COM 405: Mobile Networks – Fall 2024
Homework 1 Solutions
EPFL

1 Wireless Channel **30 points**

Consider a wireless channel where the signal traveles along two paths $d_1 = 3\ m$ and $d_2 = 3.075\ m$. The frequency of operation is $f_c = 1$ GHz and the speed of light is $c = 3 \times 10^8 m/s$. You can assume that the attenuation factor (channel magnitude) on the first path d_1 is 0.01 and on the second path d_2 is 0.008.

1. Write the exact equation of the complex channel impulse response $h(t)$.

$$\begin{aligned} h(t) = &\ 0.01 e^{-j2\pi d_1/\lambda} \delta(t - d_1/c) + 0.008 e^{-j2\pi d_2/\lambda} \delta(t - d_2/c) \\ = &\ 0.01 e^{-j20\pi} \delta(t - 10ns) + 0.008 e^{-j20.5\pi} \delta(t - 10.25ns) \\ = &\ 0.01 \delta(t - 10ns) - 0.008 j \delta(t - 10.25ns) \end{aligned}$$

2. Assuming the transmitter and receiver use a narrowband channel, what is the narrowband channel approximation of the channel impulse response.

$$h = 0.01 - 0.008j$$

3. Assume the transmitter transmittes at 0.1 mW. The receiver's noise floor is -80dBm. What is the SNR of the received signal in dB?

Received Power $= 0.1\ mW \times |h|^2 = 0.1\ mW \times 0.000164 = 1.64 \times 10^{-5} mW$.

Received Power in dBm $= 10 \log_{10}(1.64 \times 10^{-5}) = 10 \log_{10}(2 \times 10^{-5}) = -47.85$ dBm.

SNR = Received Power in dBm $-$ Noise floor in dBm $= -47.85 + 80 = 32.1$ dB.

4. Compute the SNR if we switch to a frequency of operation $f_c = 2$ GHz, the transmitter is still using a narrow band channel and transmit power of 0.1 mW, and the attenuation factor is 0.005 on the first path and 0.0048 on the second path.

$h = 0.005e^{-j2\pi 3/0.15} + 0.0048e^{-j2\pi 3.075/0.15} = 0.005 - 0.0048 = 0.0002$

Received Power $= 0.1\ mW \times |h|^2 = 4 \times 10^{-9} mW = -84$ dBm.

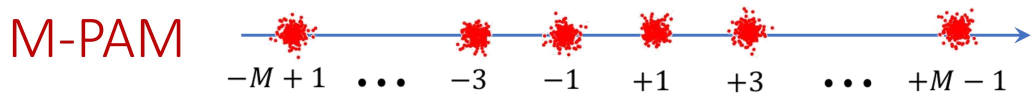
SNR $= -4$ dB.

5. What is the wireless channel phenomenon called?

Frequency Selective Fading.

2 Constellations (30 pts)

1. Consider the M-PAM constellation shown above.



(a) Derive the normalization factor that ensures the total transmitted energy is E_s. Hint, you can use the following formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\begin{aligned}
E[|x(t)|^2] &= \frac{1}{M} \times 2 \times \sum_{k=1}^{M/2} (2k-1)^2 \\
&= \frac{2}{M} \sum_{k=1}^{M/2} 4k^2 - 4k + 1 = \frac{8}{M} \sum_{k=1}^{M/2} k^2 - \frac{8}{M} \sum_{k=1}^{M/2} k + 1 \\
&= \frac{8}{M} \frac{M/2(M/2+1)(M+1)}{6} - \frac{8}{M} \frac{M/2(M/2+1)}{2} + 1 \\
&= \frac{(M+2)(M+1)}{3} - (M+2) + 1 \\
&= \frac{M^2-1}{3}
\end{aligned}$$

Hence, the normalization factor is:

$$\sqrt{\frac{M^2-1}{3}}$$

(b) What is the average number of nearest neighbor in the above constellation?

