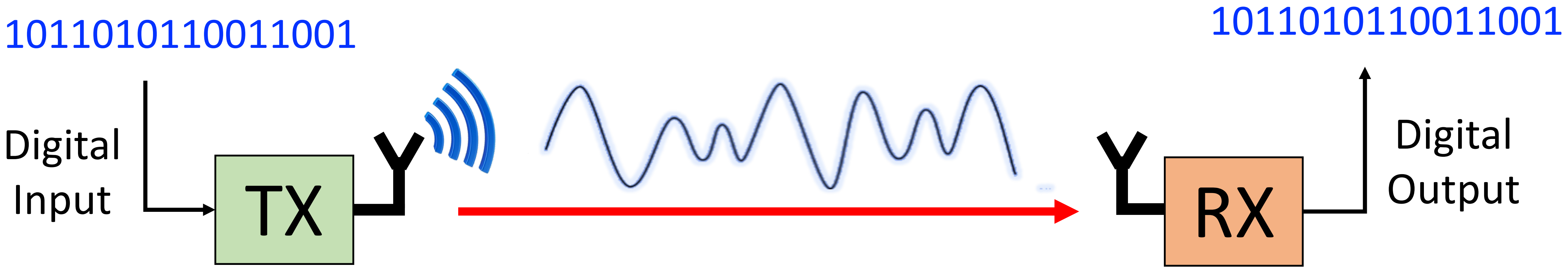
$x(2) = -1 \longrightarrow y(2) = -h + v(2)$

$\vdots$

Estimate channel: $\tilde{h} = \dfrac{1}{K} \displaystyle\sum_{k=1} \dfrac{y(k)}{x(k)}$

Correct channel: $\tilde{x}(t) = \dfrac{y(t)}{\tilde{h}}$

Single Path Channel

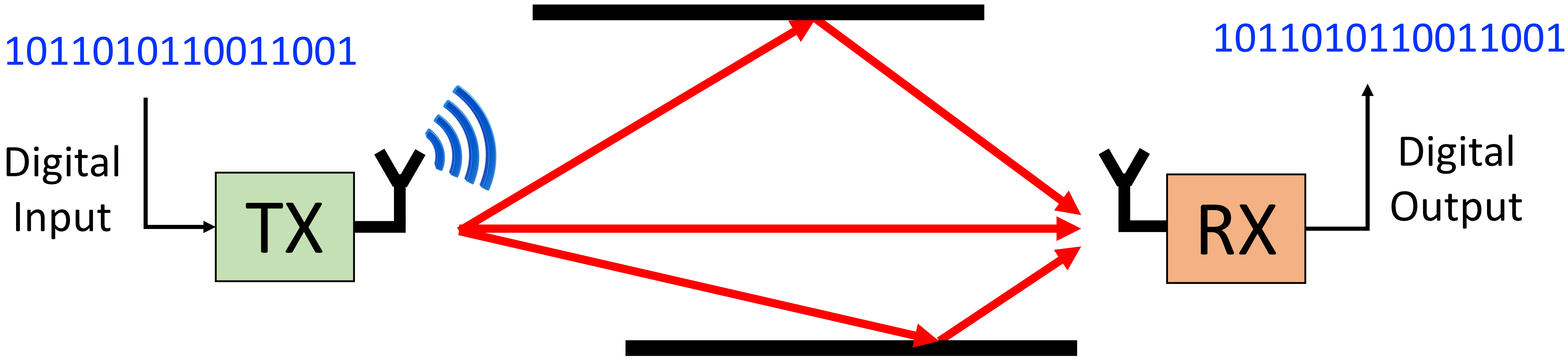


$$x(t) \longrightarrow \text{Channel} \longrightarrow y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$$

$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Assumes single path!

Multipath Channel



Multipath Propagation: radio signal reflects off objects ground, arriving at destination at slightly different times

$$y(t) = \alpha_1 e^{\phi_1}\, x(t-\tau_1) + \alpha_2 e^{\phi_2}\, x(t-\tau_2) + \alpha_3 e^{\phi_3}\, x(t-\tau_3) \cdots$$

$$y(t) = \sum_k \alpha_k e^{\phi_k}\, x(t-\tau_k) = \sum_k h(\tau_k)\, x(t-\tau_k) = h(t) * x(t)$$

$h(t)$ is channel impulse response.

Multipath Channel

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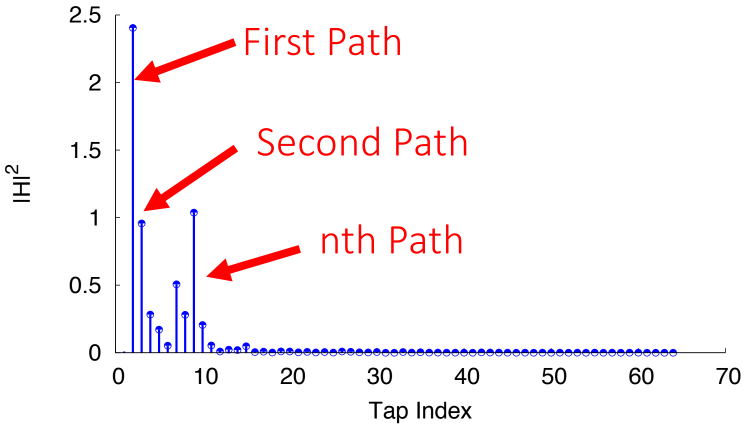
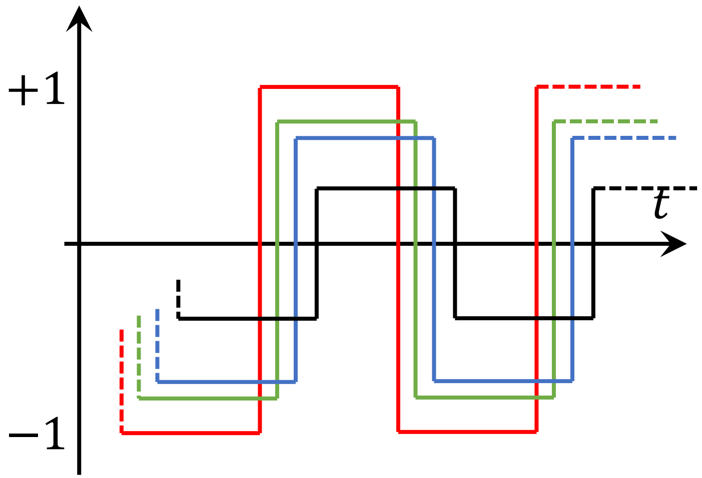
$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \; = h(t) * x(t)$$



Multi-tap Channel



ISI: Inter-Symbol-Interference

Symbols arriving along late paths interfere with following symbols.

Multipath Channel

$h(t)$ is channel impulse response.

