COM 405: Mobile Networks – Fall 2024
Homework 2 Solutions
EPFL

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1 Channel Contention 18 Points

Suppose nodes A and B are ready to send a packet. In the i^{th} round after $(i-1)$ collisions have already occurred, the two nodes can wait $0, 1, \ldots, 2^{i-1}-1$ slots until the next attempt, all 2^{i-1} choices having equal probability.

1. Find the probability q_i of a collision in the i^{th} round, given that there are collisions in the previous $(i-1)$ rounds (i.e., $q_1 = 1$, $q_2 = \frac{1}{2}$), for all $i \geq 1$.

   The probability q_i of collision in i_{th} round is independent of what happened in previous i-1 rounds. Hence there will be collision if both A and B choose the same node. Therefore the probability of collision is

   $$q_i = \frac{1}{2^{i-1}}$$

2. Find the probability p_i that exactly i rounds are needed for the first success, and compute $p_1, p_2, \ldots, p_4$.

   We will need i rounds for first success, if we collide in first i-1 rounds and finally succeed in i^{th} round. Therefore p_i is

   $$\prod_{j=1}^{i-1} q_j \times (1 - q_i)$$

   $p_1 = 0$, $p_2 = 1/2$, $p_3 = 3/8$ and $p_4 = 7/64$,

3. Now assume that after the first collision, node A "wins" the backoff and transmits successfully. After it is finished, both nodes try to transmit again (A has an infinite amount of traffic to send), causing a collision. Now compute the probability that A wins the channel for the next packet.

   After the first collision, B has contention window choice in [0,1]. However A wins the channel and its contention window comes back to 0. Now after the next collision, the contention window of A is [0,1] and contention window of B is [0,1,2,3]. We need to compute the probability that A wins this time.

   P(A wins) = P(B chooses 3)*P(A wins/B chooses 3) + P(B chooses 2)*P(A wins/B chooses 2) + P(B chooses 1)*P(A wins/B chooses 1)

   $$P(Awins) = 1/4 * 1 + 1/4 * 1 + 1.4 * 1/2$$
   $$P(Awins) = 5/8$$

2 MAC in WiFi 5 vs. WiFi 6 (16 points)

Consider an AP and 4 clients using WiFi 802.11ac with CSMA/CA. The network administrator has disable RTS/CTS. The DIFS = $50\mu s$, the SIFS = $10\mu s$, the ACK takes $30\mu s$. Assume there is only downlink traffic with constant flow of packets to each client. Also assume the MAC is fair and the AP transmits to each client in a round robin schedule. The AP on average has to backoff for 6 slots where each slot is $10\mu s$.

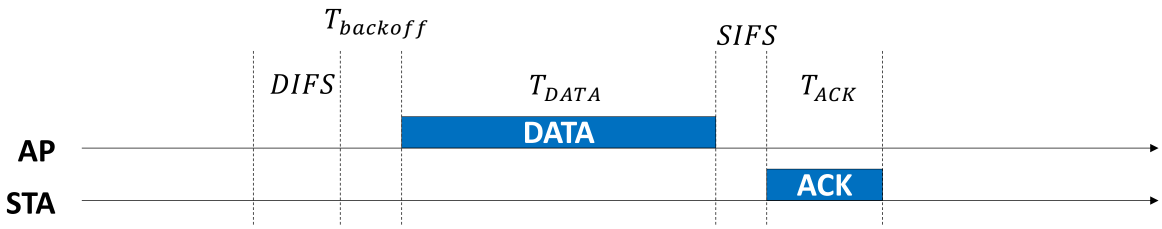

Figure 1: WiFi CSMA/CS without RTS/CTS

1. Compute the actual average throughput of a client in each of the following cases:

   (a) Packet size: 1500 bytes, Data Rate: 6.5 Mbps

   (b) Packet size: 1500 bytes, Data Rate: 390 Mbps

$$\begin{aligned}\text{Transmit packet time} &= DIFS + T_{backoff} + T_{DATA} + SIFS + T_{ACK} \\ &= 50\mu s + 6 \times 10\mu s + \frac{\text{Packet Size}}{\text{Data Rate}} + 10\mu s + 30\mu s \\ &= 150\mu s + \frac{\text{Packet Size}}{\text{Data Rate}}\end{aligned}$$

