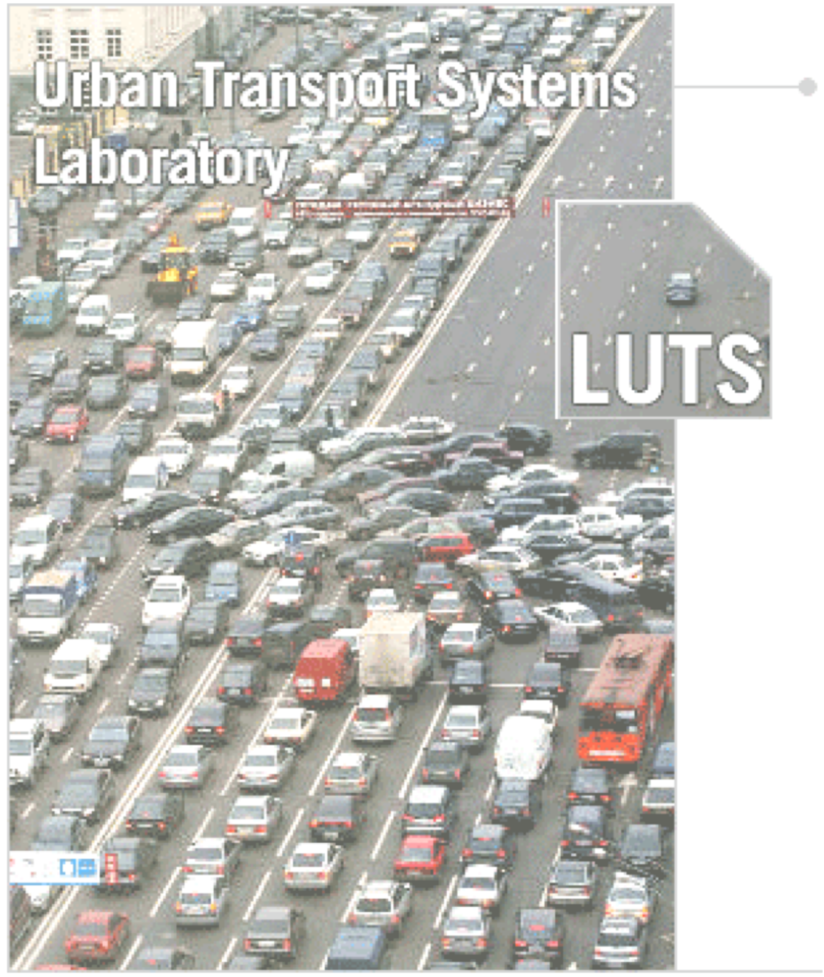
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Traffic flow modeling