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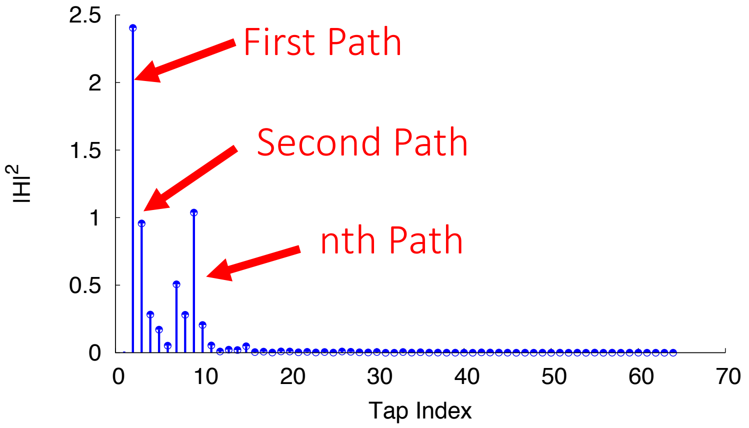
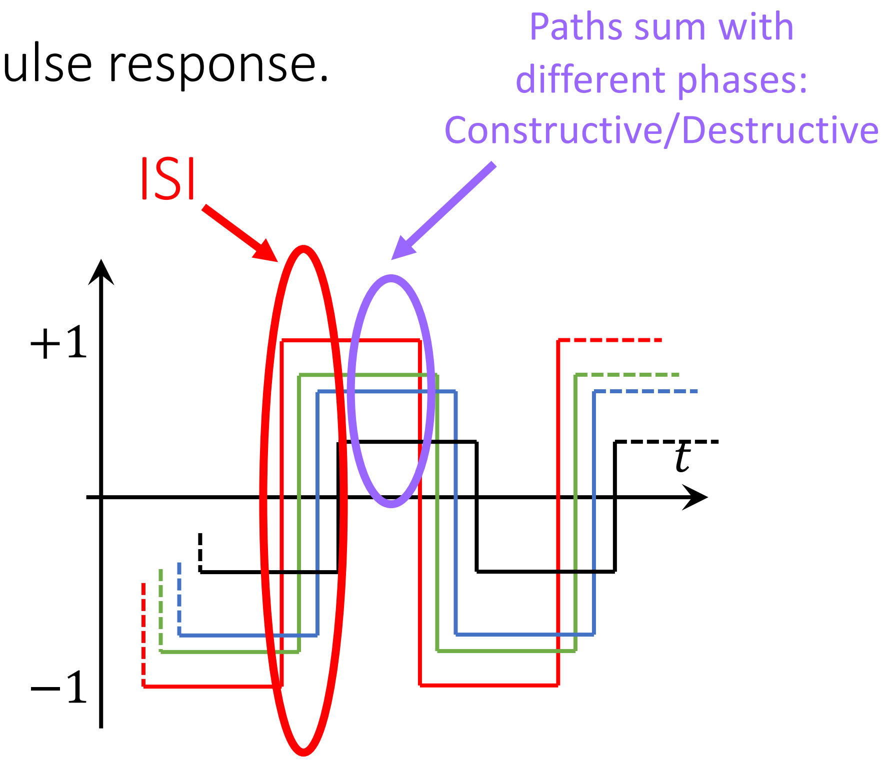


Multi-tap Channel



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Channel Fading

Symbols arriving along different paths sum up destructively

Multipath Channel

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Channel Fading: Symbols arriving along different paths sum up destructively

Example 2 paths with distance d_1 and d_2

$$h = \;h_1 + h_2 \;= \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2}\, e^{j2\pi d_2/\lambda}$$

$$= \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda}\left(1 + \frac{d_1}{d_2}\, e^{j2\pi(d_2-d_1)/\lambda}\right) \qquad \frac{d_1}{d_2} \approx 1$$

$$if\ \frac{d_2-d_1}{\lambda} \approx \frac{1}{2} \rightarrow h = \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda}\left(1 + e^{j\pi}\right) = 0$$

Destructive Interference!

Multipath Channel

Channel Fading: Symbols arriving along different paths sum up destructively

Suppose the 2 paths have distance $d_1 = 1m, d_2 = 1.06m$:

$$h = h_1 + h_2 = \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2} e^{j2\pi d_2/\lambda}$$

Compute h if the signal is transmitted at 2.5 GHz and 5 GHz. How much will power of the receiver signal differ if it is transmitted at 5 GHz instead of 2.5 GHz?

A. – 6 dB

B. 0 dB

C. 12.3 dB

D. 24.6 dB

Multipath Channel

Channel Fading: Symbols arriving along different paths sum up destructively

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Compute h if the signal is transmitted at 2.5 GHz and 5 GHz. How much will power of the receiver signal differ if it is transmitted at 5 GHz instead of 2.5 GHz?

$@f_1 = 2.5GHz\ (\lambda = 12\ cm)$: $|h| = |0.12\, e^{j\frac{2\pi}{3}} + 0.113\, e^{j\frac{5\pi}{3}}| \approx 0.0068$

$@f_2 = 5GHz\ (\lambda = 6\ cm)$: $|h| = |0.06\, e^{j\frac{4\pi}{3}} + 0.056\, e^{j\frac{4\pi}{3}}| \approx 0.116$

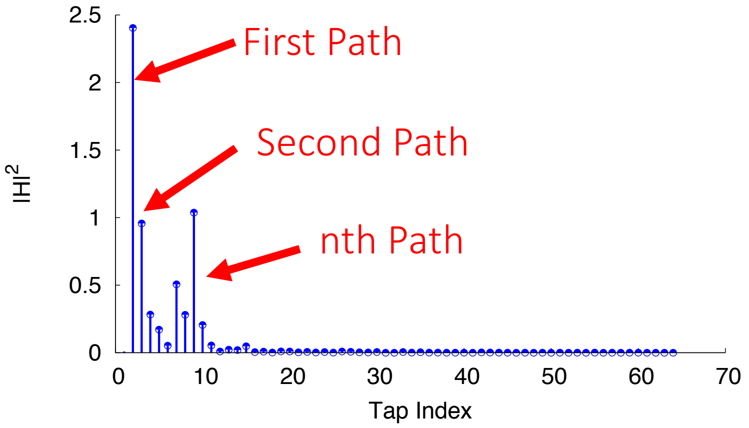
$17^2\times$
(24.6dB)

Frequency Selective Fading

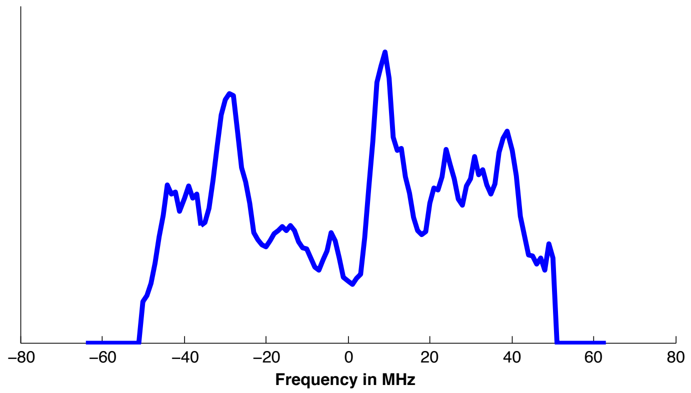
Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \;= h(t) * x(t) \qquad \Longleftrightarrow \qquad H(f)X(f)$$



Multi-tap Channel



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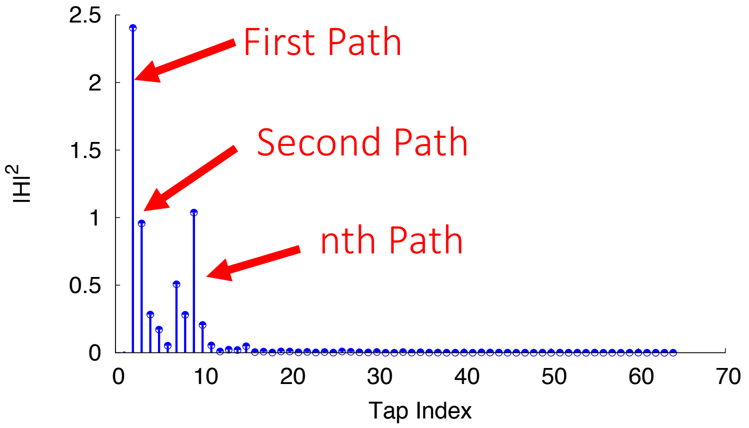

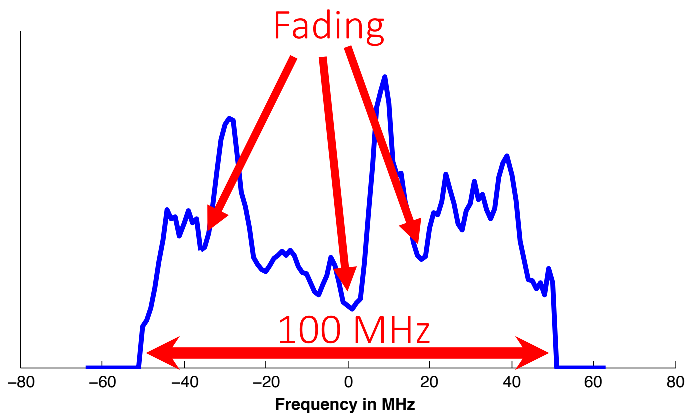

Multi-tap Channel

ISI: Inter-Symbol-Interference

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Frequency Selective Fading

Symbols arriving along different paths sum up destructively

Problematic in Wideband Channel!

