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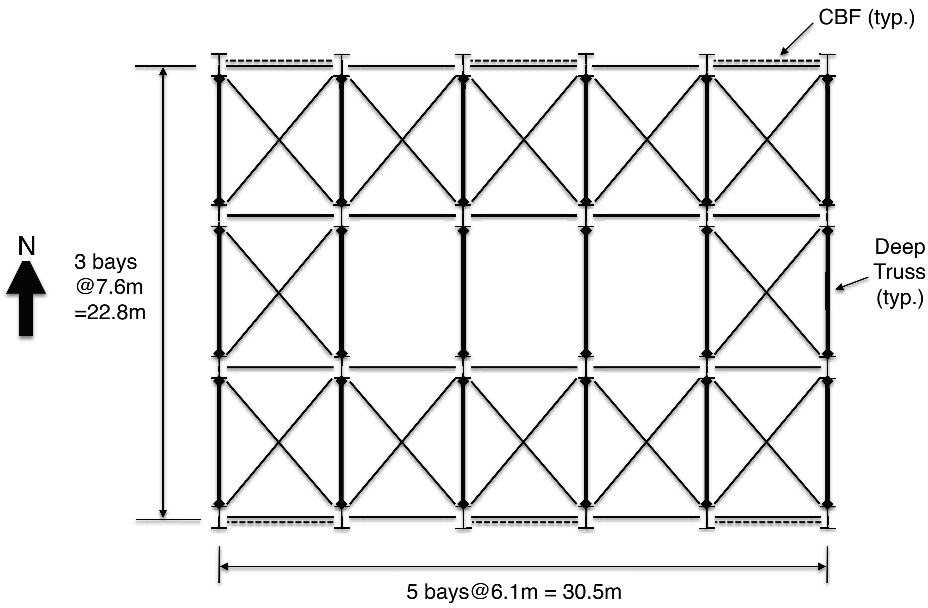
Exercise #1: Equations of motion and dynamic characteristics

Problem 1

The single-storey steel industrial building shown in Figure 1 can be idealized as a single-degree-of-freedom (SDF) system in each of its principal orthogonal directions. The columns are all I-shaped steel cross-sections ($E = 200GPa$) that are fixed at their base and connected to a roof truss system at the top. The second moments of area of the I-shaped cross sections are $I_x = 34.4x10^{-6}m^4$ and $I_y = 7.64x10^{-6}m^4$. The roof truss has a flexural stiffness that is significantly greater than that of the columns in the direction where the columns bend about their strong axis (assumed as fixed at the base) but has negligible flexural stiffness in the direction where the columns bend about their weak axis (assumed as pinned at the base). In addition, in the two perimeter frames in the east-west (EW) direction, slender X-braces, made of 25-mm-diameter circular steel rods, are installed in three bays (a total of six braces per braced perimeter frame; only the ones in tension contribute to the lateral stiffness). The total dead load acting on the roof of the structure is equal to 1.06kPa while the total dead load acting on the perimeter walls is equal to 0.48kPa.

Compute the natural period of vibration of the building in each one of its principal loading directions (N-S and E-W). List clearly your assumptions.