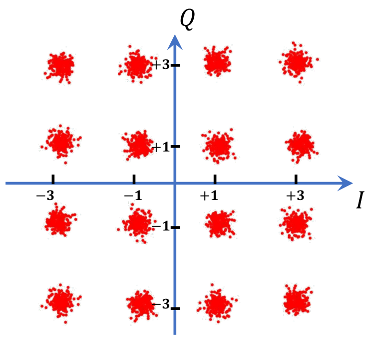
Average number of nearest neighbor:

$$\#nn = \frac{1}{M}((M-2) \times 2 + 2 \times 1) = \frac{2(M-1)}{M}$$

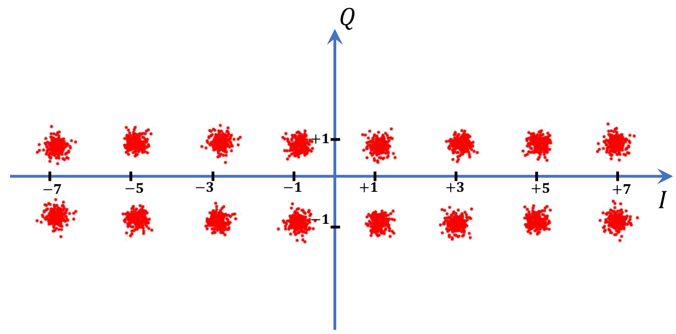
(c) What is the BER vs E_s/N_0 of the M-PAM constellation using the nearest neighbor approximation?

$$BER = \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = \#nnQ\left(\frac{2\sqrt{E_s}}{\sqrt{\frac{M^2-1}{3}}\sqrt{2N_0}}\right) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6E_s/N_0}{M^2-1}}\right)$$

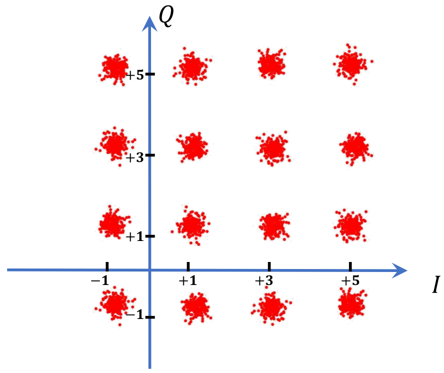
2. Consider the below 16-QAM constellations: (i) Regular 4×4 Grid. (ii) 16-QAM on a rectangular 2×8 Grid (iii) 4×4 Grid shifted so the constellation has mean of $2 + 2j$ i.e., $\mathrm{E}[x(t)] = 2 + 2j$. (iv) 4×4 Grid rotated around $(0, 0)$ by $\pi/2$.



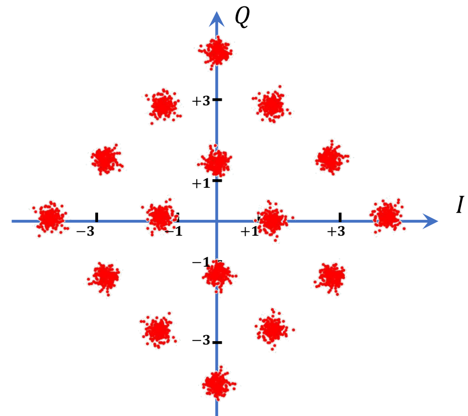
(i) 4×4 Grid



(ii) 2×8 Grid



(iii) 4×4 Grid shifted to mean $2 + 2j$



(iv) 4×4 Grid rotated by $\frac{\pi}{2}$

(a) Suppose $E_s = 1$, for each of the four constellations, compute the minimum distance d_{min} between any two constellation points. (Hint: make sure to first normalize the total transmitted energy to 1).

For 4×4 Grid: $E[|x(t)|^2] = \frac{1}{16}(4 \times 2 + 8 \times 10 + 4 \times 18) = 10 \rightarrow d_{min} = 2/\sqrt{10}$

For 2×8 Grid: $E[|x(t)|^2] = \frac{1}{16}(4 \times 2 + 4 \times 10 + 4 \times 26 + 4 \times 50) = 22 \rightarrow d_{min} = 2/\sqrt{22}$

For shifted Grid: $E[|x(t)|^2] = \frac{1}{16}(4 \times 2 + 4 \times 10 + 1 \times 18 + 4 \times 26 + 2 \times 34 + 1 \times 50) = 18 \rightarrow d_{min} = 2/\sqrt{18}$

For rotated Grid, the magnitude of each constellation point does not change. Hence, the d_{min} does not change and $d_{min} = 2/\sqrt{10}$

(b) For each of the rectangular, shifted, and rotated constellations, state whether their BER performance will be better, the same or worse than the regular 4×4 Grid. Justify your answer.

