FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes RESSLab

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Exercise #5: Response spectrum

Problem 1

A one-story reinforced-concrete building is idealized for structural analysis as a massless frame supporting a dead load of 5000 kg at the beam level. The frame is 8 m wide and 4 m high. Each column, fixed at the base, has a 25 cm-square cross section. The Young's modulus of concrete is 20 GPa, and the damping ratio of the building is estimated as 5%. If the building is to be designed for the design spectrum of Fig. 1.1 scaled to a peak ground acceleration of $0.5g$, determine the design values of lateral deformation and bending moments in the columns for two conditions:
(a) The cross section of the beam is much larger than that of the columns, so the beam may be assumed as rigid in flexure.
(b) The beam cross section is much smaller than the columns, so the beam stiffness can be ignored. Comment on the influence of beam stiffness on the design quantities.

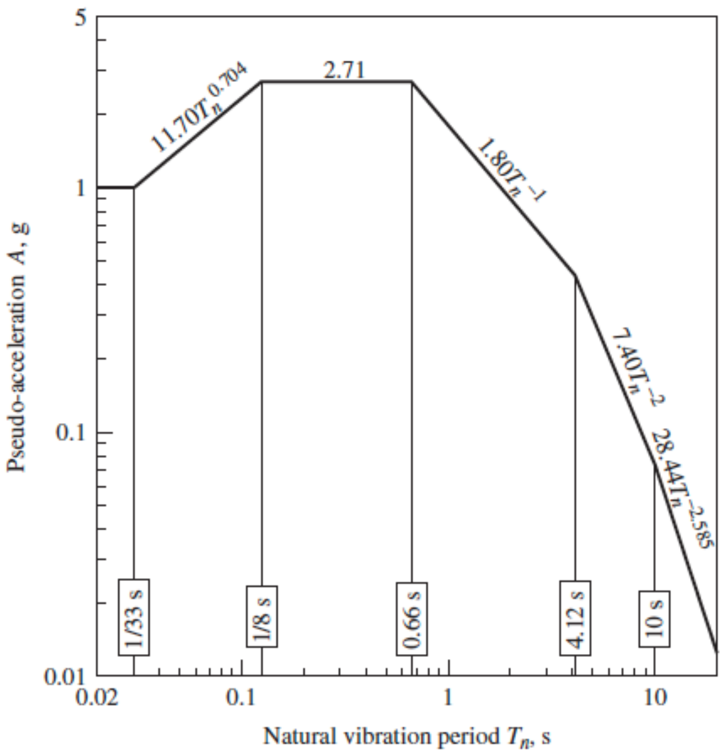


Figure 1.1 Elastic pseudo-acceleration design spectrum for ground motions with $\ddot{u}_{go} = 1g, \dot{u}_{go} = 122\, cm/s, u_{go} = 91 cm, \zeta = 5\%$

Solution

a) Frame with rigid beam ($EI_b = \infty$)

In this case the lateral stiffness is due to bending in the columns. As such,

$$k = 2\left[\frac{12EI_c}{h^3}\right] = \frac{24\left(\frac{20kN}{mm^2}\cdot 250^4/12\,(mm^4)\right)}{4000^3(mm^3)} = 2.44\frac{kN}{mm}$$

The seismic weight, $W = 5000\,kg = 50\,kN$. Therefore, the seismic mass is as follows:

$$m = \frac{W}{g} = \frac{50}{9810} = 0.0051\frac{(kNsec^2)}{mm}$$

As such, the natural period of the oscillator is as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.0051}{2.44}} = 0.287sec$$

and,

$$\omega_n = \frac{2\pi}{T_n} = \frac{2\pi}{0.287} = 21.89\frac{rads}{sec}$$

For $T_n = 0.287sec$ and $\zeta = 5\%$, Figure 1.1 gives $2.71g$ ($1/8\,sec < T_n < 0.66\,sec$) but should be scaled by 0.50 (as the problem states), therefore, $A/g = 1.355$.

$$D = \frac{A}{\omega_n^2} = \frac{1.355\cdot 9810}{21.89^2} = 27.75mm$$

Design quantities:

Lateral deformation, $u_o = 27.75mm$

Lateral force, $f_{So} = \left(\frac{W}{g}\right)\cdot A = 0.0051\cdot 1.355\cdot 9810 = 67.75kN$

Bending moment at top and bottom of the columns,

$$M = \left(\frac{f_{So}}{2}\right)\cdot\frac{h}{2} = \left(\frac{67.75}{2}\right)\cdot\left(\frac{4000}{2}\right) = 67750kN\cdot mm$$

Bending moments in the columns are shown in the diagram (next page) for the fixed end columns.

b) a) Frame with flexible beam ($EI_b = 0$) – no flexural stiffness

In this case,

$$k = 2\left[\frac{3EI_c}{h^3}\right] = \frac{6\left(\frac{20kN}{mm^2} \cdot 250^4/12\,(mm^4)\right)}{4000^3(mm^3)} = 0.61\frac{kN}{mm}$$