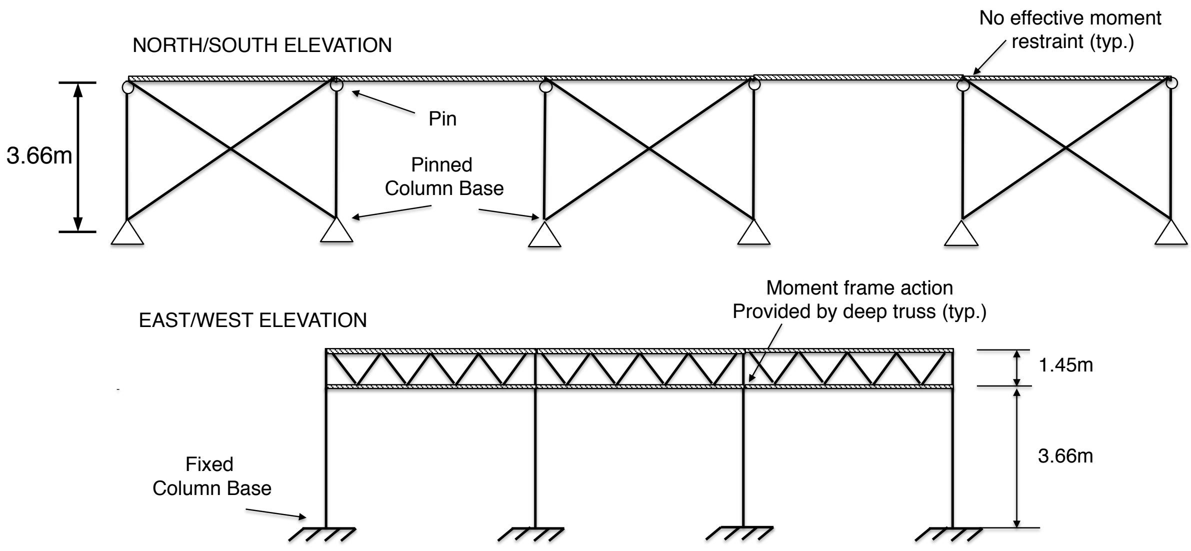


Figure 1. Single-storey industrial building

Problem 2

Compute the elastic stiffness and derive the equation of motion for the frame shown in Figure 2.1. The flexural rigidity of the beam and columns is noted in the figure. The mass lumped at the beam is m; otherwise, assume the frame to be massless and neglect damping.

1. Assume, $EI_b = \infty$
2. Assume that EI_b is not infinite.

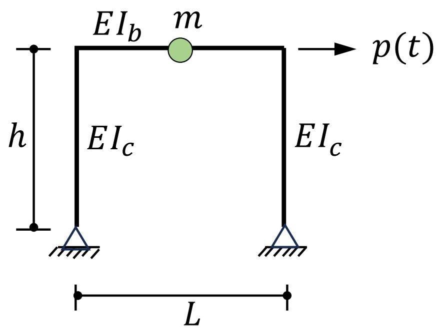
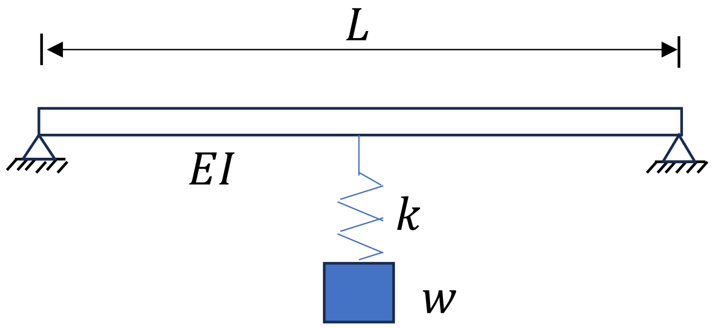


Figure 2. Portal frame

Problem 3

Determine the natural frequency of a weight w suspended from a spring at the midpoint of a simply supported beam. The length of the beam is L, and its flexural rigidity is EI. The spring is k. Assume the beam to be massless.



Problem 1 - Solution

To compute the natural period in each of the principal directions, the mass associated with these SDF systems must first be determined. It is assumed that the total dead load acting on the roof as well as the dead load corresponding to the top half of the perimeter walls will contribute to the total mass that is lumped at the top of the SDF system. First the weight is computed as follows:

W = 1.06kPa x 30.5m x 22.8m + 0.48kPa x 2 x (30.5m + 22.8m) x 1.83m = 830kN.

Which corresponds to a mass, $m = W/\mathrm{g} = 85\mathrm{kNs^2/m}$.

Stiffness in North-South Direction (NS)

The building's lateral stiffness in this direction is provided by six moment-resisting frames. Since the columns are fixed at their base and the flexural stiffness of the roof truss is much greater than the stiffness of the columns bending about their strong axis, we can determine that the lateral stiffness provided by each column in the moment frame is:

$$k_i = \frac{12EI_x}{h^3} = \frac{12\left(200\times10^6 kN/m^2\right)\left(34.4\times10^{-6} m^4\right)}{\left(3.66m\right)^3} = 1684kN/m$$

The total stiffness of the building in the NS direction is therefore:

$$k_{NS} = \sum_{i=1}^{24} k_i = 24\left(1684kN/m\right) = 40416kN/m$$

Stiffness in the east-west direction (EW)

The building's lateral stiffness in this direction is provided by 12 diagonal braces (six in each perimeter frame). However, because of the high slenderness of the bracing members, it is expected that the braces will buckle in compression at a very low load and will not therefore contribute significantly to the stiffness of the building in this direction (i.e., can be assumed as tension-only braces). As such, only the stiffness provided by a total of six braces (three in each perimeter frame) acting in tension for any given direction of loading is considered in this calculation.