An intro to LWR Theory

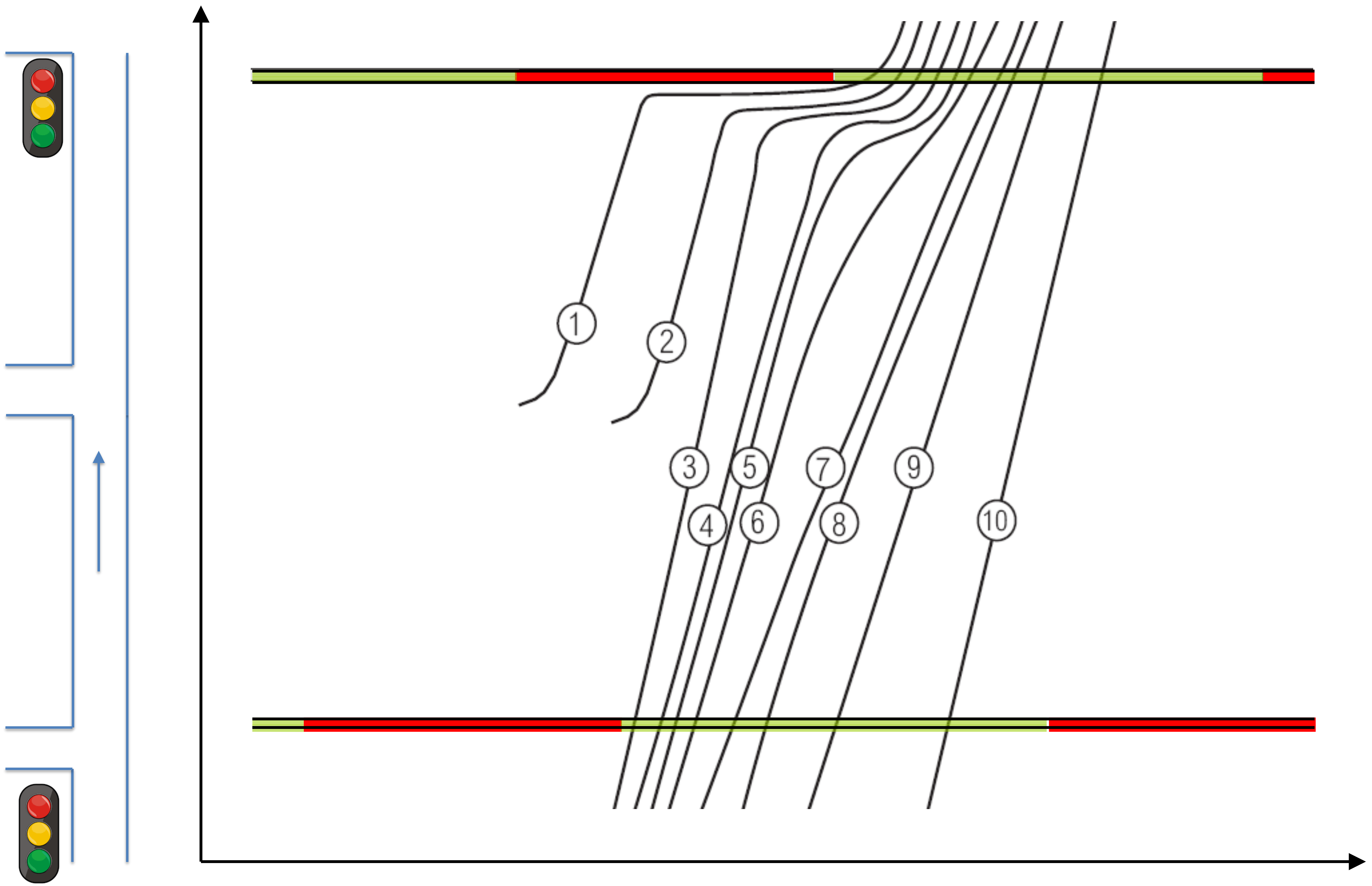


Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis

Welcome

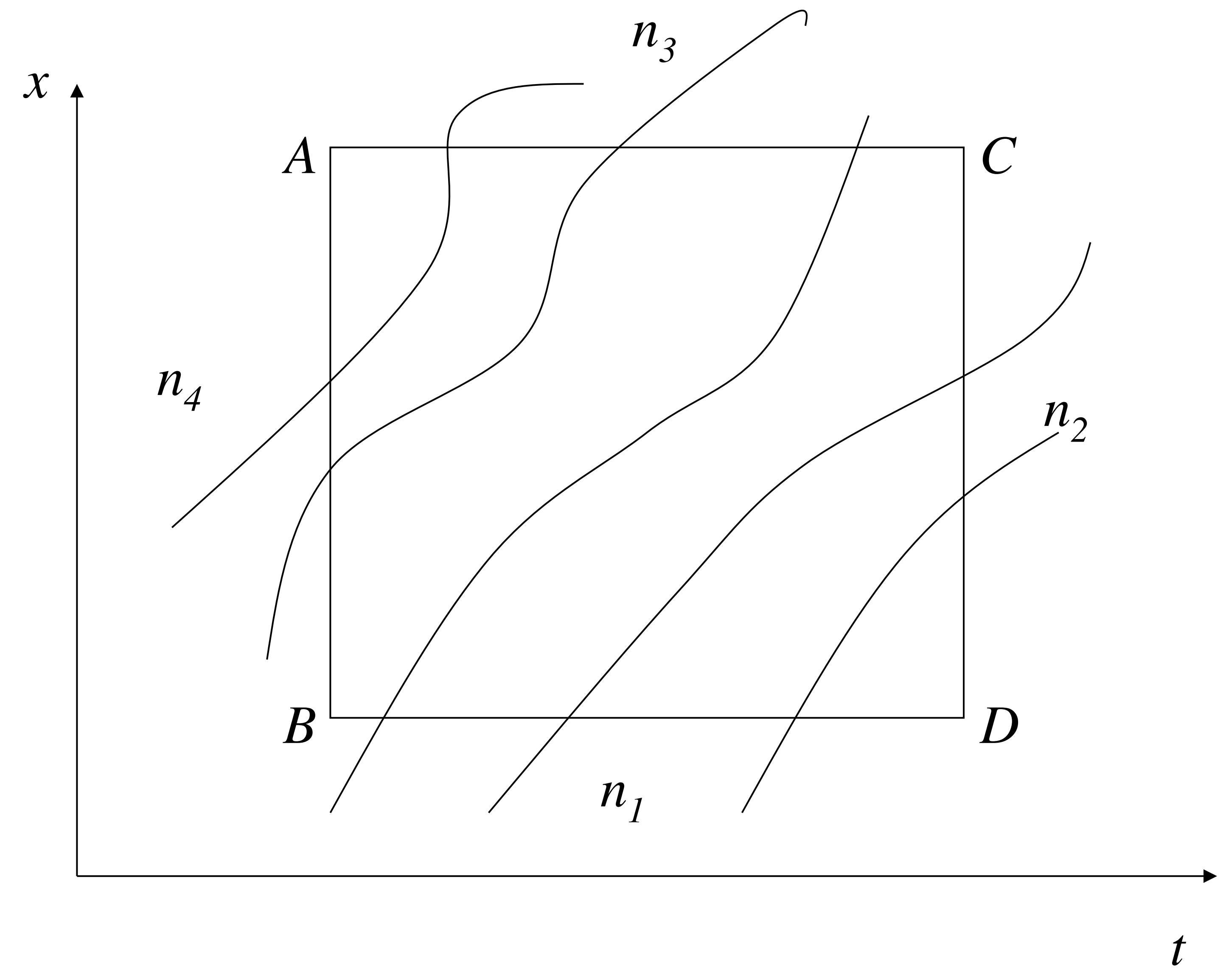
Trajectories of vehicles at a traffic light



Differential form of conservation law

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0$$

<u>A simple proof</u>



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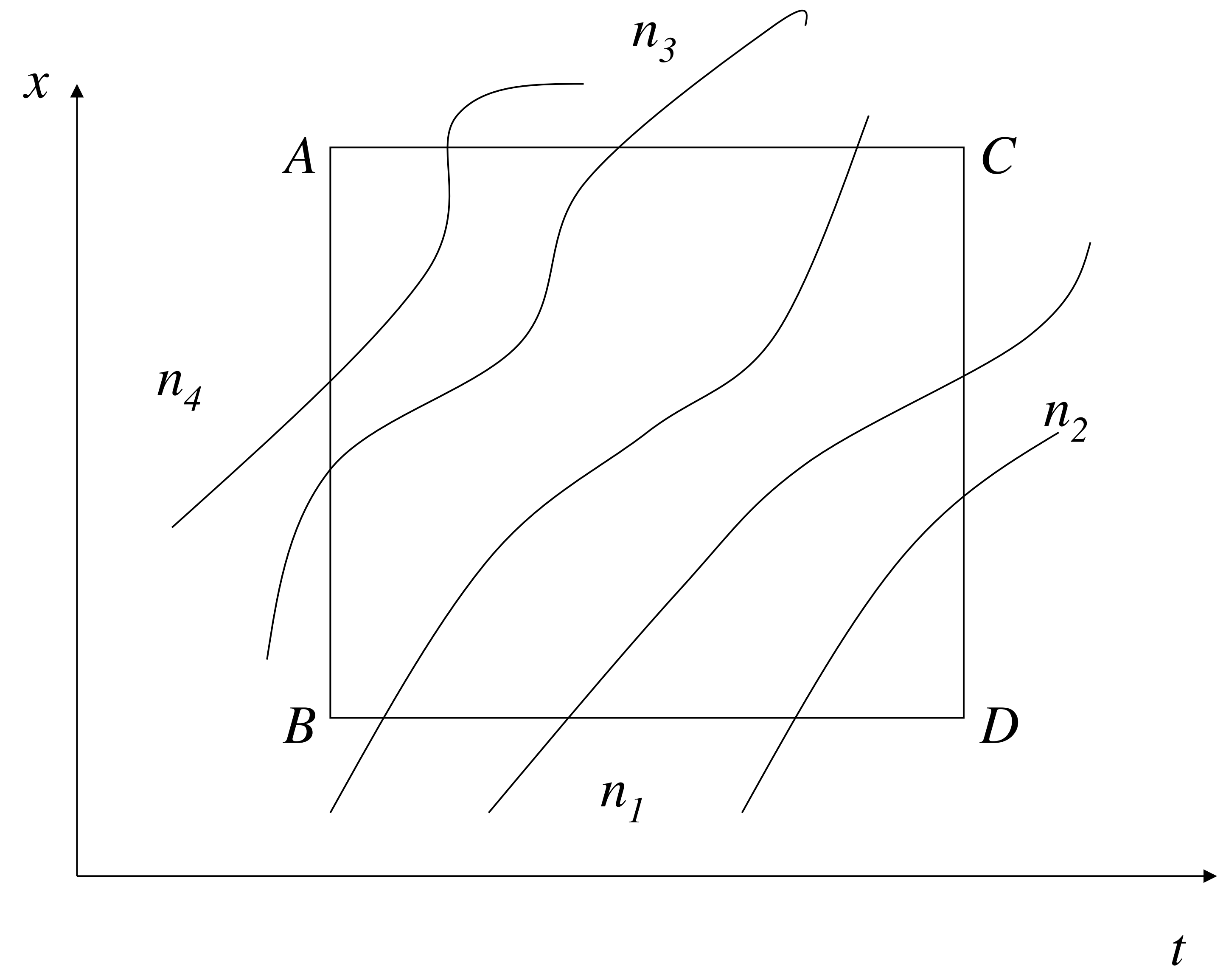