Narrowband Channel

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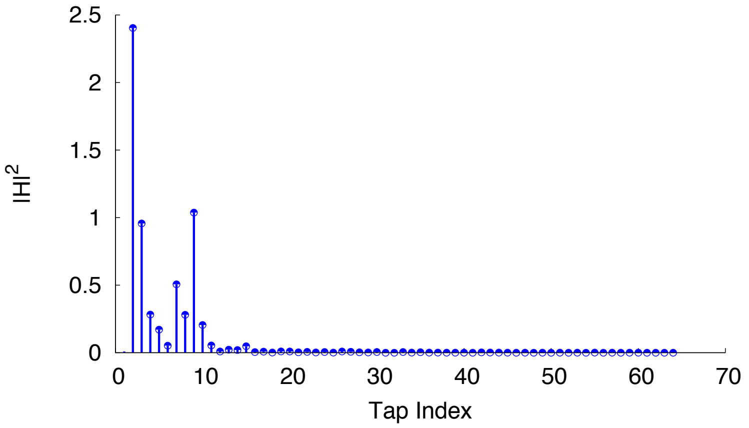

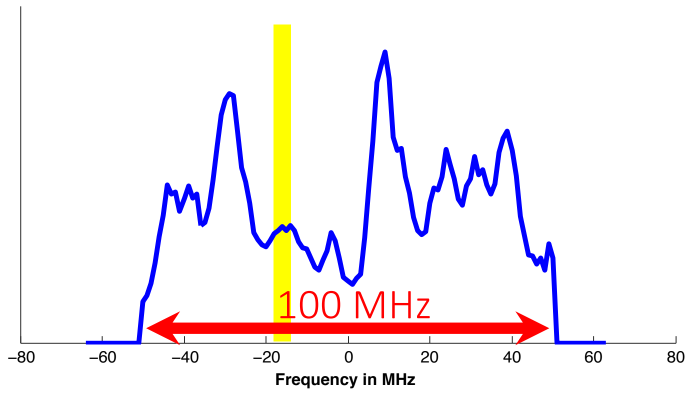

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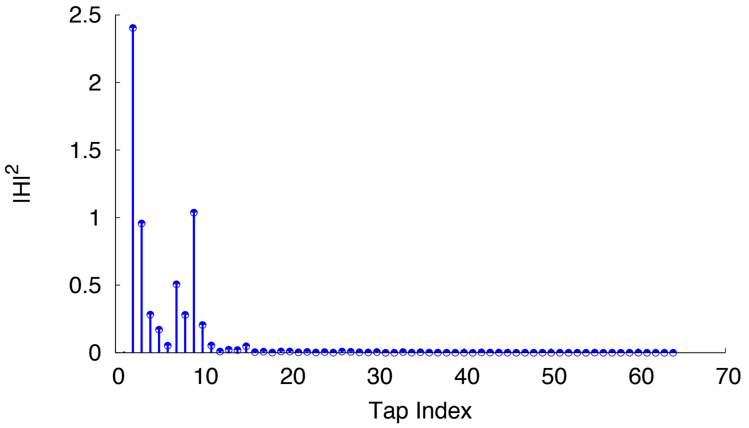

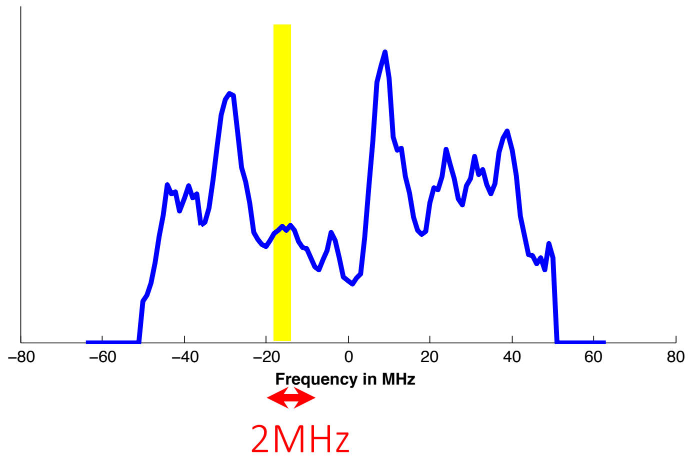

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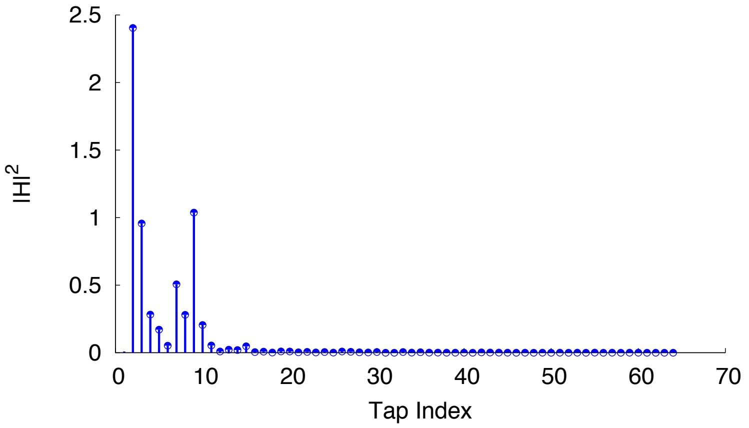

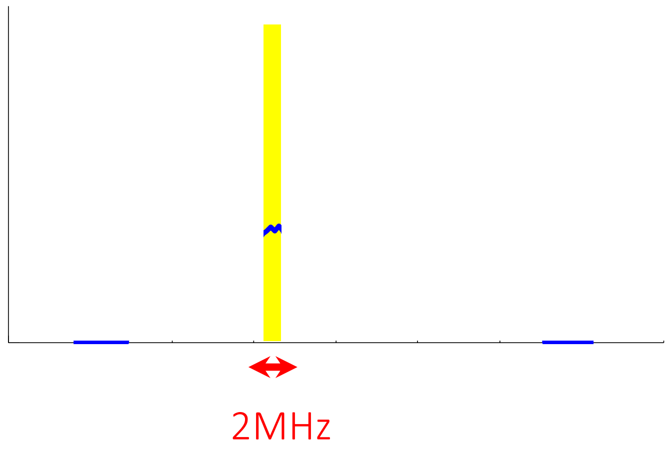

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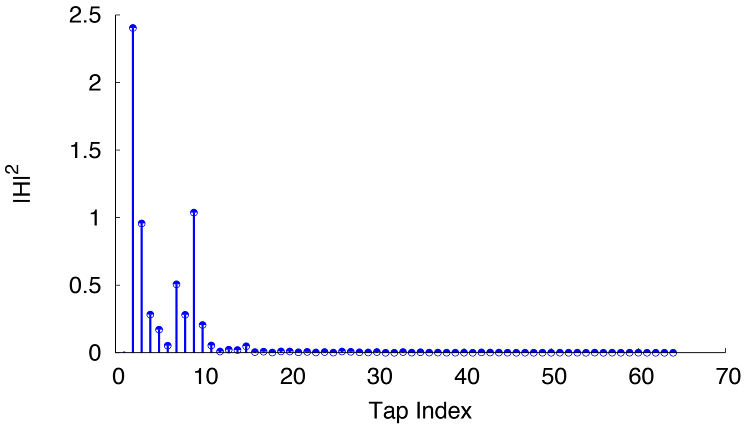
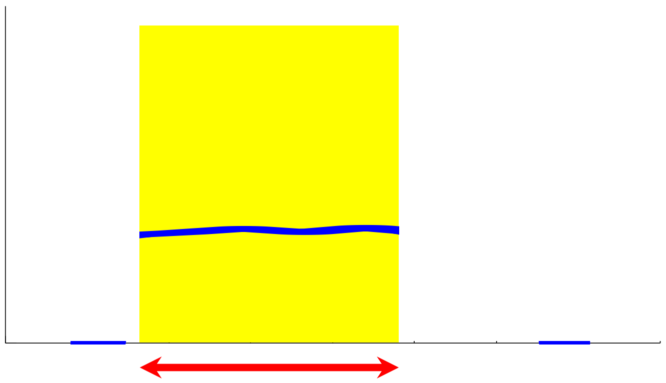