Assume that the length of the brace is L_b. Each diagonal brace provides the building with a lateral stiffness in this direction of:

$$k_i = \frac{EA}{L_b}\cos^2\theta$$

in which,

$$A = \frac{\pi d^2}{4} = 490\times10^{-6} m^2, L_b = \sqrt{3.66^2 + 6.10^2} = 7.11m$$

$$\theta = \tan^{-1}\left(\frac{3.66}{6.10}\right) = 0.858$$

Therefore,

$$k_i = \frac{\left(490\times10^{-6}m^2\right)\left(200\times10^6 kN/m^2\right)}{7.11}\cos^2\left(31^o\right) = 10147kN/m$$

The total stiffness provided by the braces acting in tension in the EW direction is therefore:

$$k_{EW-braces} = 6\times10147kN/m = 60881kN/m$$

Finally, the natural periods can be calculated as follows:

$$T_{NS} = 2\pi\sqrt{\frac{m}{k_{NS}}} = 2\pi\sqrt{\frac{85kNs^2/m}{40416kN/m}} = 0.288\,sec$$

$$T_{EW} = 2\pi\sqrt{\frac{m}{k_{EW-braces}}} = 2\pi\sqrt{\frac{85kNs^2/m}{60881kN/m}} = 0.235\,sec$$

Problem 2 Solution (sketches courtesy of Ms. Aline Bönzli)

1. When the steel beam rigidity is infinite, the lateral stiffness of the frame is only attributed to the column deformations. As such,

$$k = 2 \cdot k_{column} = 2 \cdot \frac{3EI_c}{h^3} = \frac{6EI_c}{h^3}$$

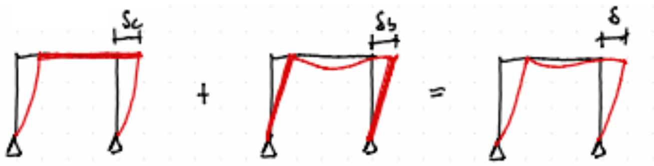
The equation of motion in this case,

$$m\ddot{u} + \left(\frac{6EI_c}{h^3}\right)u = p(t)$$

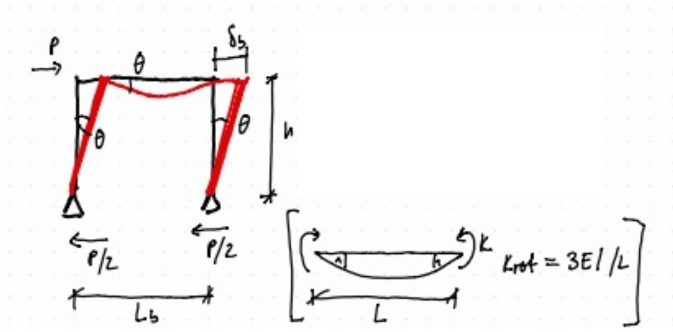
The displacement in this case due to a force P applied at the left upper corner would be:

$$\delta_c = \frac{P}{k} = \frac{Ph^3}{6\,E\,I_c}$$

2. When the steel beam contributes to the overall lateral stiffness, the total lateral deflection is attributed to bending in the columns and bending in the beam. As such,



To calculate the deflection due to bending in the beams, assume that the steel columns are rigid and rotate about the pin supports as shown below:



$$\delta_b \cong \theta h = \frac{M}{K_{rot}}h$$

The moment in the beam can be calculated from the moment in the column as follows:

$$M = \frac{P}{2}h = \frac{Ph}{2}$$

The beam is in contraflexure (double curvature), as such,

$$K_{rot} = 3EI_b/(L_b/2) = \frac{6EI_b}{L_b}$$

Therefore,

$$\delta_b = \frac{M}{K_{rot}} h = \frac{Ph/2}{6EI_b/L_b} h = \frac{Ph^2 L_b}{12EI_b}$$

$$\delta = \delta_c + \delta_b = \frac{F}{\left(2 \cdot \frac{3EI_c}{h^3}\right)} + \frac{FL_b h^2}{12EI_b} = \frac{Fh^3}{6EI_c} + \frac{FL_b h^2}{12EI_b} = \frac{Fh^2}{6E}\left(\frac{h}{I_c} + \frac{L_b}{2I_b}\right)$$