$$n_1 + n_4 = n_3 + n_2$$

$$q_{BD} * \Delta T + k_{AB} * \Delta L = q_{AC} * \Delta T + k_{CD} * \Delta L$$

$$0 = (q_{AC} - q_{BD}) * \Delta T + (\, k_{CD} - k_{AB}) * \Delta L$$

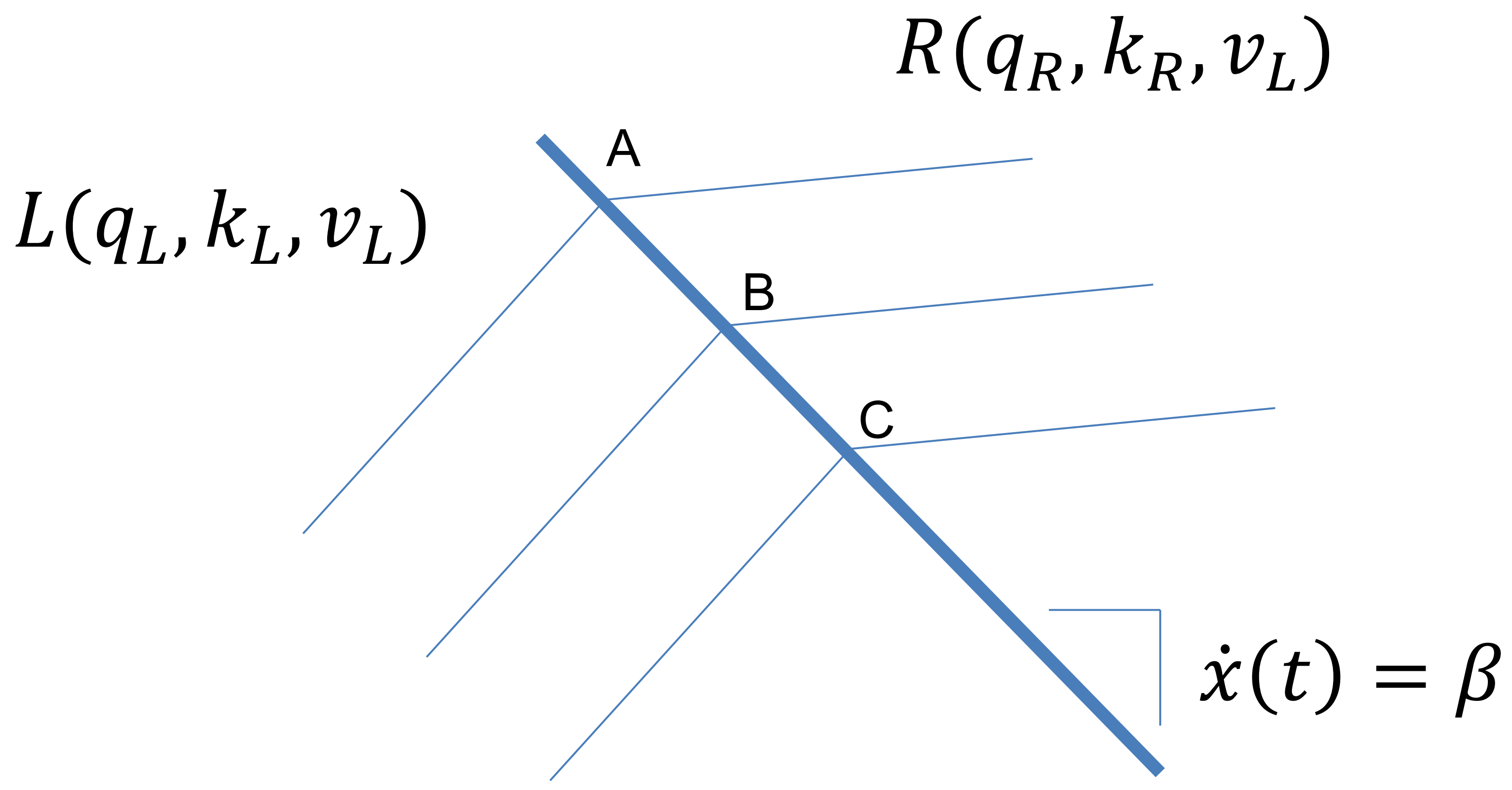
$$\frac{\Delta k}{\Delta T} + \frac{\Delta q}{\Delta L} = 0$$

Shockwaves and conservation law

- A shock wave is a drastic change in traffic density that propagates through a traffic stream.