The 4×4 Grid has significantly larger d_{min} than the 2×8 Grid and the shifted constellation and hence it will have significantly better BER performance. It will, however, have exactly the same BER performance as the rotated constellation.

(c) If the BER performance is the same, is there another reason why we prefer to use the 4×4 Grid over the other constellations.

The 4×4 Grid has much simpler decoding boundaries that are independent for I and Q compared to the rotated Grid. Hence, it is easier to implement in practice.

(d) **(Bonus)** Formally prove that any constellation where the mean of the constellation points $\mathrm{E}[x(t)] \neq 0$ is suboptimal in terms of its BER performance.

We will prove this by contradiction. Suppose a constellation with a mean $\mathrm{E}[x(t)] \neq 0$, has the optimal BER performance compared to any constellation with the same number of points.

Let $\mu = \mathrm{E}[x(t)]$ and $P = \mathrm{E}[|x(t)|^2]$. Let d be the minimum distance between any two constellation points before normalization, then $d_{min} = d\sqrt{E_s/P}$.

Consider this new shifted constellation: $x'(t) = x(t) - \mu$. This new constellation has a zero mean since $\mathrm{E}[x'(t)] = \mathrm{E}[x(t) - \mu]] = 0$. It also has the same minimum distance before normalization d since it is simply a shifted version of the previous constellation.

Now we have,

$$\begin{aligned}
E[|x'(t)|^2] &= E[|x(t) - \mu|^2] \\
&= E[|x(t)|^2 - \mu^* x(t) - \mu x^*(t) + |\mu|^2|] \\
&= E[|x(t)|^2] - \mu^* E[x(t)] - \mu E[x^*(t)] + |\mu|^2 \\
&= P - \mu^*\mu - \mu\mu^* + |\mu|^2 \\
&= P - |\mu|^2
\end{aligned}$$

The minimum distance of this new constellation after normalization is

$$d'_{min} = d\sqrt{E_s/E[|x'(t)|^2]} = d\sqrt{E_s/(P - |\mu|^2)]} > d\sqrt{E_s/P]} \geq d_{min}.$$

Hence, the new constellation has a larger d_{min} with the same number of nearest neighbor. Hence, it will give better BER performance which is a contradiction.

Thus, if there is any constellation where the mean is not zero, I can form a better constellation by shifting the points so the mean is zero.

3 OFDM in WiFi 5 vs. WiFi 6 **40 points**

Consider 802.11ax (WiFi 6) and 802.11ac (WiFi 5) using OFDM with the parameters shown below:

WiFi	802.11ax	802.11ac
Bandwidth	80 MHz	80 MHz
N	1024	256
DC Bins	$-2, -1, 0, +1, +2$	$-1, 0, +1$
Guard Bins	-512 to -501 and $+501$ to $+511$	-128 to -123 and $+123$ to $+127$
Pilot Bins	$\pm 24, \pm 92, \pm 158, \pm 226, \pm 266, \pm 334, \pm 400, \pm 468$	$\pm 11, \pm 39, \pm 75, \pm 103$
Cyclic Prefix	$1.6 \mu s$	$1.6 \mu s$
Preamble	8 symbols: 4 for packet detection, 2 for CFO estimation, 2 for channel estimation.	

For both WiFi 5 and WiFi 6, the transmitter can choose between 3 modulation and coding schemes:

- MCS 0: BPSK with coding rate 1/2
- MCS 1: 4 QAM with coding rate 3/4
- MCS 2: 64 QAM with coding rate 5/6

1. Compute the data rate for each of the above modulation schemes for WiFi 5 and WiFi 6. (You should ignore the preamble in this part and you must show your detailed calculation to get the credit for this part.)