Narrowband

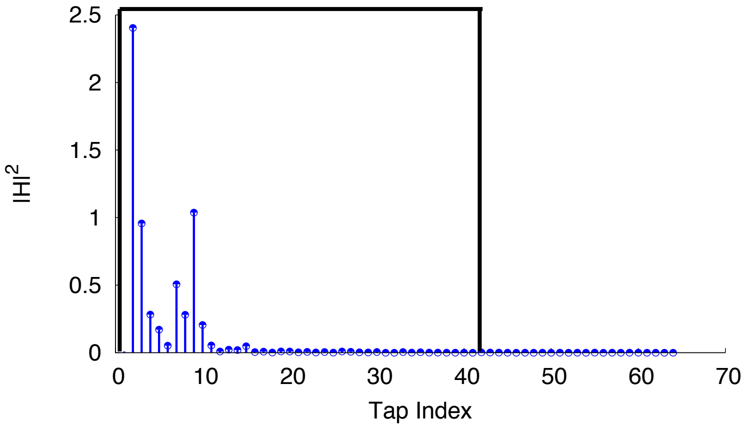
Symbol time: $T \propto \dfrac{1}{\text{Bandwidth}}$ $\gg \tau_k$

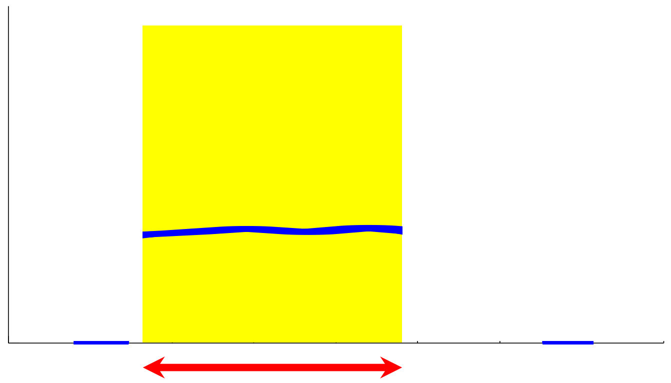
Flat Channel

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \;= h(t) * x(t) \qquad \Leftrightarrow \qquad H(f)X(f)$$





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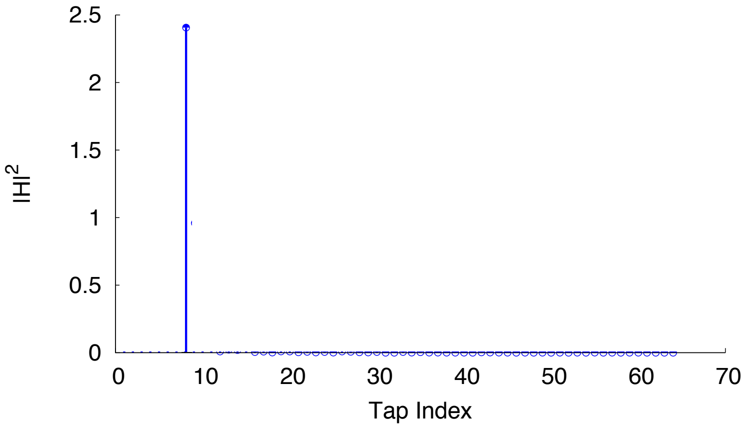
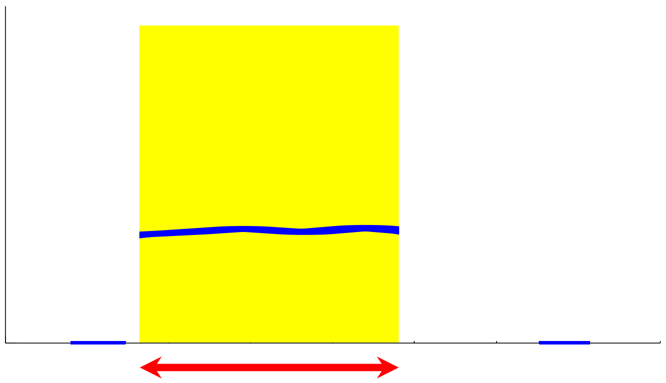




Narrowband

Symbol time: $T \propto \frac{1}{\text{Bandwidth}}$ $\gg \tau_k$

Flat Channel

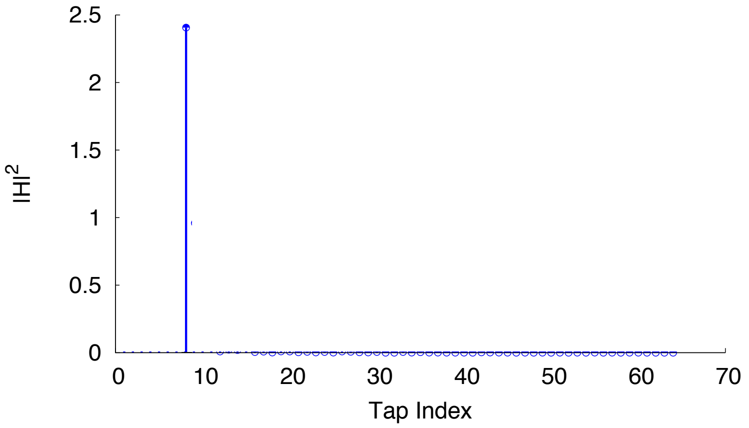
$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \ \approx \sum_k h(\tau_k)\, x(t) \ = \left(\sum_k h(\tau_k)\right) x(t) \ = hx(t)$$

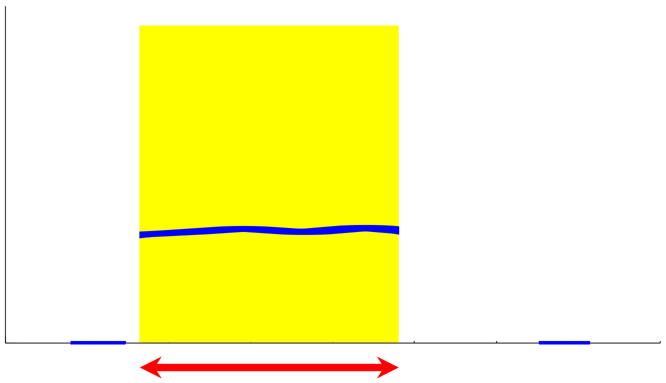
Narrowband Channel is Approximated by a Single Tap Channel

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t) \;= h\, x(t) \qquad \Leftrightarrow \qquad h\, X(f)$$