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \qquad (1)$$

- Shocks only happen for increasing density profile $k_R > k_L$ (unique solution).
- Instantaneous speed adjustment (infinite acceleration)
- $\dot{x}(t) = \frac{q_R - q_L}{k_R - k_L}$ (Rankine-Hugoniot condition)



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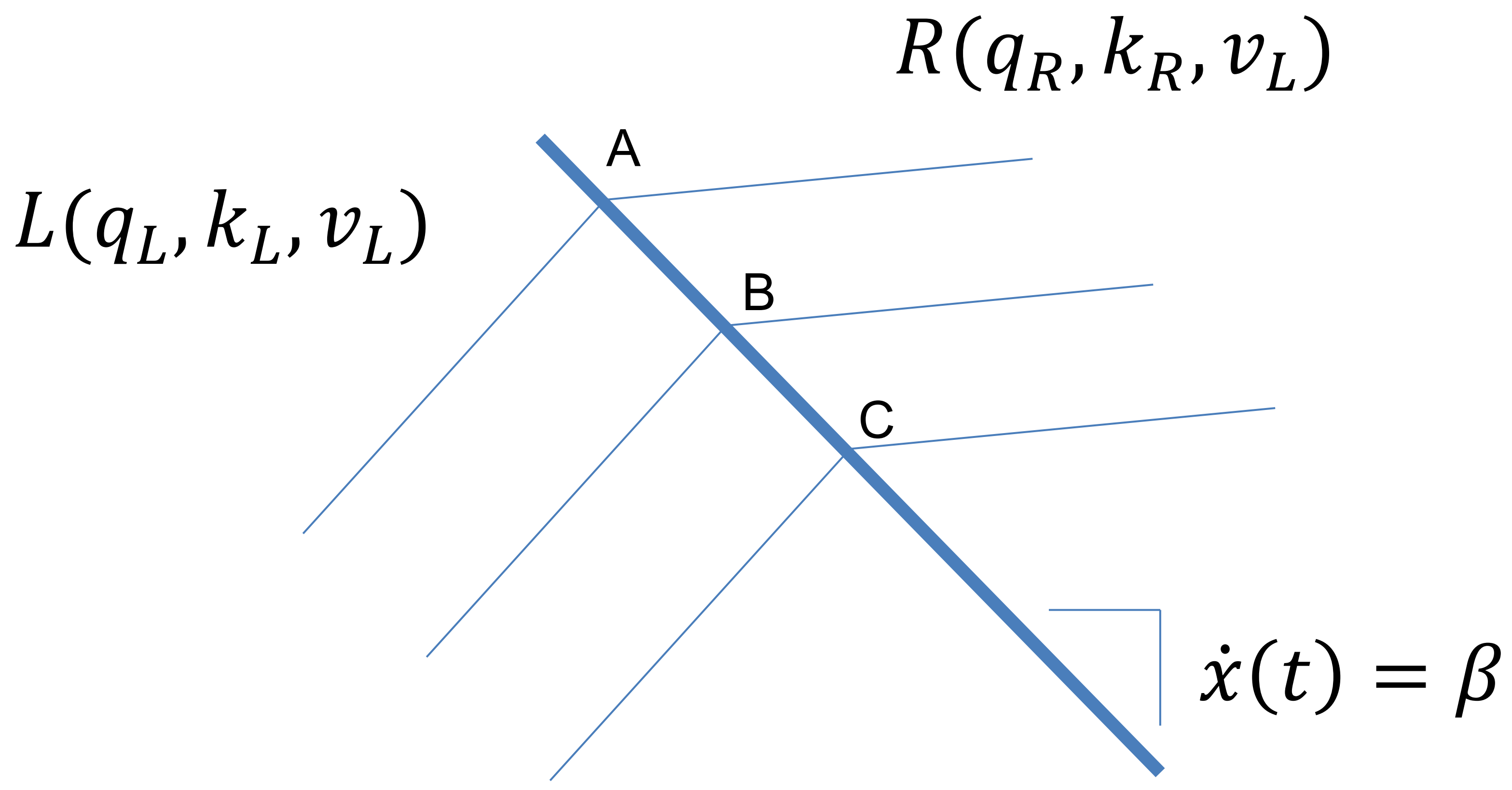
A simple proof of R-H:

$$(\beta + v_L) * t = s_L$$

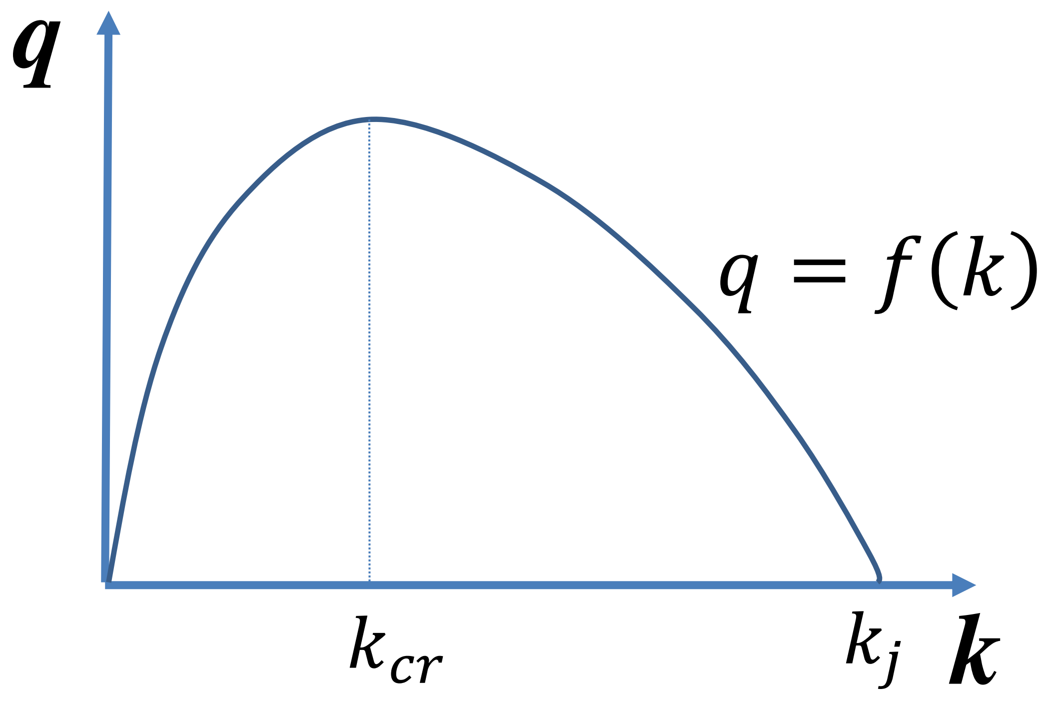
$$(\beta + v_R) * t = s_R$$



Basics of LWR Theory

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \qquad (1)$$

$$\dot{x}(t) = \frac{q_R - q_L}{k_R - k_L} \qquad (2)$$



$$q = f(k) \qquad (3)$$

$$\frac{\partial k(x,t)}{\partial t} + f'(k)\frac{\partial k(x,t)}{\partial x} = 0 \qquad (4)$$