Data Rate = Bandwidth $\times$ Modulation Rate $\times$ Coding Rate $\times (1-$ Overhead)

Overhead $= \frac{\#GuardBins + \#PilotBins + \#DCBins + CP}{N + CP}$

For WiFi 6, Overhead $= \frac{23+16+5+128}{1024+128} = \frac{172}{1152}$.

MCS 0: Data Rate $= 80 \times 10^6 \times 1 \times 1/2 \times (980/1152) = 34.02$ Mbps.
MCS 1: Data Rate $= 80 \times 10^6 \times 2 \times 3/4 \times (980/1152) = 102.08$ Mbps.
MCS 2: Data Rate $= 80 \times 10^6 \times 6 \times 5/6 \times (980/1152) = 340.27$ Mbps.

For WiFi 5, Overhead $= \frac{11+8+3+128}{256+128} = \frac{150}{384}$.

MCS 0: Data Rate $= 80 \times 10^6 \times 1 \times 1/2 \times (234/384) = 24.37$ Mbps.
MCS 1: Data Rate $= 80 \times 10^6 \times 2 \times 3/4 \times (234/384) = 73.12$ Mbps.
MCS 2: Data Rate $= 80 \times 10^6 \times 6 \times 5/6 \times (234/384) = 243.75$ Mbps.

2. Which standard has higher data rates? WiFi 5 or WiFi 6? Given that the standards have the same bandwidth and support the same modulation and coding schemes, explain why one standard is able to provide higher data rates?

   WiFi 6 has higher data rates. This is because the larger symbol (FFT) size reduces the overhead of the CP, Guard bins, Pilots, and DC bins.

3. Suppose each transmitted packet must contain 1500 bytes of data bits. What is the overhead of the preamble in WiFi 5 and WiFi 6 for each of the 3 modulation and coding schemes? Only consider the overhead of the preamble.

$$\text{Preamble Overhead } = \frac{\text{Total Preamble Time}}{\text{Packet Time}}$$

$$\text{Total Preamble Time} = 8 \times \text{Symbol Time} + CP$$

$$\text{Packet Time} = \text{Total Preamble Time} + \#\text{Data Symbols} \times (\text{Symbol Time } + CP)$$

$$\#\text{DataSymbols} = \left\lceil \frac{\#\text{Coded Bits}}{\#\text{Bits in OFDM Symbol}} \right\rceil = \left\lceil \frac{\#\text{Data Bits/Coding Rate}}{\text{Modulation Rate} \times \#\text{Data Bins}} \right\rceil$$

For WiFi 6,

Symbol Time $= 1024/(80 \times 10^6) = 12.8\mu s \rightarrow$ Total Preamble Time $= 104\mu s$

Data Bins $= 1024 - 5 - 23 - 16 = 980$.

MCS 0: #DataSymbols $= \left\lceil \frac{1500\times8/(1/2)}{1\times980} \right\rceil = 25$

$\rightarrow$ Packet Time$= 104\mu s + 25 \times 14.4\mu s = 464\mu s$
$\rightarrow$ Preamble Overhead$= 22.4\%$

MCS 1: #DataSymbols $= \left\lceil \frac{1500\times8/(3/4)}{2\times980} \right\rceil = 9$

$\rightarrow$ Packet Time $= 104\mu s + 9 \times 14.4\mu s = 233.6\mu s$
$\rightarrow$ Preamble Overhead$= 44.5\%$

MCS 2: #DataSymbols $= \left\lceil \frac{1500\times8/(5/6)}{6\times980} \right\rceil = 3$

$\rightarrow$ Packet Time $= 104\mu s + 3 \times 14.4\mu s = 147.2\mu s$
$\rightarrow$ Preamble Overhead$= 70.6\%$

For WiFi 5,

Symbol Time $= 256/(80 \times 10^6) = 3.2\mu s \rightarrow$ Total Preamble Time $= 27.2\mu s$

Data Bins $= 256 - 3 - 11 - 8 = 234$.