Narrowband

Flat Channel

Symbol time: $T \propto \dfrac{1}{\text{Bandwidth}} \gg \tau_k$

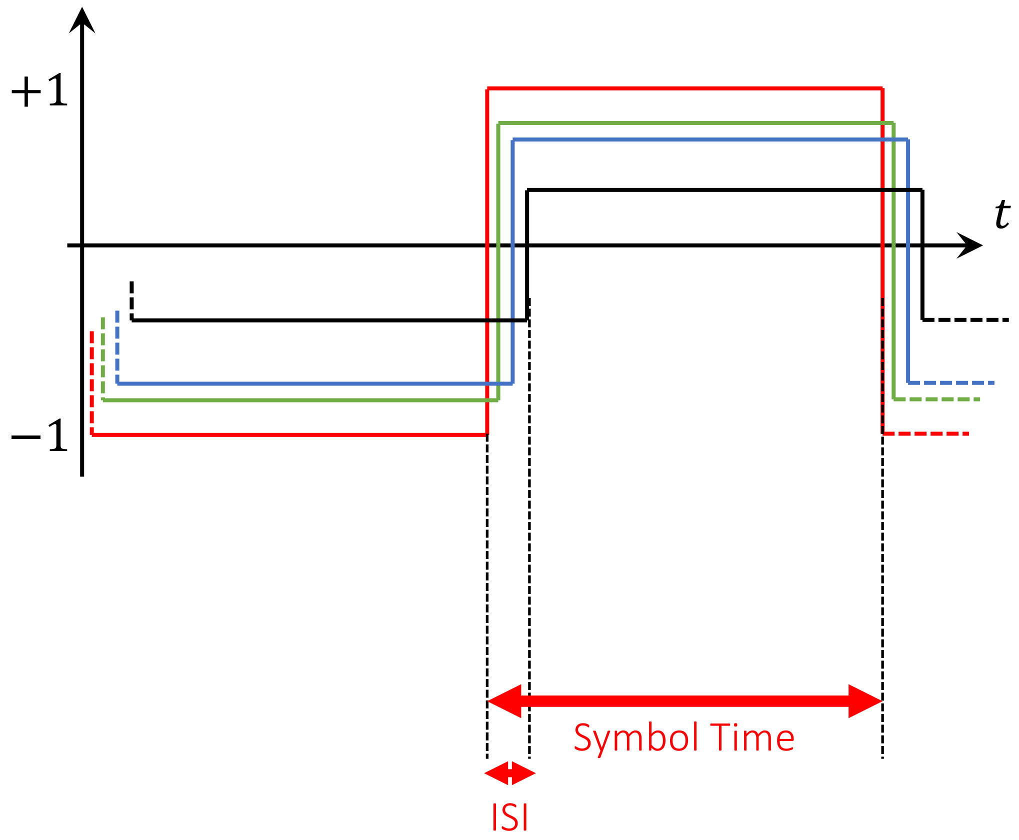
$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \approx \sum_k h(\tau_k)\, x(t) = \left(\sum_k h(\tau_k)\right) x(t) = hx(t)$$

Narrowband Channel is Approximated by a Single Tap Channel

Narrowband vs. Wideband Channel

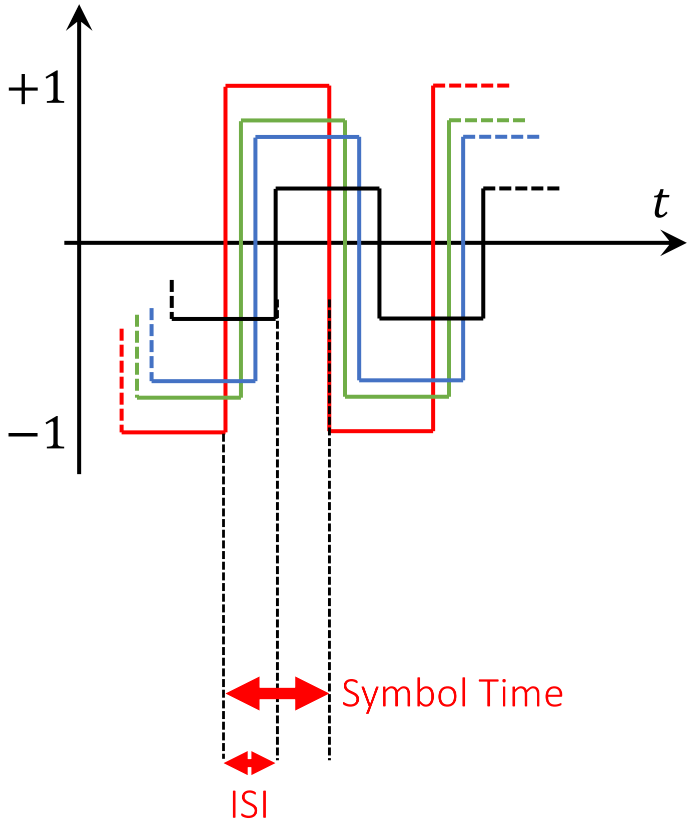
Narrowband Channel

$$h\,x(t)$$



Wideband Channel

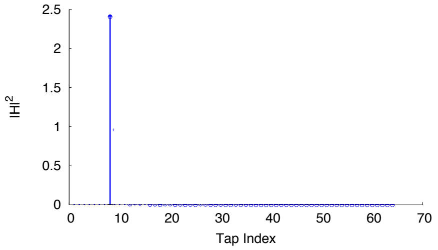
$$h(t) * x(t)$$

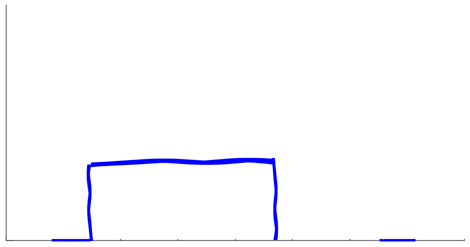


Narrowband vs. Wideband Channel

Narrowband Channel

$$h\,x(t) \quad \Longleftrightarrow \quad h\,X(f)$$



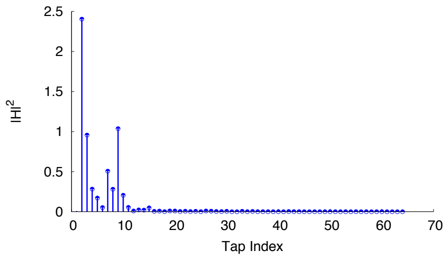


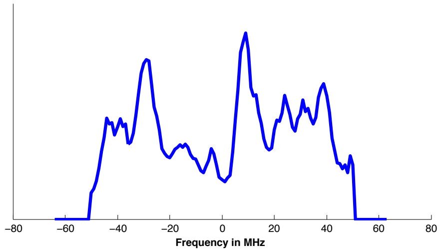
≈ Single Tap

≈ Flat Channel

Wideband Channel

$$h(t) * x(t) \quad \Longleftrightarrow \quad H(f)\,X(f)$$





Multi-tap Channel (ISI)

Frequency Selective Channel

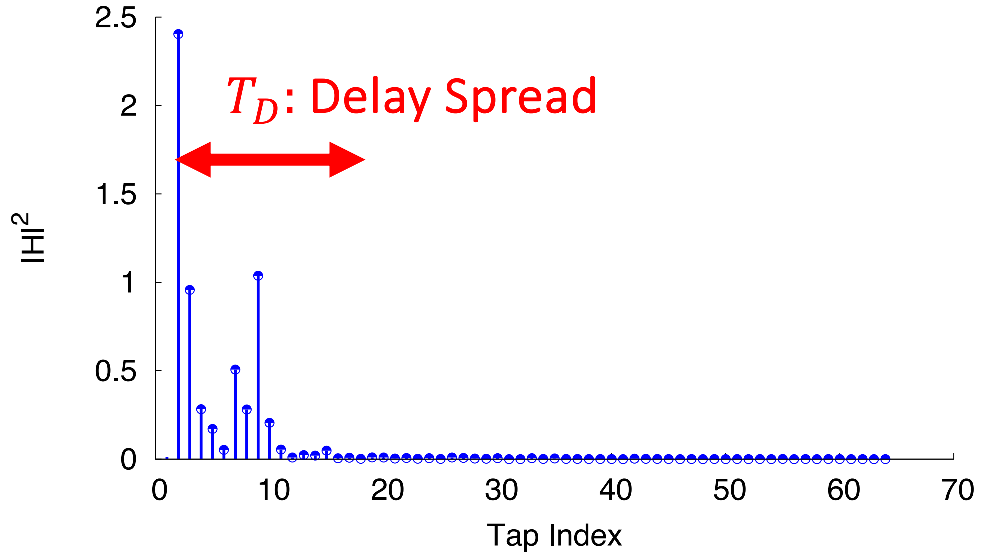
Need to correct for ISI to be able to decode correctly!