$$k_t + c(k)k_x = 0 \qquad (4')$$

$c(k)$:characteristic speed

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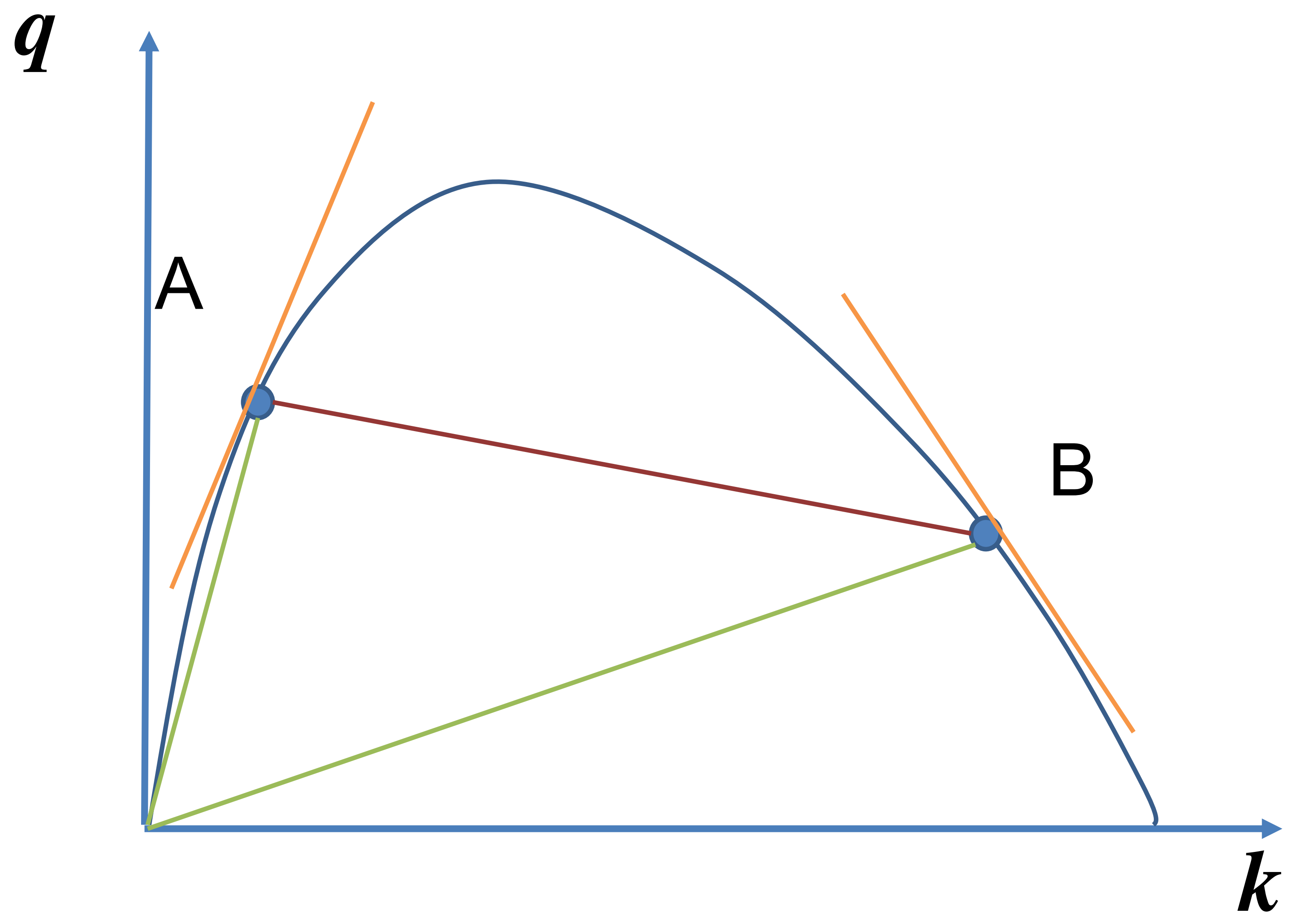

$$\frac{\partial k(x,t)}{\partial t} + f'(k)\frac{\partial k(x,t)}{\partial x} = 0 \quad (4)$$

$$k_t + c(k)k_x = 0 \quad (4')$$

Lemma: An observer that travels at speed $c(k)$ sees no change in density

Proof: $$\frac{dk(x,t)}{dt} = k_t + k_x\frac{dx}{dt} = k_t + k_x c(k) = 0.$$

$c(k)$:characteristic speed

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$$\dot{x}(t) = \frac{q_R - q_L}{k_R - k_L} \qquad (2)$$