For each user, however, the time to receive a packet is $4\times$ the time it takes the AP to transmit it. Hence, the actual throughput per client on average is:

$$\text{Throughput} = \frac{\text{Packet Size}}{4 \times \text{Transmit packet time}} = \frac{1}{\frac{600\mu s}{\text{Packet Size}} + \frac{4}{\text{Data Rate}}}$$

The above equation gives us:

(a) 1.502 Mbps

(b) 16.60 Mbps

2. The network operator upgrades the AP and clients to 802.11ax with OFDMA. RTS/CTS is still disabled and all the network parameters are the same. Each of the Block ACK Request and Block ACK takes $30\mu s$. Assume that the AP allocates equal number of subcarriers to each client and ignore overhead in terms or guard subcarriers.

   Compute the actual average throughput of a client in each of the following cases:

   (a) Packet size: 1500 bytes, Single User Data Rate: 6.5 Mbps

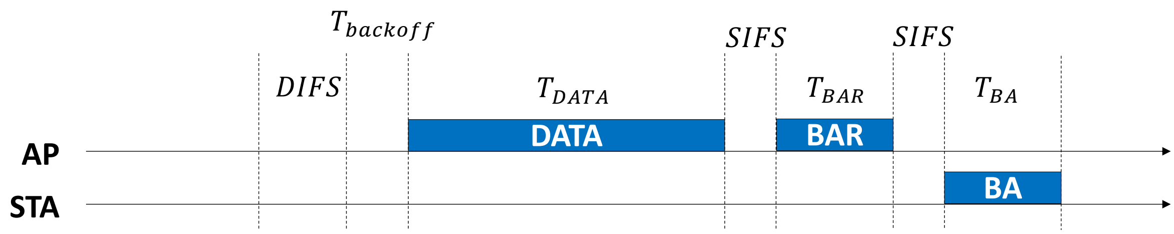

Figure 2: WiFi OFDMA without RTS/CTS

(b) Packet size: 1500 bytes, Single User Data Rate: 390 Mbps

$$\begin{aligned}
\text{Transmit packet time} = \quad & DIFS + T_{backoff} + T_{DATA} + SIFS + T_{BAR} + SIFS + T_{BA} \\
= \quad & 50\mu s + 6 \times 10\mu s + \frac{\text{Packet Size}}{\text{Data Rate}/4} + 10\mu s + 30\mu s + 10\mu s + 30\mu s \\
= \quad & 190\mu s + \frac{4 \times \text{Packet Size}}{\text{Data Rate}}
\end{aligned}$$

In the above, the data rate is divided by 4 since each user gets 1/4 of the subcarriers. Hence, the actual throughput per client on average is:

$$\text{Throughput} = \frac{\text{Packet Size}}{\text{Transmit packet time}} = \frac{1}{\frac{190\mu s}{\text{Packet Size}} + \frac{4}{\text{Data Rate}}}$$

The above equation gives us:

(a) 1.584 Mbps

(b) 38.33 Mbps

3. In general when does OFDMA yield substantial throughput gain and why?

OFDMA give substantial throughput gain for high data rate packets since it amortizes the overhead of the CSMA protocol. Since the overhead is low at low data rates, the gain is not substantail.

4. Does it still make sense to use OFDMA in cases where there is no throughput gain? Explain why or why not.

Yes, since it can help reduce the latency.

3 MIMO **26 points**

1. Consider a 3×3 MIMO system:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) (2 points) How many packets can be transmitted in parallel if the channel matrix $\mathbf{H} = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 3 \\ 6 & 5 & -3 \end{bmatrix}$

> This matrix is not invertible since it has a 0 determinant. It also has a rank $2 < 3$. Hence, at most 2 packets can be transmitted in parallel using this MIMO system.

(b) (6 points) Now suppose the channel matrix is complex: $\mathbf{H} = \begin{bmatrix} 1+j & 2 & 3 \\ 8 & 9-j & 2+j \\ 6+2j & 5 & -3 \end{bmatrix}$.