Estimating and Correcting for multi-tap channel is hard

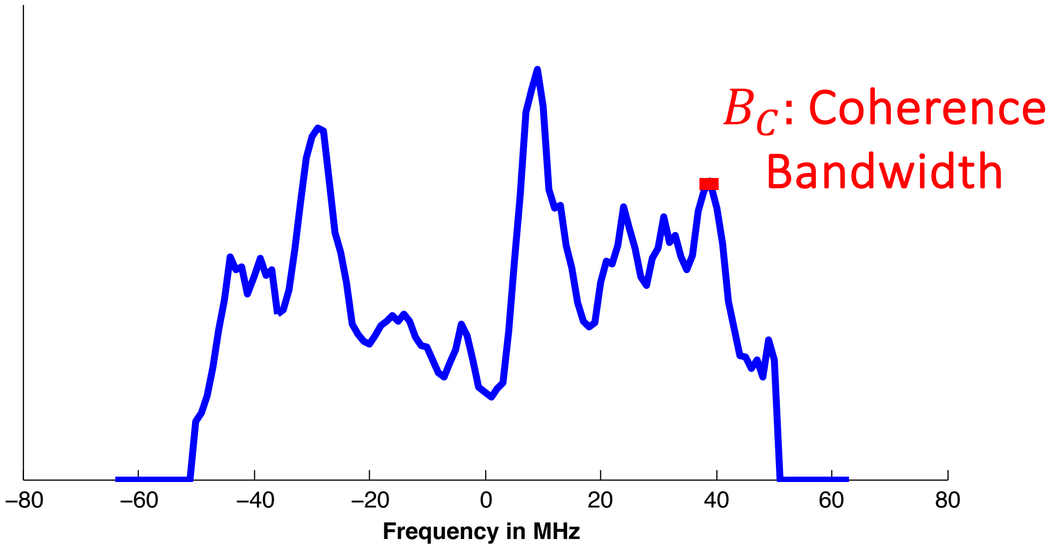
Multipath Wideband Channel

$$h(t) * x(t) \quad \Leftrightarrow \quad H(f)\,X(f)$$

Multi-tap Channel (ISI)



Frequency Selective Channel



$$T_D = 1/B_C$$

Delay Spread: Spread of the channel in time i.e. time delay between first and last path received above noise floor

Coherence Bandwidth: region where the channel frequency response is flat (coherent).

Time Varying Channel

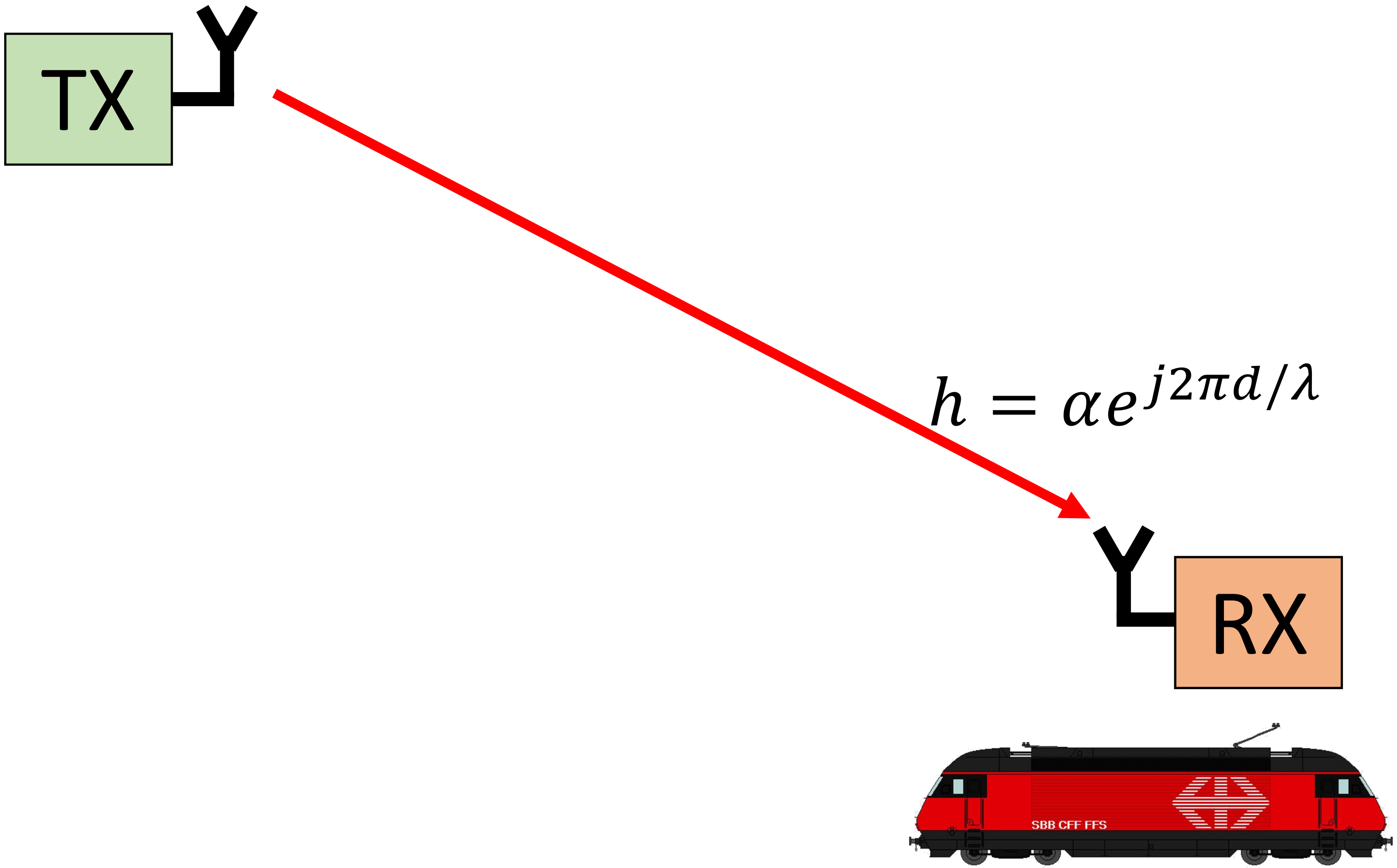
So far, considered only static channels, i.e. the channel does not change.
But the channel $h(t)$ cannot be the same forever

Reasons why $h(t)$ changes:

- **Environment changes:**
  - Reflectors move
  - People moving around
- **Transmitter or Receiver Moves:**
  - Reflectors move
  - People moving around