The seismic weight, $W = 5000\ kg\ =\ 50\ kN$. Therefore, the seismic mass is as follows:

$$m = \frac{W}{g} = \frac{50}{9810} = 0.0051\frac{(kNsec^2)}{mm}$$

As such, the natural period of the oscillator is as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.0051}{0.61}} = 0.574sec$$

and,

$$\omega_n = \frac{2\pi}{T_n} = \frac{2\pi}{0.574} = 10.94\frac{rads}{sec}$$

For $T_n\ =\ 0.574sec$ and $\zeta = 5\%$, Figure 1.1 gives $2.71g$ $(1/8\,sec\ <\ T_n\ <\ 0.66\,sec)$ but should be scaled by 0.50 (as the problem states), therefore, $A/g\ =\ 1.355$.

$$D = \frac{A}{\omega_n^2} = \frac{1.355 \cdot 9810}{10.94^2} = 111.0mm$$

Design quantities:

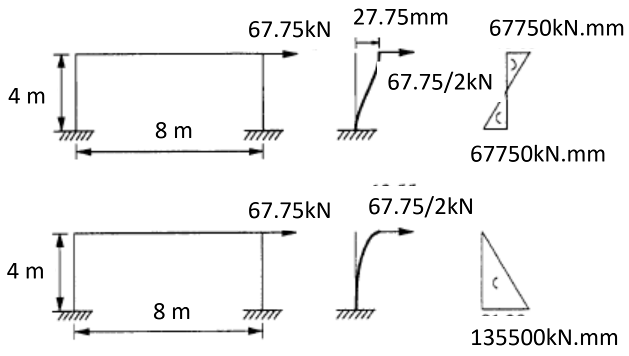
Lateral deformation, $u_o = 111.0mm$

Lateral force, $f_{So} = \left(\frac{W}{g}\right) \cdot A = 0.0051 \cdot 1.355 \cdot 9810 = 67.75kN$

Bending moment at top and bottom of the columns,

$$M = \left(\frac{f_{So}}{2}\right) \cdot h = \left(\frac{67.75}{2}\right) \cdot (4000) = 135500kN \cdot mm$$

Bending moments in the columns are shown in the diagram for the second case.



Discussion: Interestingly, the more flexible case (beam with zero stiffness) although it elongates the natural period of the SDF system, the expected absolute acceleration is the same. For the given mass, this is why the inertia force, f_{So} is the same in both cases. However, the more flexible SDF system deflects about 4 times more than the stiffer one. This could potentially cause lateral stability problems. Because of the zero stiffness at the top end of the columns, these behave like cantilevers and the maximum moment at the base in this case is 2 times more than the fixed-end column.

Problem 2

The system shown in Figure 2.1 consists of a bin mounted on a rigid platform supported by four columns $8\ m$ long. The mass of the platform, bin, and contents is 50,000 kg and may be taken as a point mass located 2 m above the bottom of the platform. The columns are braced in the longitudinal direction, that is, normal to the plane of the paper, but are unbraced in the transverse direction as shown in the figure. The column properties are: $A = 120\ cm^2$, $E = 200\ GPa$, $I = 80{,}000\ cm^4$, and $W_{el} = 2800\ cm^3$. Taking damping ratio to be 5%, find the peak lateral displacement and the peak stress in the columns due to gravity and the earthquake characterized by the design spectrum of Fig. 2.2 scaled to $1/3g$ acting in the transverse direction. Take the columns to be fixed at the base and at the rigid platform. Neglect axial deformation of the column and gravity effects on the lateral stiffness.