We want to decode the packets in x_1, x_2, and x_3 using the projection method described in class. What are the vectors $\vec{h}_{23}^{\perp}$, $\vec{h}_{13}^{\perp}$ and $\vec{h}_{12}^{\perp}$ we need to project on to decode the packets.

> To decode x_1, we project on a direction orthogonal to the plane containing $\vec{h}_2$ and $\vec{h}_3$. Hence, $\vec{h}_{23}^{\perp}$ is the cross product of $\vec{h}_2$ and $\vec{h}_3$
>
> $$\vec{h}_{23}^{\perp} = \vec{h}_2 \times \vec{h}_3 = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} \times \begin{bmatrix} h_{13} \\ h_{23} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 2 \\ 9-j \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2+j \\ -3 \end{bmatrix} = \begin{bmatrix} -37-2j \\ 21 \\ -23+5j \end{bmatrix}$$
>
> Similarly, we can compute,
>
> $$\vec{h}_{13}^{\perp} = \vec{h}_1 \times \vec{h}_3 = \begin{bmatrix} -34-10j \\ 21+9j \\ -23+3j \end{bmatrix} \qquad \vec{h}_{12}^{\perp} = \vec{h}_1 \times \vec{h}_2 = \begin{bmatrix} -16-12j \\ 7-j \\ -6+8j \end{bmatrix}$$

2. Consider a 1×3 SIMO system where n is the noise:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0.02-0.01j \\ 0.05 \\ -0.03+0.04j \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

Suppose $E[|x|^2] = E_s = 100$ and $E[|n_1|^2] = E[|n_2|^2] = E[|n_3|^2] = N_0 = 0.05$.

(a) (3 points) The receiver decodes using $y = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3$. What values of α_1, α_2, and α_3 maximize the SNR?

Maximum Ratio Combining (MRC) will maximize the SNR and lead to the optimal performance. Hence, we set $\alpha_1 = h_1^* = 0.02 + 0.01j$, $\alpha_2 = h_2^* = 0.05$, and $\alpha_3 = h_3^* = -0.03 - 0.04j$

(b) (3 points) Compute the SNR in dB on each antenna if we decode using y_1, y_2, and y_3 independently.

$$SNR = \frac{|h|^2 E_s}{N_0}$$

For antenna 1, $SNR = 1 \rightarrow 0$ dB.
For antenna 2, $SNR = 5 \rightarrow 6.98$ dB.
For antenna 3, $SNR = 5 \rightarrow 6.98$ dB.

(c) (2 points) Compute the SNR if we combine the signals using the scheme derived in part (a).

$SNR = \frac{\left(|h_1|^2+|h_2|^2+|h_3|^2\right)E_s}{N_0} = 11 \rightarrow 10$ dB.

(d) (2 points) Compute the SNR if we combine the signals by setting $\alpha_1 = \alpha_2 = \alpha_3 = 1$.

$SNR = \frac{\left(|h_1+h_2+h_3|^2\right)E_s}{3N_0} = \frac{5}{3} \rightarrow 2.12$ dB.

3. Consider a 2×1 MISO system:

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.02 - 0.01j & -0.03 + 0.04j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) (2 points) The transmitter wants to transmit x. It sets $x_1 = \alpha_1 x$ and $x_2 = \alpha_2 x$. What values of α_1 and α_2 maximize the SNR at the receiver?

Maximum Ratio Combining (MRC) will maximize the SNR and lead to the optimal performance. However, we need to normalize to ensure that the total transmit power does not increase. Hence, we set:

$\alpha_1 = \frac{h_1^*}{\sqrt{|h_1|^2+|h_2|^2}} = (0.02 + 0.01j)/\sqrt{0.003}$

$\alpha_2 = \frac{h_1^*}{\sqrt{|h_1|^2+|h_2|^2}} = (-0.03 - 0.04j)/\sqrt{0.003}$

(b) (4 points) Suppose the transmitter uses Alamouti Codes to transmit. Suppose it is using 16 QAM and the first symbol is $(1 + j)/\sqrt{10}$ and the second symbol is $(-3 + j)/\sqrt{10}$. What values will the transmitter transmit on x_1 and x_2 in the first and second time slot?