MCS 0: #DataSymbols $= \left\lceil \frac{1500\times8/(1/2)}{1\times234} \right\rceil = 103$

$\rightarrow$ Packet Time$= 27.2\mu s + 103 \times 4.8\mu s = 521.6\mu s$
$\rightarrow$ Preamble Overhead$= 5.2\%$

MCS 1: #DataSymbols $= \left\lceil \frac{1500\times8/(3/4)}{2\times234} \right\rceil = 35$

$\rightarrow$ Packet Time $= 27.2\mu s + 35 \times 4.8\mu s = 195.2\mu s$
$\rightarrow$ Preamble Overhead$= 13.9\%$

MCS 2: #DataSymbols $= \left\lceil \frac{1500\times8/(5/6)}{6\times234} \right\rceil = 11$

$\rightarrow$ Packet Time $= 27.2\mu s + 11 \times 4.8\mu s = 80\mu s$
$\rightarrow$ Preamble Overhead$= 34\%$

4. Compute the actual data rate for each of the 3 modulation and coding schemes in WiFi 5 and WiFi 6 while accounting for the preamble overhead.

Actual Data Rate = 1500×8/Packet Time

For WiFi 6,

MCS 0: Data Rate = $12000/464\mu s = 25.86$ Mbps

MCS 1: Data Rate = $12000/233.6\mu s = 51.37$ Mbps

MCS 2: Data Rate = $12000/147.2\mu s = 81.52$ Mbps

For WiFi 5,

MCS 0: Data Rate = $12000/521.6\mu s = 23$ Mbps

MCS 1: Data Rate = $12000/195.2\mu s = 61.47$ Mbps

MCS 2: Data Rate = $12000/80\mu s = 150$ Mbps

Alternatively, a less accurate way to compute the actual data rate is:

Actual Data Rate = Data Rate Computed in part 1 $\times(1-$ Preamble Overhead).

For WiFi 6,

MCS 0: Data Rate $34.02 \times (1 - 0.224) = 26.39$ Mbps

MCS 1: Data Rate $102.08 \times (1 - 0.445) = 56.65$ Mbps

MCS 2: Data Rate $340.27 \times (1 - 0.706) = 100$ Mbps

For WiFi 5,

MCS 0: Data Rate $24.37 \times (1 - 0.052) = 23.1$ Mbps

MCS 1: Data Rate $73.12 \times (1 - 0.139) = 62.95$ Mbps

MCS 2: Data Rate $243.75 \times (1 - 0.34) = 160.875$ Mbps

The discrepancy is due to taking the ceiling in computing the number of data symbols needed.

5. Which standard has higher actual data rates? WiFi 5 or WiFi 6? Explain why one standard performs better?

   > WiFi 5 has higher data rates except for MCS0. This is because the larger symbol (FFT) size in WiFi 6 increases the overhead of the preamble while reducing the number of data symbols needed to transmit the data. The overhead of the preamble dominates the overhead of the CP, Guard bins, Pilots, and DC bins.

6. For each of WiFi 5 and WiFi 6, what is the maximum absolute value of the CFO that can be estimated given the above OFDM parameters i.e., a larger CFO would not be estimated correctly during the coarse CFO estimation? (Hint: Phase wraps around every 2π and CFO can be negative or positive.)

   > $\Delta f_c = \frac{\angle A}{2\pi N T_s}$
   >
   > $-\pi \leq \angle A \leq \pi \rightarrow |\angle A| \leq \pi$
   >
   > $|\Delta f_c| \leq \frac{\pi}{2\pi N T_s} = 80MHz/2N$
   >
   > Hence, the maximum CFO that can be estimated is 156.25 kHz for WiFi 5 and 39 kHz for WiFi 6.

7. Name one advantage of using WiFi 5 to estimate CFO over WiFi 6 and one advantage of using WiFi 6 to estimate CFO over WiFi 5.

   > WiFi 5 can estimate and tolerate larger CFO.
   >
   > WiFi 6 can get more accurate estimates of the CFO since the larger N in $\Delta f_c = \frac{\angle A}{2\pi N T_s}$ averages the noise more.