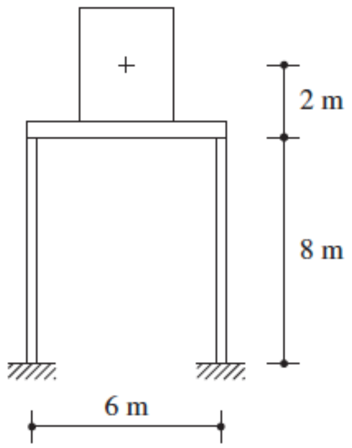


Figure 2.1

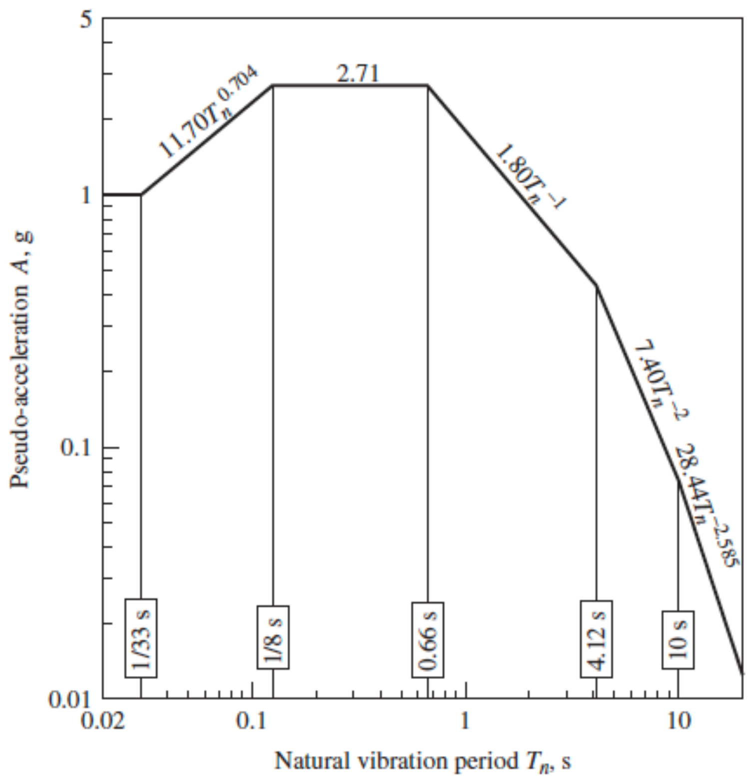


Figure 2.2 Elastic pseudo-acceleration design spectrum for ground motions with $\ddot{u}_{go} = 1g, \dot{u}_{go} = 122\ cm/s\ , u_{go} = 91 cm, \zeta = 5\%$

Solution

1. Determine first the structural properties of the SDF system:

Lateral stiffness (you have to account for 4 columns as the frame is 3-D):

$$k = 4\left[\frac{12EI_c}{h^3}\right] = \frac{48\left(\frac{200kN}{mm^2} \cdot 800000000(mm^4)\right)}{8000^3(mm^3)} = 15.0\frac{kN}{mm}$$

The seismic weight, $W = 50000\ kg\ = 500\ kN$. Therefore, the seismic mass is as follows:

$$m = \frac{W}{g} = \frac{500}{9810} = 0.051\frac{(kNsec^2)}{mm}$$

As such, the natural period of the oscillator is as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.051}{15.0}} = 0.366sec$$

and,

$$\omega_n = \frac{2\pi}{T_n} = \frac{2\pi}{0.366} = 17.16\frac{rads}{sec}$$

For $T_n\ =\ 0.518sec$ and $\zeta = 5\%$, Figure 1.1 gives $2.71g$ $(1/8\,sec\ <\ T_n\ <\ 0.66\,sec)$ but should be scaled by 1/3 (as the problem states), therefore, $A/g\ =\ 0.903$.

$$D = \frac{A}{\omega_n^2} = \frac{0.903 \cdot 9810}{17.16^2} = 30.1mm$$

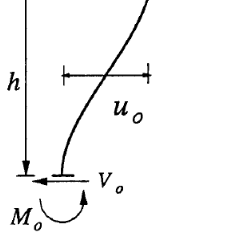
Design quantities:

Lateral deformation, $u_o = 30.1mm$

Column:

Shear, $V_o = k_c u_o = \frac{12EI}{h^3}u_o = \frac{12\left(\frac{200kN}{mm^2} \cdot 800000000(mm^4)\right)}{8000^3(mm^3)}30.1 = 112.9kN$

Bending moment at top and bottom of the columns,

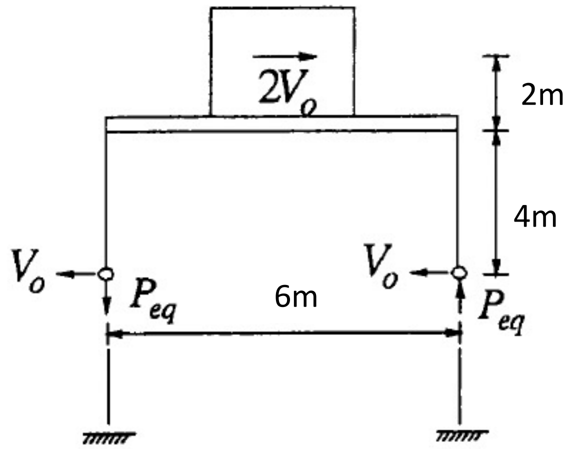


$$M = (V_o) \cdot \frac{h}{2} = (112.9kN) \cdot \left(\frac{8000}{2}\right) = 451666.67kN \cdot mm$$

The stress in the column due to flexure in this case is as follows:

$$\sigma_M = \frac{M}{W_{el}} = \frac{451666.67kN.mm}{2800000mm^3} = 0.161\frac{kN}{mm^2} = 161MPa$$

The stress due to axial forces:



The sketch represents half of the structure, i.e., one pair of columns with rigid platform and rigid column bases; there is a point of inflection at mid-height of each column. Hence, taking moments about one of the inflection points, we get the column axial forces due to earthquake to be as follows,

$$P_{eq} = 2V_o\frac{(4000 + 2000)}{6000} = 2 \cdot 112.9kN\frac{(4000 + 2000)}{6000} = 225.8kN$$

The axial force in each column due to the gravity load is as follows,

$$P_{grav} = \frac{W}{4} = \frac{500}{4} = 125kN$$

Therefore, the total axial stress in each column is as follows:

$$\sigma_a = \frac{N_{grav} + N_{eq}}{A} = \frac{(225.8 + 125)kN}{12000mm^2} = 0.0292\frac{kN}{mm^2} = 29.2MPa$$

The total resultant stress per column is as follows:

$$\sigma = \sigma_M + \sigma_a = 161 + 29.5 = 190.5MPa$$