During the first time slot, the transmitter will transmit $x[1]/\sqrt{2} = (1+j)/\sqrt{20}$ on antenna 1 and $x[2]/\sqrt{2} = (-3+j)/\sqrt{20}$ on antenna 2.

During the first time slot, the transmitter will transmit $-x^*[2]/\sqrt{2} = (3+j)/\sqrt{20}$ on antenna 1 and $x^*[1]/\sqrt{2} = (1-j)/\sqrt{20}$ on antenna 2.

(c) (2 points) Name one advantage and one disadvantage of using Alamouti codes compared to scheme derived in the first part.

Advantage: Alamouti does not require knowing the channel at the transmitter and hence does not require feedback from the receiver which adds a lot of overhead.

Disadvantage: Alamouti gives lower SNR due to scaling the signals by $\sqrt{2}$. It loses 3 dB compared to MRC.

4 Scheduling (40 points)

1. Consider a wireless TDMA system with 4 mobile client connected to the base station. The data rates of the four mobile clients C_1, C_2, C_3, and C_4 are $R_1 = 20$ Mbps, $R_2 = 50$ Mbps, $R_3 = 70$ Mbps, and $R_4 = 140$ Mbps respectively. The base station divides time into equal time slots of length T and schedules a single mobile client in each time slot. Compute the throughput of each client and the whole network throughput when the following scheduling strategy is used:

   (a) Round-Robin Scheduling

   In round-robin scheduling, clients are scheduled in cyclic order one after the other. This means that each client will be assigned 1 time slot every 4 time slots. Then, the througput of each client is:

   $$S_1 = \frac{R_1}{4} = 5\ Mbps, S_2 = \frac{R_2}{4} = 12.5\ Mbps, S_3 = \frac{R_3}{4} = 17.5\ Mbps, S_4 = \frac{R_4}{4} = 35\ Mbps$$

   The total network throughput $S_T = S_1 + S_2 + S_3 + S_4 = 70\ Mbps$

   (b) Max-Throughput Scheduling

   In max-throughput, the clients are scheduled to maximize the total network throughput . Then, the througput of each client is:

   $$S_1 = 0, S_2 = 0, S_3 = 0, S_4 = 140\ Mbps$$

   The total network throughput $S_T = S_1 + S_2 + S_3 + S_4 = 140\ Mbps$

   (c) Max-Min Scheduling

   The scheduler assigns the time slots to clients so they have the same minimum throughput. Assume client i is assigned x_i percentage of the slots. Then, we need:

   $$x_i R_i = x_j R_j$$

   Further, we have $\sum_i x_i = 1$. We end up with the following set of 4 linear equations:

   $$\begin{array}{ll} x_1 + x_2 + x_3 + x_4 = 1 & 20x_1 = 50x_2 \\ 20x_1 = 70x_3 & 20x_1 = 140x_4 \end{array}$$

   Solving these equations we get (Note that the last 3 equations are reduntant since they are not linearly dependent):

   $$x_1 = 35/64, x_2 = 14/64, x_3 = 10/64, x_4 = 5/64$$

   Then, the throughput of each terminal is: $S_i = 20x_1 = 175/16 = 10.9375\ Mbps$
   The total network throughput is: $S_T = 4S_i = 43.75\ Mbps$

2. Recall that the Proportional fair (PF) scheduler is a compromise scheduling policy, trying to balance the completing interests of maximizing the total network throughput and providing all clients with a minimal level of service.

   Let M be the number of clients and S_i the long run throughput of client i. The PF scheduler aims to maximize the following objective function:

$$\sum_{i=1}^{M} \ln S_i$$

   The throughput in slot $t-1$ is denoted as $S_i[t-1]$. $S_i[t]$ is updated as follows:

$$S_i[t] = \left(1 - \frac{1}{\tau}\right) \times S_i[t-1] + \frac{1}{\tau} \times R_i[t] I(i,t)$$

   where τ is a constant > 1, $R_i[t]$ is the data rate of user i at time t, and $I(i,t)$ is the indicator function, i.e., $I(i,t) = 1$ if client i is scheduled at time t, and 0 otherwise.