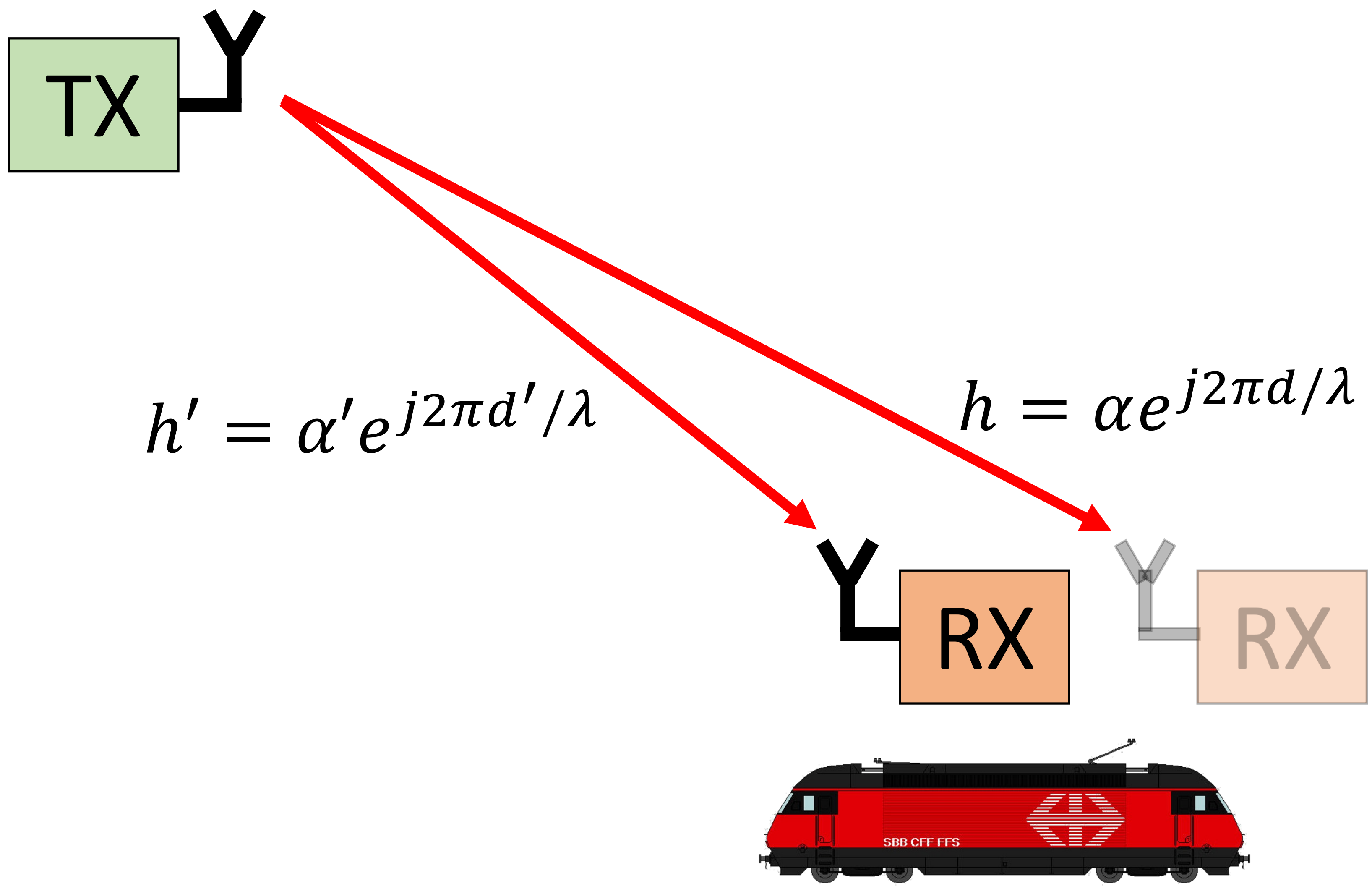
$\rightarrow$ Doppler Spread/Shift

Doppler Shift

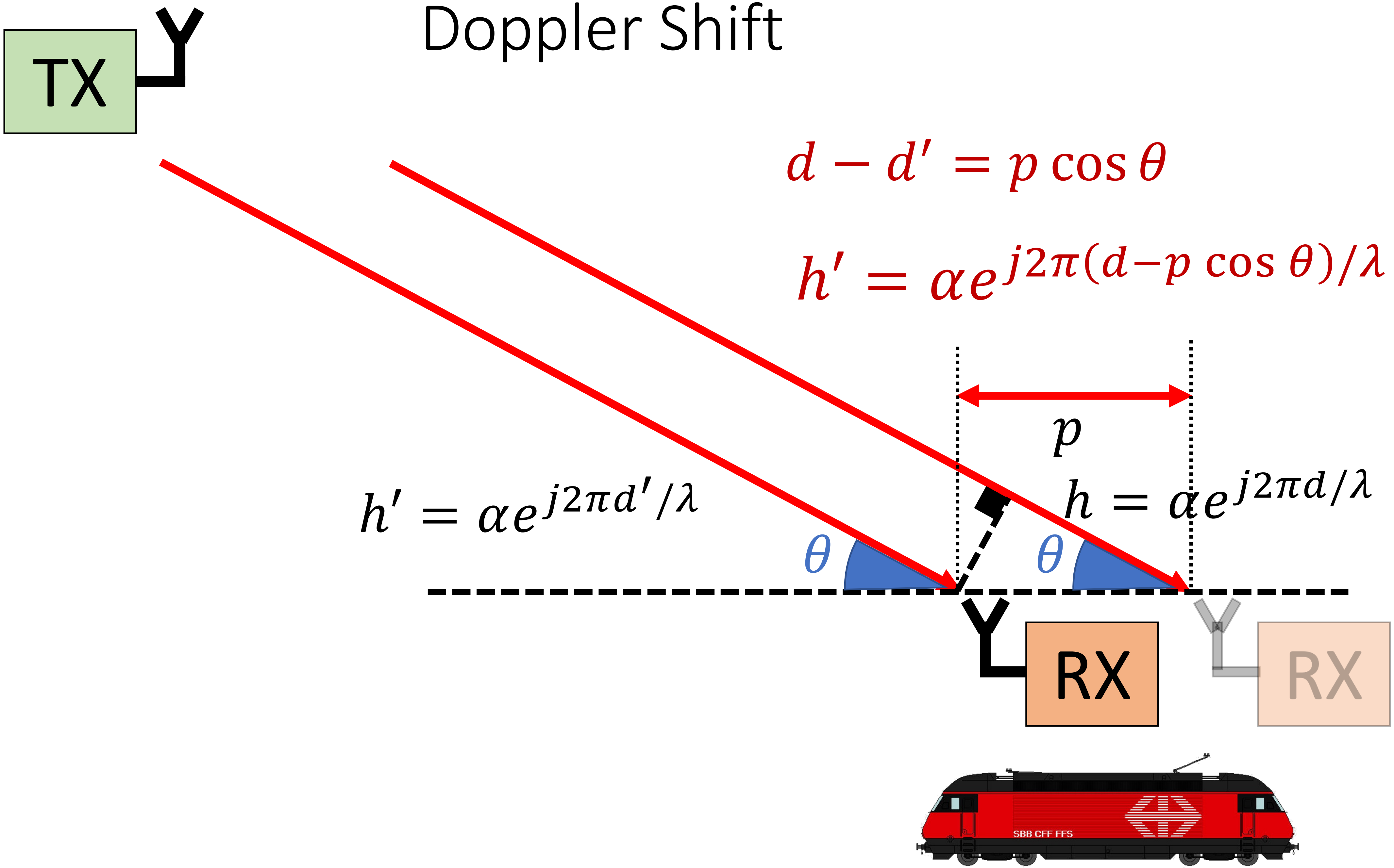


$$h = \alpha e^{j2\pi d/\lambda}$$

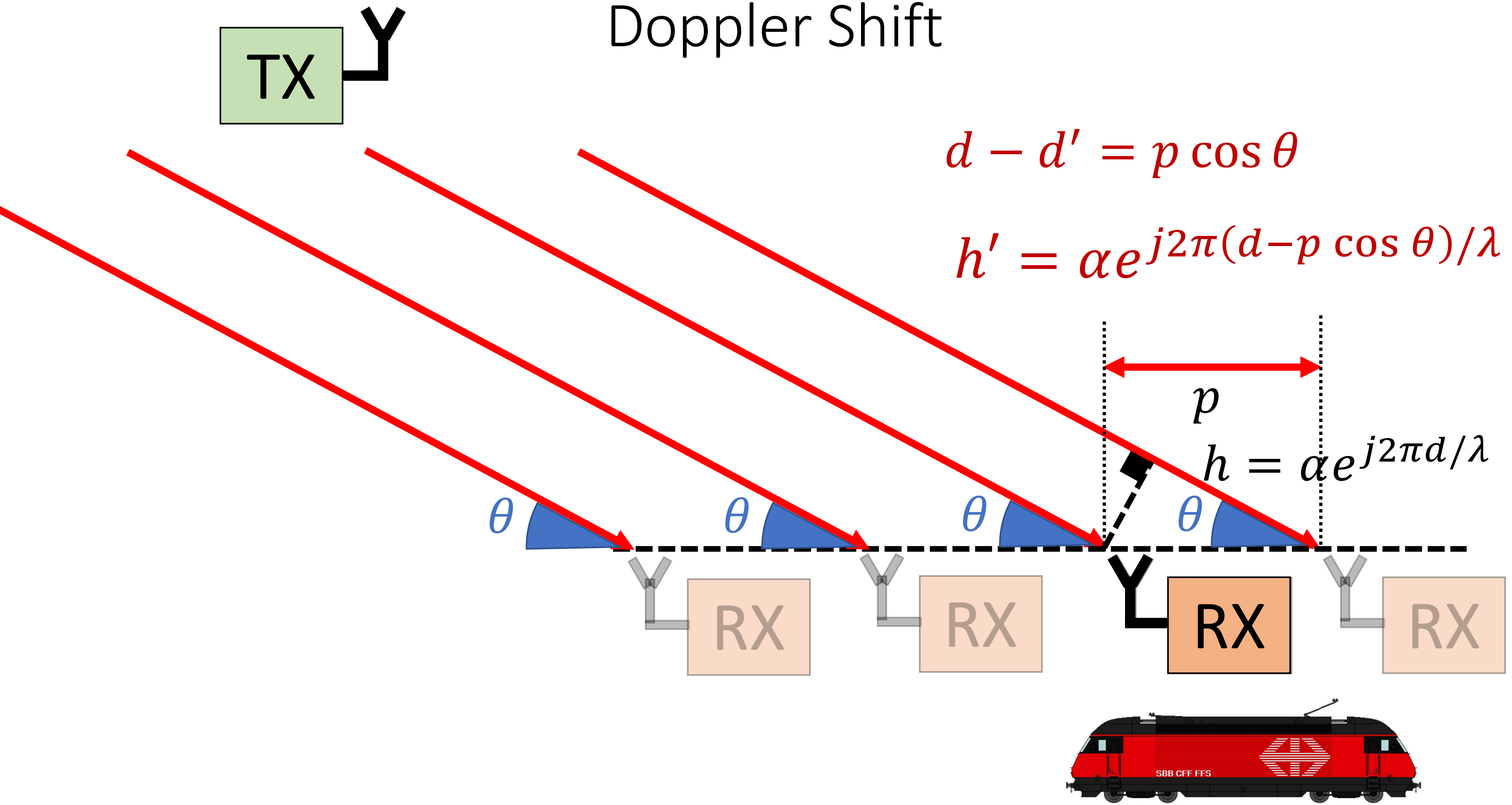
Doppler Shift



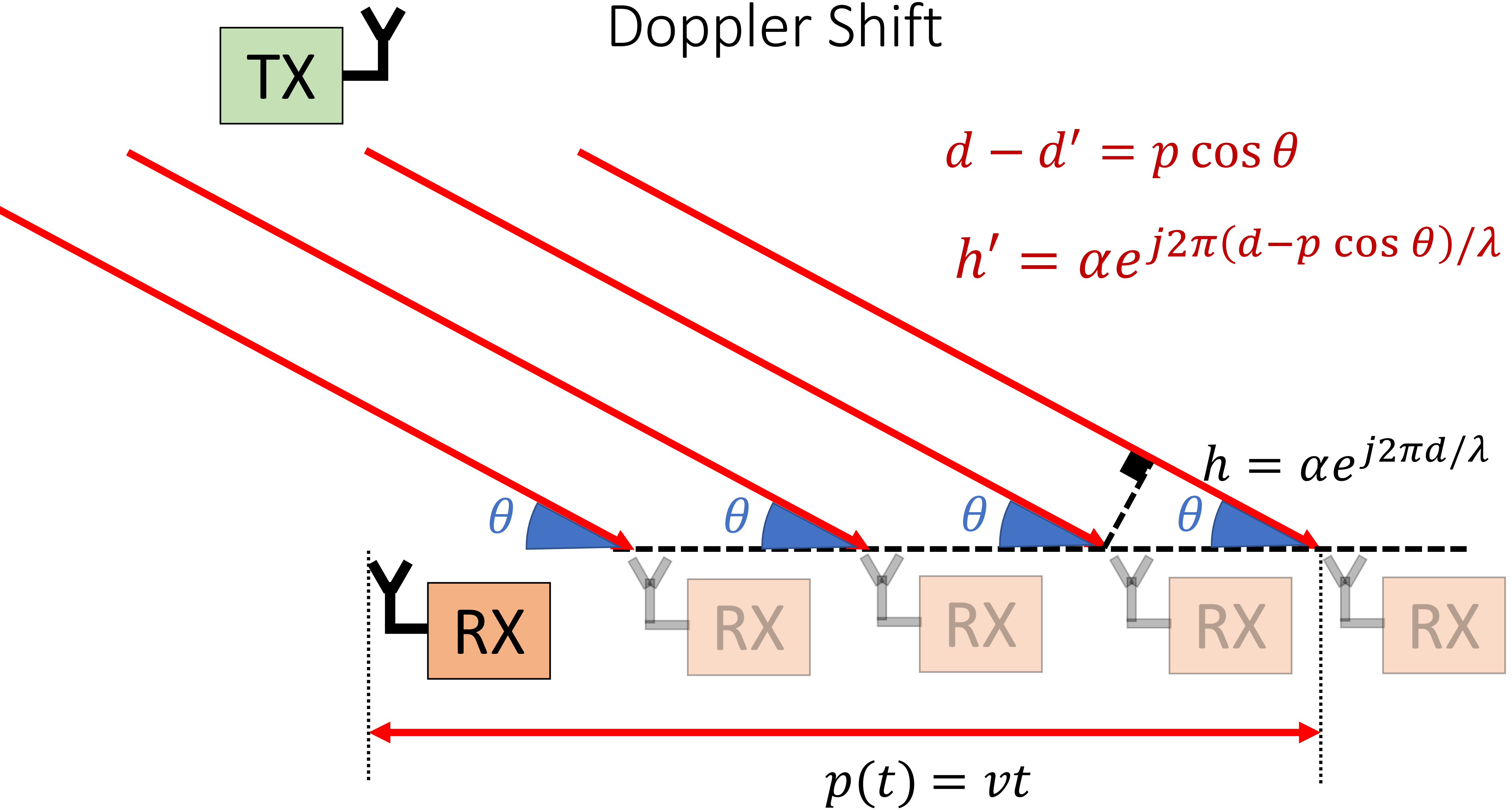
Doppler Shift



Doppler Shift



Doppler Shift



Doppler Shift

$$h' = \alpha e^{j2\pi(d - vt\cos\theta)/\lambda}$$

$$= \alpha e^{j2\pi d/\lambda} e^{-j2\pi(v\cos\theta/\lambda)t}$$

$$= h e^{-j2\pi\Delta f t}$$

Doppler Shift: Frequency shift caused by the movement in TX/RX or reflectors

$$B_D = \frac{v\cos\theta}{\lambda}$$

Time Varying Channel

$$h(\tau, t)$$

Delay of different paths

Time Changing