Therefore, the lateral stiffness is,

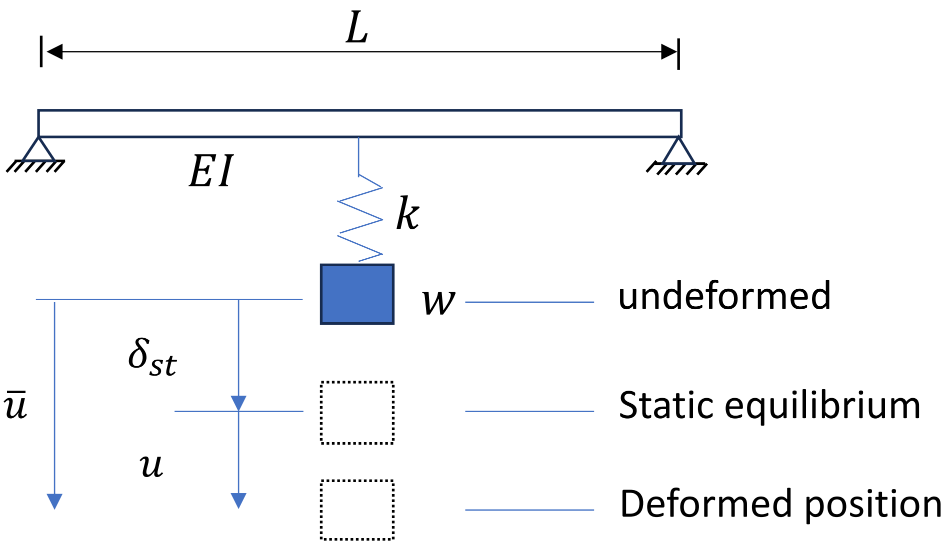
$$K = \frac{F}{\delta} = \frac{6E}{h^2} \frac{1}{\left[\frac{h}{I_c} + \frac{L_b}{2I_b}\right]}$$

The equation of motion in this case,

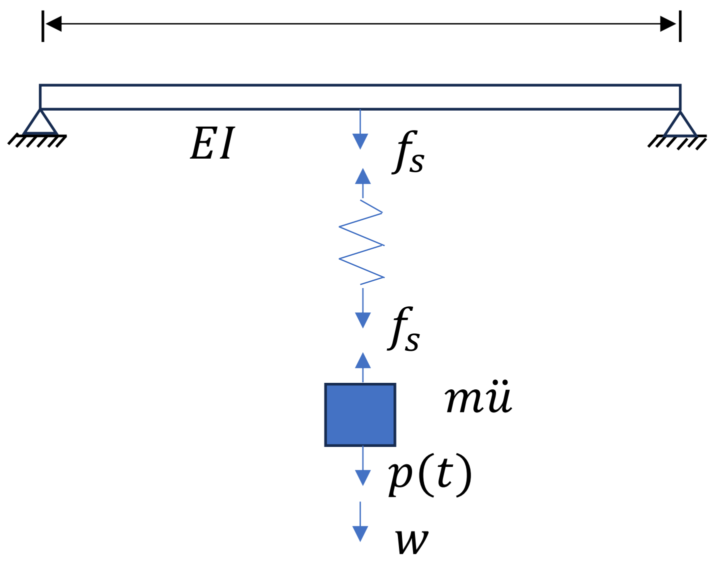
$$m\ddot{u} + \left(\frac{6E}{h^2} \frac{1}{\left[\frac{h}{I_c} + \frac{L_b}{2I_b}\right]}\right) u = p(t)$$

Problem 3 Solution

First we should consider the different states of equilibrium.



Then a free body diagram between the beam, spring and weight,



The equation of motion is as follows,

$$m\ddot{\bar{u}} + f_s = w + p(t) \quad (1)$$

The restoring force, $f_s = k_e\bar{u}$ (k_e is the effective stiffness)

Hence,

$$m\ddot{\bar{u}} + k_e\bar{u} = w + p(t) \quad (2)$$

$$\bar{u} = \delta_{spring} + \delta_{beam} \tag{3}$$

$$f_s = k\delta_{spring} = k_{beam}\delta_{beam} \tag{4}$$

As such substituting to Equation (3),

$$\frac{f_s}{k_e} = \frac{f_s}{k} + \frac{f_s}{k_{beam}} \tag{5}$$

And

$$k_e = \frac{kk_{beam}}{k+k_{beam}} = \frac{k\left(\frac{48EI}{L^3}\right)}{k+\frac{48EI}{L^3}} \tag{6}$$

Finally,

$$\omega_n = \sqrt{\frac{k_e}{w/g}}$$