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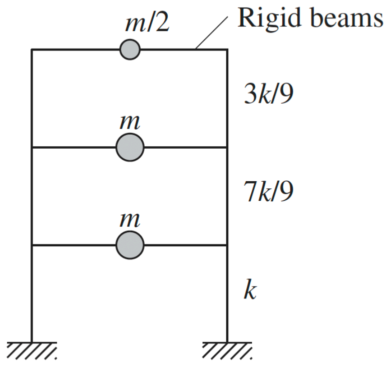
Exercise #10: Damping models

Problem 1

The properties of a three-story shear building are given in Figure 1.1. These include the floor masses, story stiffnesses, natural vibration frequencies and modes. Derive a Rayleigh damping matrix such that the damping ratio is 5% for the first and third modes. Compute the damping ratio for the second mode.



$m = 45{,}000$ kg $\quad k = 300$ kN/cm

$\omega_n = 12.17, 25.82, 39.44$ rad/s

$$\phi_1 = \begin{Bmatrix} 0.4837 \\ 0.9673 \\ 1.4510 \end{Bmatrix}, \quad \phi_2 = \begin{Bmatrix} -0.7454 \\ -0.7454 \\ 1.4907 \end{Bmatrix}, \quad \phi_3 = \begin{Bmatrix} -1.1970 \\ 0.8550 \\ -0.3420 \end{Bmatrix}$$

Figure 1.1 Schematic and properties of the building

Solution

We should first setup the mass and stiffness matrices of the structure – pay attention on how the degrees of freedom are setup for those matrices (3rd line will refer to the degree-of-freedom at the top floor of the structure).

As such,

$$\mathbf{m} = \frac{1}{g}\cdot\begin{bmatrix} W & & \\ & W & \\ & & \frac{W}{2} \end{bmatrix} = \frac{1}{9810\,mm/s^2}\cdot\begin{bmatrix} 441.3 & & \\ & 441.3 & \\ & & \frac{441.3}{2} \end{bmatrix} kN = \begin{bmatrix} 0.0450 & & \\ & 0.0450 & \\ & & 0.0225 \end{bmatrix}\left(\frac{kNs^2}{mm}\right)$$

The corresponding stiffness matrix is as follows (See Week #7 in-class exercise)

$$\mathbf{k} = \frac{k}{9}\begin{bmatrix} 16 & -7 & 0 \\ -7 & 10 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \frac{30}{9}\begin{bmatrix} 16 & -7 & 0 \\ -7 & 10 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 53.3333 & -23.3333 & 0 \\ -23.3333 & 33.3333 & -10.0000 \\ 0 & -10.0000 & 10.0000 \end{bmatrix}(kN/mm)$$

The above matrices are going to be used within the Rayleigh damping matrix formulation to construct the damping matrix once we derive the stiffness and mass proportional coefficients.

Therefore, to determine a_0 and a_1, we consider the frequencies for the 1st and 3rd modes as the problem states:

$$\begin{bmatrix} 1/12.17 & 12.17 \\ 1/39.44 & 39.44 \end{bmatrix}\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = 2\begin{Bmatrix} 0.05 \\ 0.05 \end{Bmatrix}$$

Since both modes have the same damping ratio ($\xi_1 = \xi_3 = 5\%$), we have:

$$a_0 = \xi\frac{2\omega_1\omega_3}{\omega_1 + \omega_3} = 0{,}93\ s^{-1}$$

$$a_1 = \xi\frac{2}{\omega_1 + \omega_3} = 0{,}00194\ s$$

Therefore, the corresponding damping matrix is as follows:

$$\mathbf{c} = a_0\mathbf{m} + a_1\mathbf{k}$$

$$\Rightarrow \mathbf{c} = 0.93 s^{-1}\cdot\begin{bmatrix} 0.0450 & & \\ & 0.0450 & \\ & & 0.0225 \end{bmatrix}\frac{kNs^2}{mm} + 0{,}00194\ s\cdot\begin{bmatrix} 53.3333 & -23.3333 & 0 \\ -23.3333 & 33.3333 & -10.0000 \\ 0 & -10.0000 & 10.0000 \end{bmatrix}\frac{kN}{mm}$$

$$\Rightarrow \mathbf{c} = \begin{bmatrix} 0.145317 & -0.045267 & 0 \\ -0.045267 & 0.106517 & -0.0194 \\ 0 & -0.0194 & 0.040325 \end{bmatrix}\frac{kNs}{mm}$$

Finally, in order to compute the damping ratio for the second mode:

$$\zeta_2 = \frac{a_0}{2}\cdot\frac{1}{\omega_2} + \frac{a_1}{2}\cdot\omega_2 = \frac{0.93 s^{-1}}{2}\cdot\frac{1}{25.82\ s^{-1}} + \frac{0{,}00194\ s}{2}\cdot 25.82\ s^{-1} = 0.0430 = 4.3\%$$

As expected, the damping ratio for the second mode (between first and third) would be less than 5% in this case.