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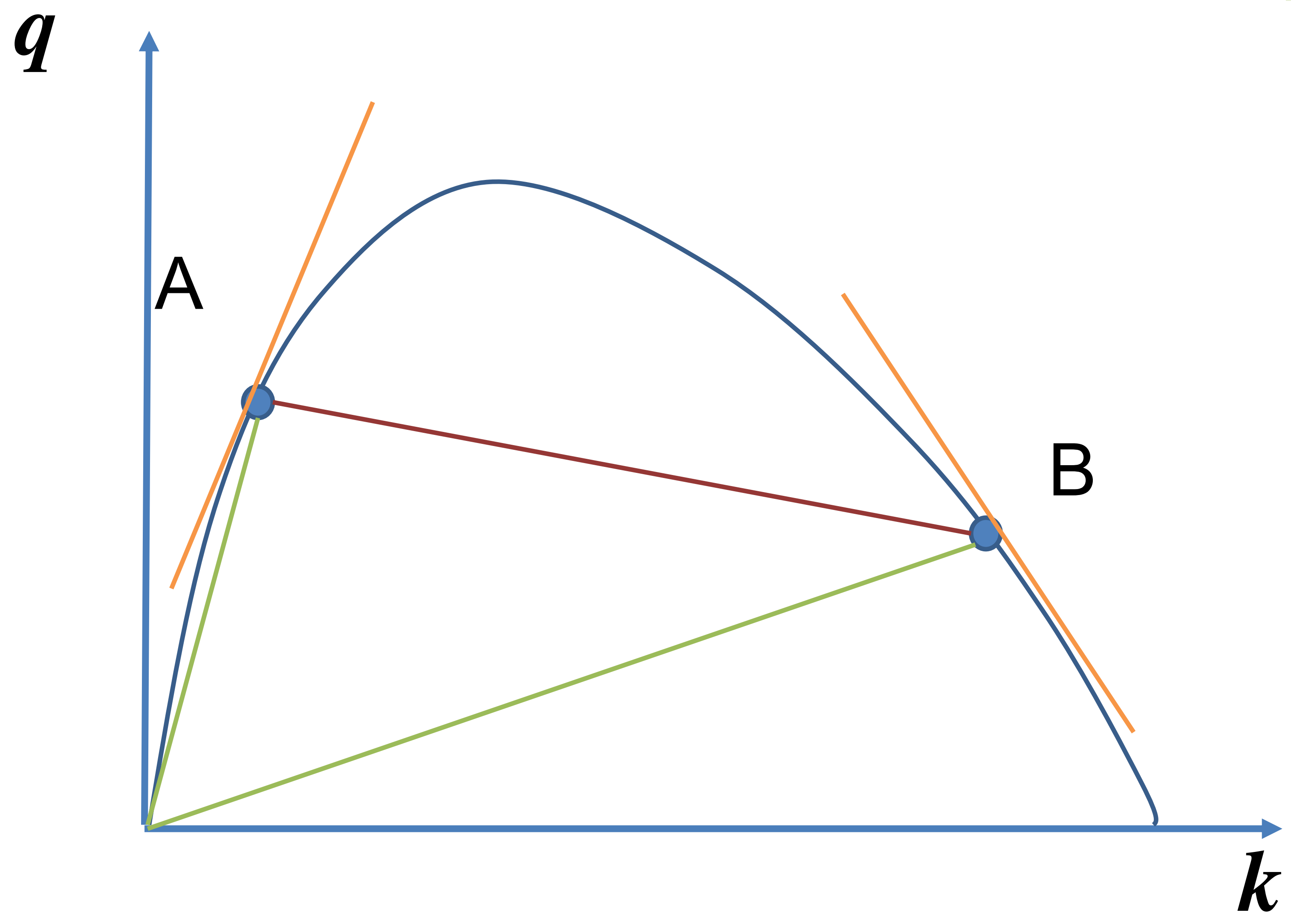
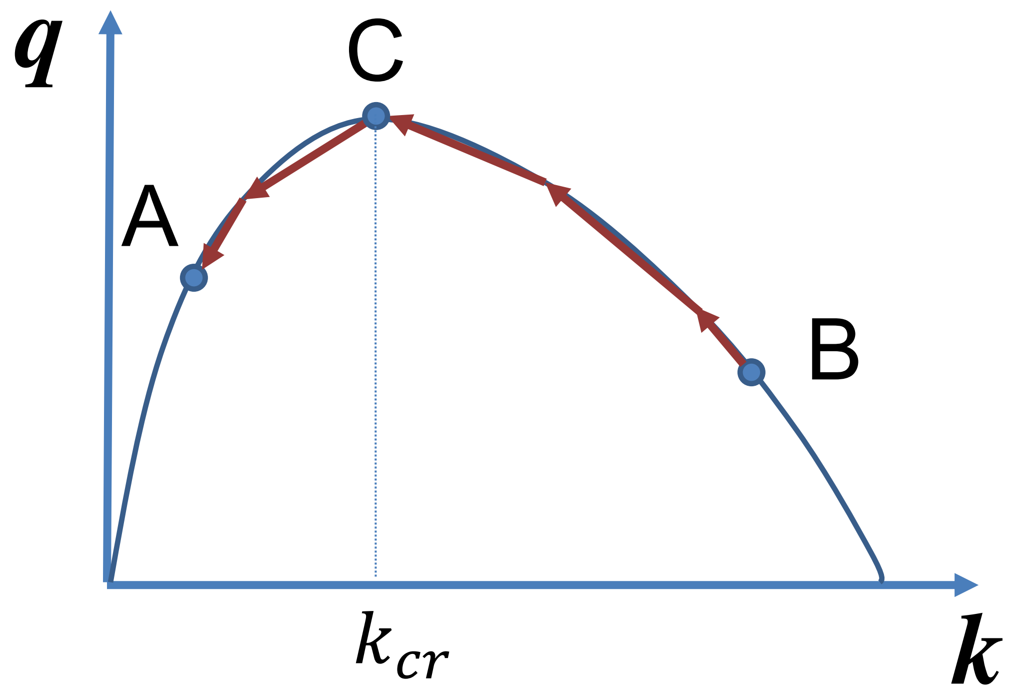
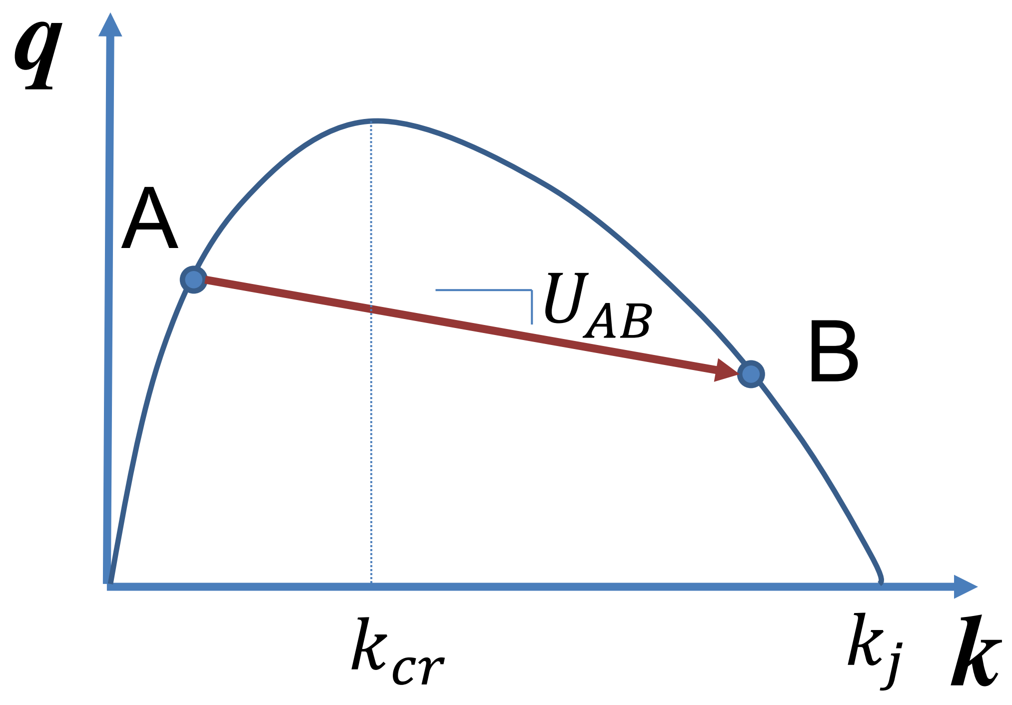
Entropy condition: If there is a decreasing density profile, there are multiple solutions. Entropy condition helps to choose a single solution. Flow is maximized for a given density, meaning that a path following FD states is chosen.

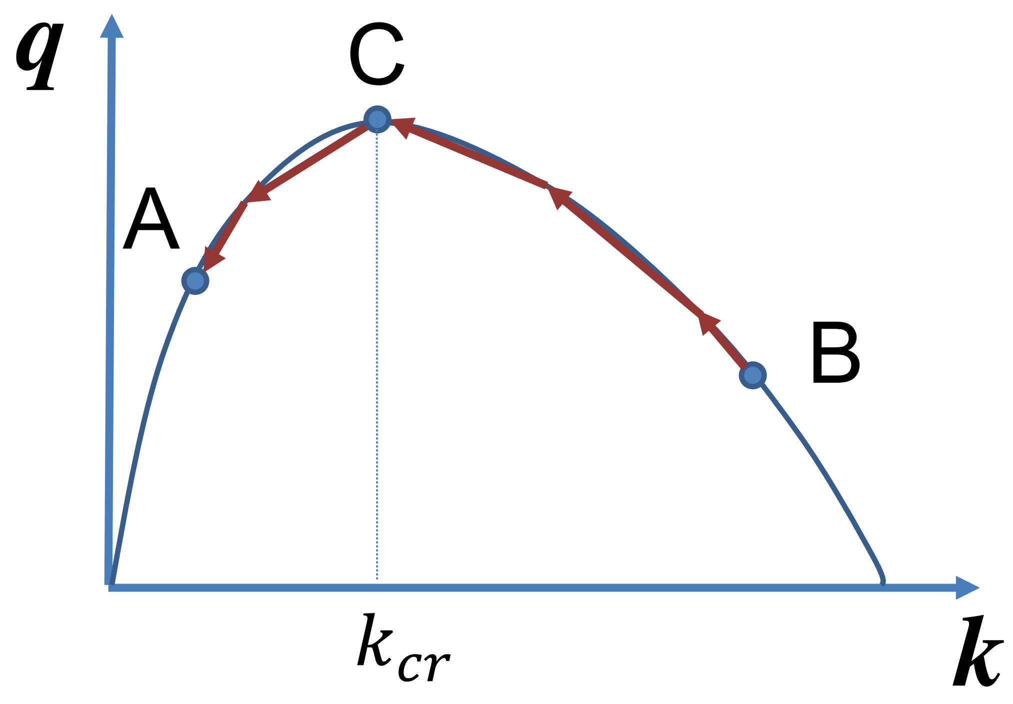


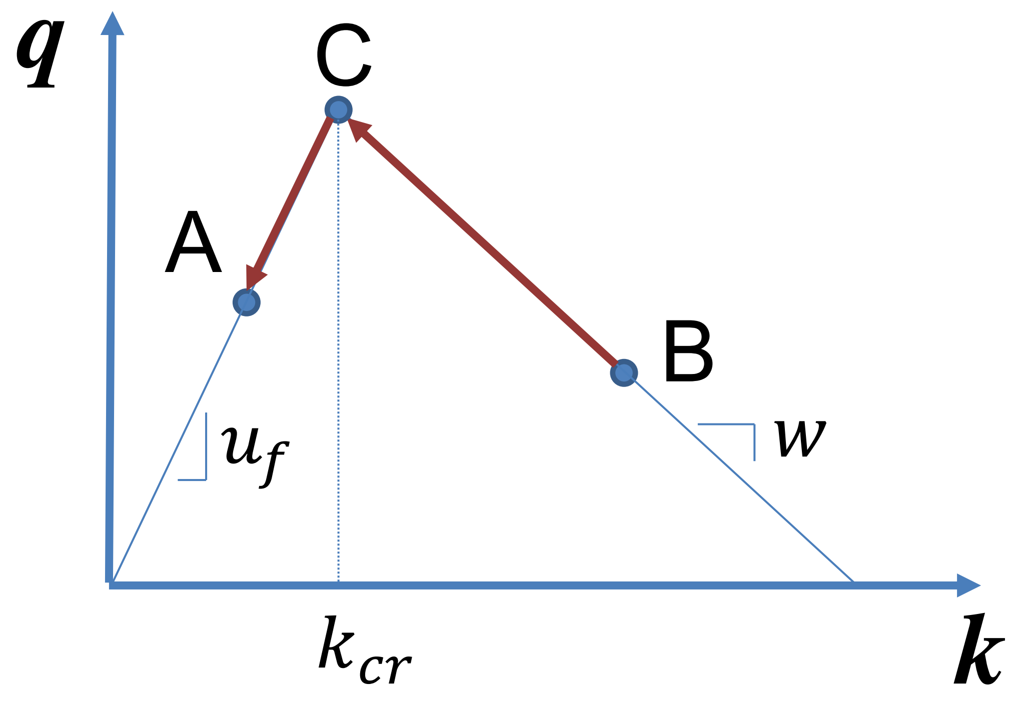


Principles of LWR theory

- All possible states that can be observed in a road segment should belong in a FD (stationary conditions)
- When a traffic stream is moving from A→B with $k_A < k_B$ it follows a linear interface between A and B in the FD
- When a traffic stream is moving from B→A with $k_A < k_B$ follows a path **on the FD** (entropy conditions)
- If additionally $k_A < k_{cr} < k_B$ then the path passes through capacity state C (queues at active bottlenecks discharge at capacity)
- The speed of a shockwave between two states is $U_{AB} = \frac{q_B - q_A}{k_B - k_A}$
- If FD is triangular, then from B→A, there are only characteristics with speeds u_f and/or w







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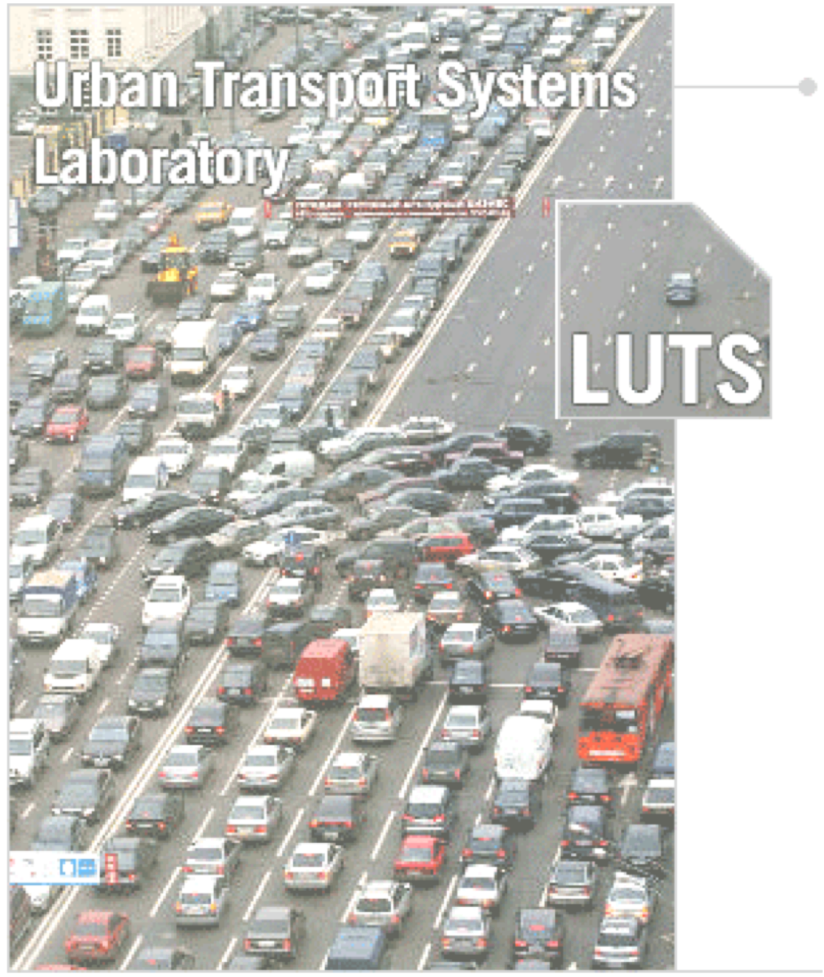
Traffic flow modeling