   Prove that in order to maximize the object function, the client with the highest $R_i[t]/S_i[t-1]$ should be scheduled.

$$\begin{aligned} \sum_{i=1}^{M} \ln(S_i) = & \sum_{i=1}^{M} \ln\left((1-1/\tau) \times S_i[t-1] + (1/\tau) \times R_i[t] I(i,t)\right) \\ = & \sum_{i=1}^{M} \ln\left((1-1/\tau) S_i[t-1]\left(1 + \frac{(1/\tau) R_i[t] I(i,t)}{(1-1/\tau) S_i[t-1]}\right)\right) \\ = & \sum_{i=1}^{M} \ln\left((1-1/\tau) S_i[t-1]\right) + \sum_{i=1}^{M} \ln\left(1 + \frac{R_i[t] I(i,t)}{(\tau-1) S_i[t-1]}\right) \end{aligned}$$

   Let:

$$\begin{aligned} A = & \sum_{i=1}^{M} \ln\left((1-1/\tau) S_i[t-1]\right) \\ B = & \sum_{i=1}^{M} \ln\left(1 + \frac{R_i[t] I(i,t)}{(\tau-1) S_i[t-1]}\right) \end{aligned}$$

   A is a constant that is independent of which client gets scheduled in time slot t. Hence, maximizing the objective function is equivalent to maximizing B. Let i^* be the client scheduled at time t, then $I(i^*, t) = 1$ and $I(i,t) = 0$ for $i \neq i^*$. Then,

$$B = \ln\left(1 + \frac{1}{\tau - 1} \frac{R_{i^*}[t]}{S_{i^*}[t-1]}\right)$$

   Hence, maximizing B is equivalent to maximizing $R_i[t]/S_i[t-1]$.

3. Consider a wireless TDMA system with 2 mobile clients. The base station divides time into equal time slots of length T and schedules a single mobile client in each time slot using a proportional fair (PF) scheduler. The transmission continues for the next two time slots $t = 2$ and $t = 3$. The

data rates of the two mobile clients C_1 and C_2 are for all t, $R_1[t] = 32$ Mbps and $R_2[t] = 64$ Mbps respectively. The average throughput is updated using the following equation:

$$S_i[t] = \left(1 - \frac{1}{t}\right) \times S_i[t-1] + \frac{1}{t} \times R_i[t] I(i,t)$$

where $I(i,t) = 1$ if client i is scheduled in time slot t and $I(i,t) = 0$ if client i is not scheduled.

Assume $S_1[1] = 16$ Mbps and $S_2[1] = 48$ Mbps. Calculate the average throughput of the two clients in the following two time slots:$S_1[2], S_2[2], S_1[3], S_2[3]$.

In time slot $t = 2$, we schedule:

$$i[2] = \arg\max_i \left(\frac{R_i[t]}{S_i[t-1]}\right) = \arg\max_i \left(\frac{32}{16}, \frac{64}{48}\right) = 1$$

Since client 1 is scheduled at $t = 2$, we get:

$$\begin{aligned} S_1[2] &= 1/2 \times S_1[1] + 1/2 \times R_1[2] \times 1 = 1/2 \times 16 + 1/2 \times 32 = 24\ Mbps \\ S_2[2] &= 1/2 \times S_2[1] + 1/2 \times R_2[2] \times 0 = 1/2 \times 48 + 0 = 24\ Mbps \end{aligned}$$

In time slot $t = 3$, we schedule:

$$i[3] = \arg\max_i \left(\frac{R_i[t]}{S_i[t-1]}\right) = \arg\max_i \left(\frac{32}{24}, \frac{64}{24}\right) = 2$$

Since client 2 is scheduled at $t = 3$, we get:

$$\begin{aligned} S_1[3] &= 2/3 \times S_1[2] + 1/3 \times R_1[3] \times 0 = 2/3 \times 24 + 0 = 16\ Mbps \\ S_2[3] &= 2/3 \times S_2[2] + 1/3 \times R_2[3] \times 1 = 2/3 \times 24 + 1/3 \times 64 = 112/3 = 37.333\ Mbps \end{aligned}$$