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Exercise #4: Earthquake response of single-degree-of-freedom systems

Problem 1

The concrete frame shown in the figure is located on sloping ground and carries 1220kN of seismic weight. The beam is made much stiffer than the columns and can be assumed as rigid. The cross sections of the concrete column ($E = 30\text{GPa}$) are 250mm x 250mm and their respective lengths are shown in the figure. Determine the following:

1. The natural vibration period of the frame.
2. The base shear in the two columns at the instant of peak response due to the ElCentro ground motion. Assume the damping ratio to be 2%.

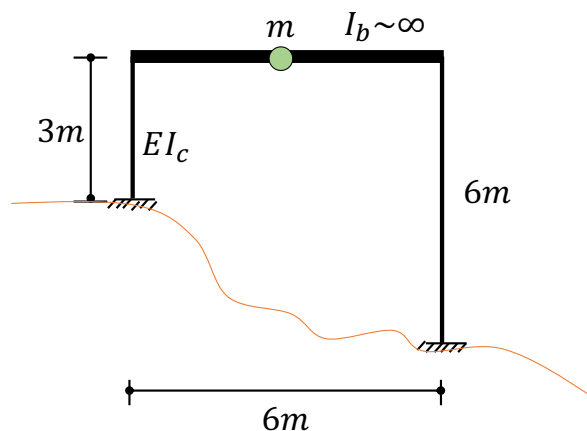


Figure 3.1

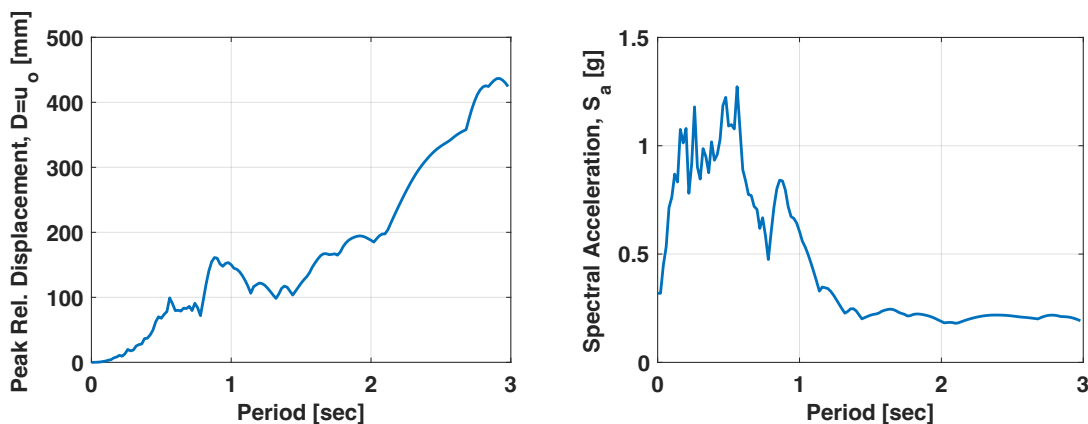


Figure 3.2 response spectra for ElCentro ground motion (2% damping ratio)

Problem 1 - Solution

To compute the natural vibration period of the frame, the seismic mass and lateral stiffness shall be calculated.

The seismic mass is as follows:

$$m = \frac{W}{g} = \frac{1220kN}{9810mm/s^2} = 0.12436kN - s^2/mm$$

The beam's flexural stiffness is infinite. Therefore, the two columns only contribute to the lateral translational stiffness of the frame. The two columns are fixed at both ends, therefore,

$$K_{left} = \frac{12EI_c}{h_{left}^3} = \frac{12 \cdot \frac{30kN}{mm^2} \cdot 325520833mm^4}{3000mm^2} = 4.34 \frac{kN}{mm}$$

$$K_{right} = \frac{12EI_c}{h_{right}^3} = \frac{12 \cdot \frac{30kN}{mm^2} \cdot 325520833mm^4}{6000mm^2} = 0.543 \frac{kN}{mm}$$

NOTE: The second moment of area of the concrete column cross sections shall be calculated. Because both cross sections are square ($b = 250mm$), $I_c = 250^4/12 = 325520833mm^4$.

Therefore, the total lateral translational stiffness of the frame is,

$$K = K_{left} + K_{right} = 4.88 \frac{kN}{mm}$$

Therefore,

$$T_n = 2\pi \sqrt{\frac{m}{K}} = 2\pi \sqrt{\frac{0.12436}{4.88}} = 1.00sec$$

From the displacement response spectrum of the ElCentro ground motion, at $T_n = 1sec$, $u_o = 150mm$, for 2% damping ratio.

Therefore, we can compute the corresponding base shear forces for each column as follows:

$$V_{left} = K_{left} \cdot u_o = 4.34 \left(\frac{kN}{mm} \right) \cdot 150mm = 651kN$$

$$V_{right} = K_{right} \cdot u_o = 0.543 \left(\frac{kN}{mm} \right) \cdot 150mm = 81.4kN$$

Note that most of the shear force is distributed to the stiffer element of the two.

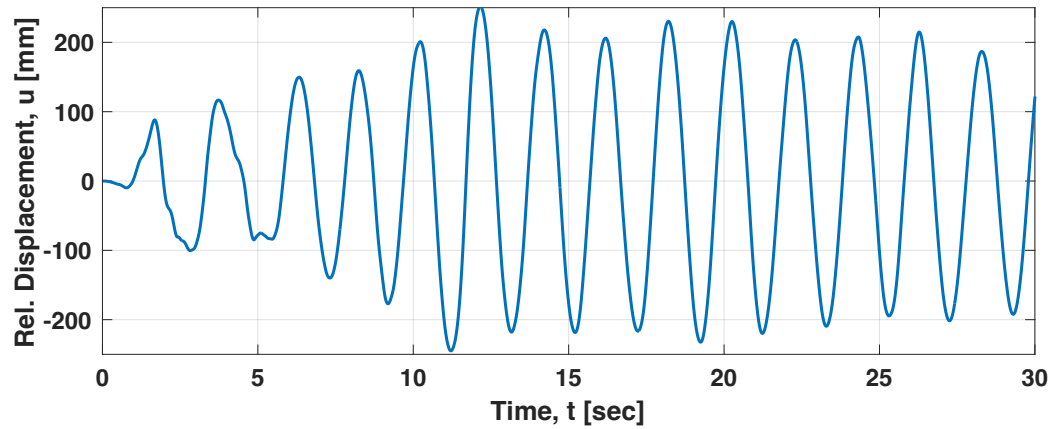
Problem 2

Implement the numerical algorithm of the Central Difference Method into a programming language of your choice and compute the deformation response $u(t)$ for $0 \leq t \leq 15\text{sec}$ for an SDF system with natural period of $T_n = 2\text{sec}$ and damping ratio of 0% and 5% to ElCentro 1940 ground motion.

Problem 2 – Solution

The code in MATLAB is provided in Moodle. The following plots are presented for 0% and 5% damping ratio:

2% damping ratio:



5% damping ratio:

