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Exercise #7: MDF systems

Problem 1

Using the definition of stiffness and mass influence coefficients, formulate the equations of motion for the three-story shear frame with lumped masses shown in the figure below. The beams are rigid in flexure, and the flexural rigidity of the columns is as shown. Neglect the axial deformations in all elements.

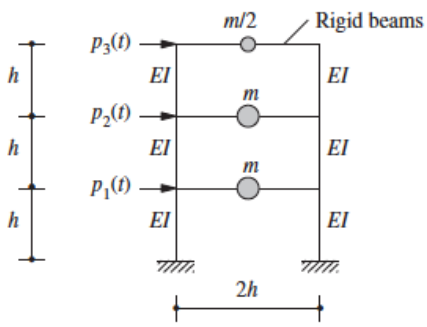


Figure 1.1

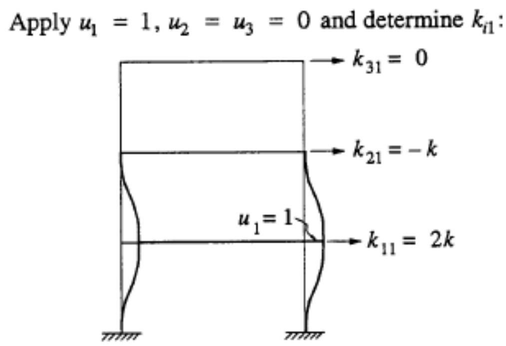
Solution

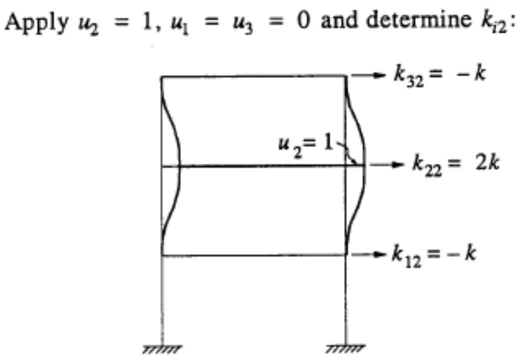
Since the beams are rigid in flexure and axial deformation is neglected in columns, three degrees-of-freedom (DOFs) associated with each story represent the properties of this three-story shear building. The corresponding story masses and story stiffness are as follows:

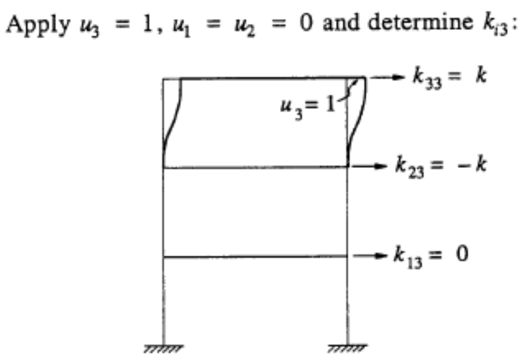
$$m_1 = m, \qquad m_2 = m, \qquad m_3 = \frac{m}{2}$$

$$k_1 = k_2 = k_3 = 2\left(\frac{12EI}{h^3}\right) = \frac{24EI}{h^3} = k$$

Determine the stiffness matrix:







The stiffness matrix is,

$$\mathbf{k} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

The mass matrix is as follows:

$$\mathbf{m} = m\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0.5 \end{bmatrix}$$

The equations of motion are as follows for the case without damping,

$$m\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0.5 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{Bmatrix}$$