$h(\tau, 0)$: Channel Impulse Response at Time $t = 0$.

$h(0, t)$: Channel of the first path over time.

$h(t)$: Channel Impulse Response of static channel.

Time Varying Channel

$$h(\tau, t)$$

Delay of different paths

Time Changing

Delay Spread

$$T_D = \Delta\tau$$

$$\Updownarrow$$

Coherence Bandwidth

$$B_C = 1/T_D$$

Doppler Spread

$$B_D = \Delta f$$

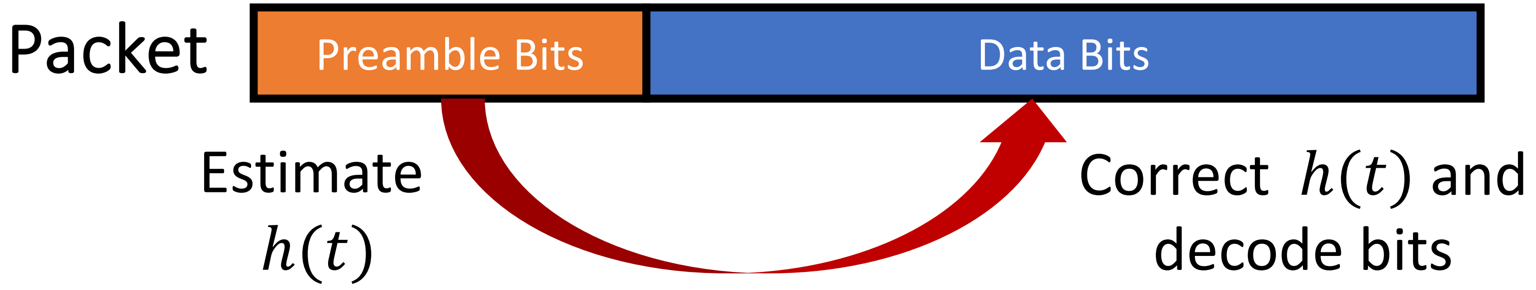
$$\Updownarrow$$

Coherence Time

$$T_C = 1/B_D$$

Time Varying Channel

Channel Coherence Time : Time period over which we can assume the channel is static.



Assumption: Channel Coherence Time >> Time to transmit packet

\+ Lower Overhead → higher throughput

– Risk of channel changing → Cannot decode bits