Examples of LWR Theory



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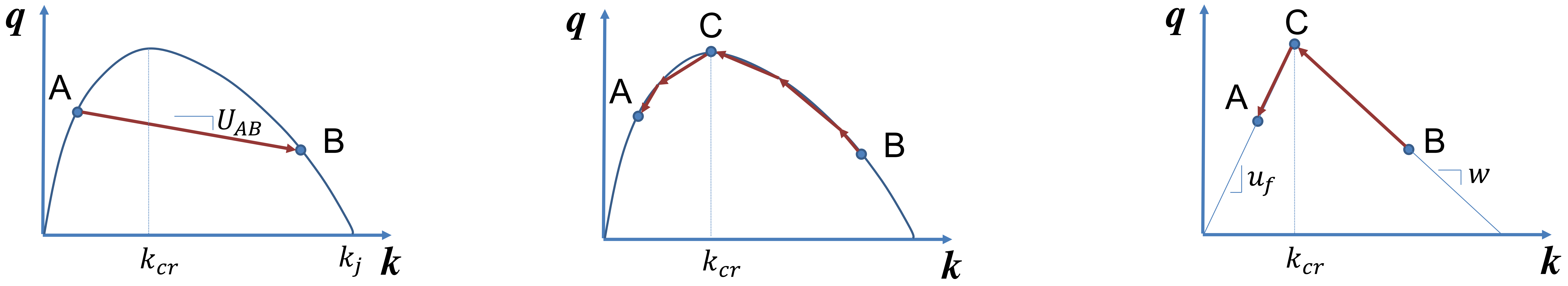
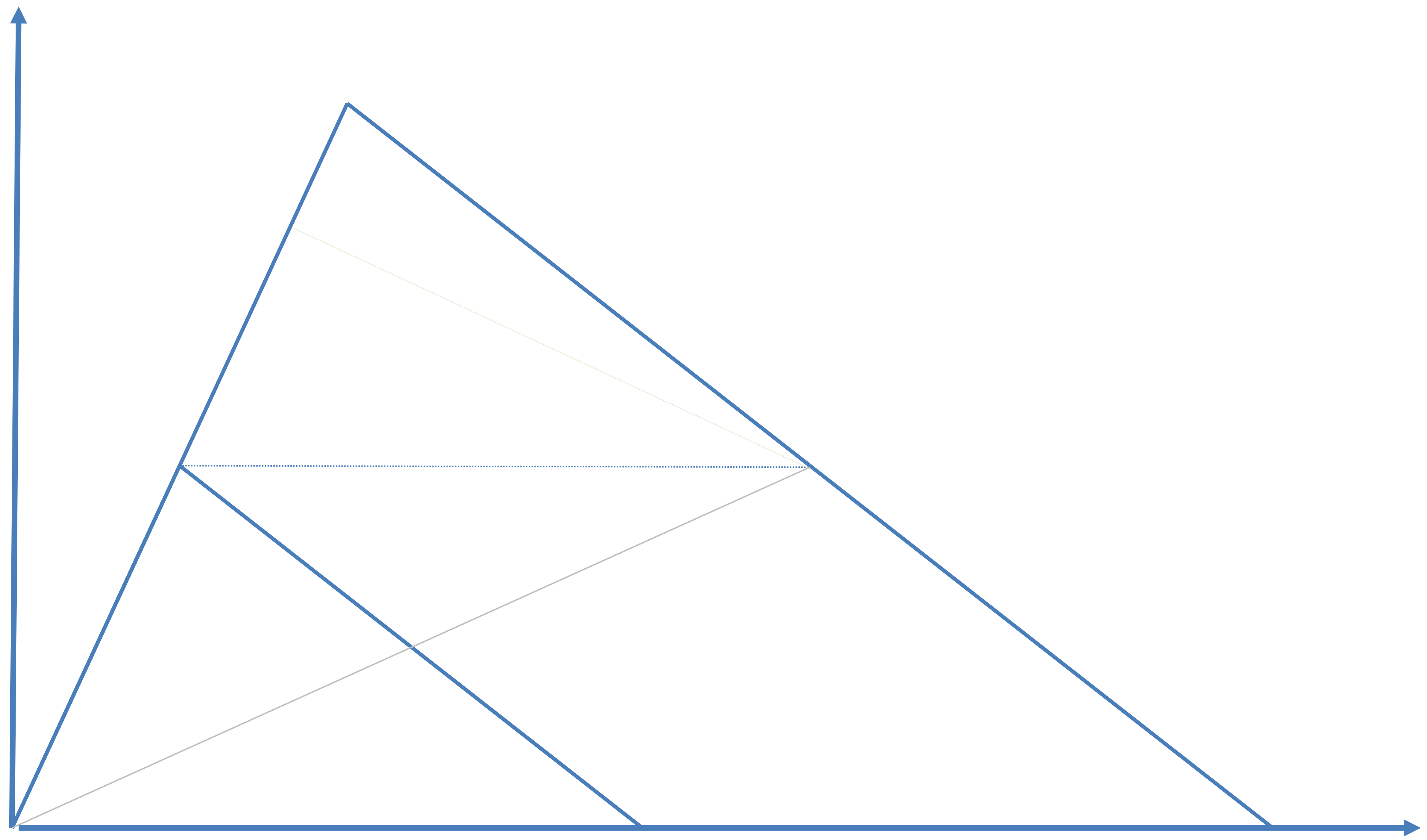


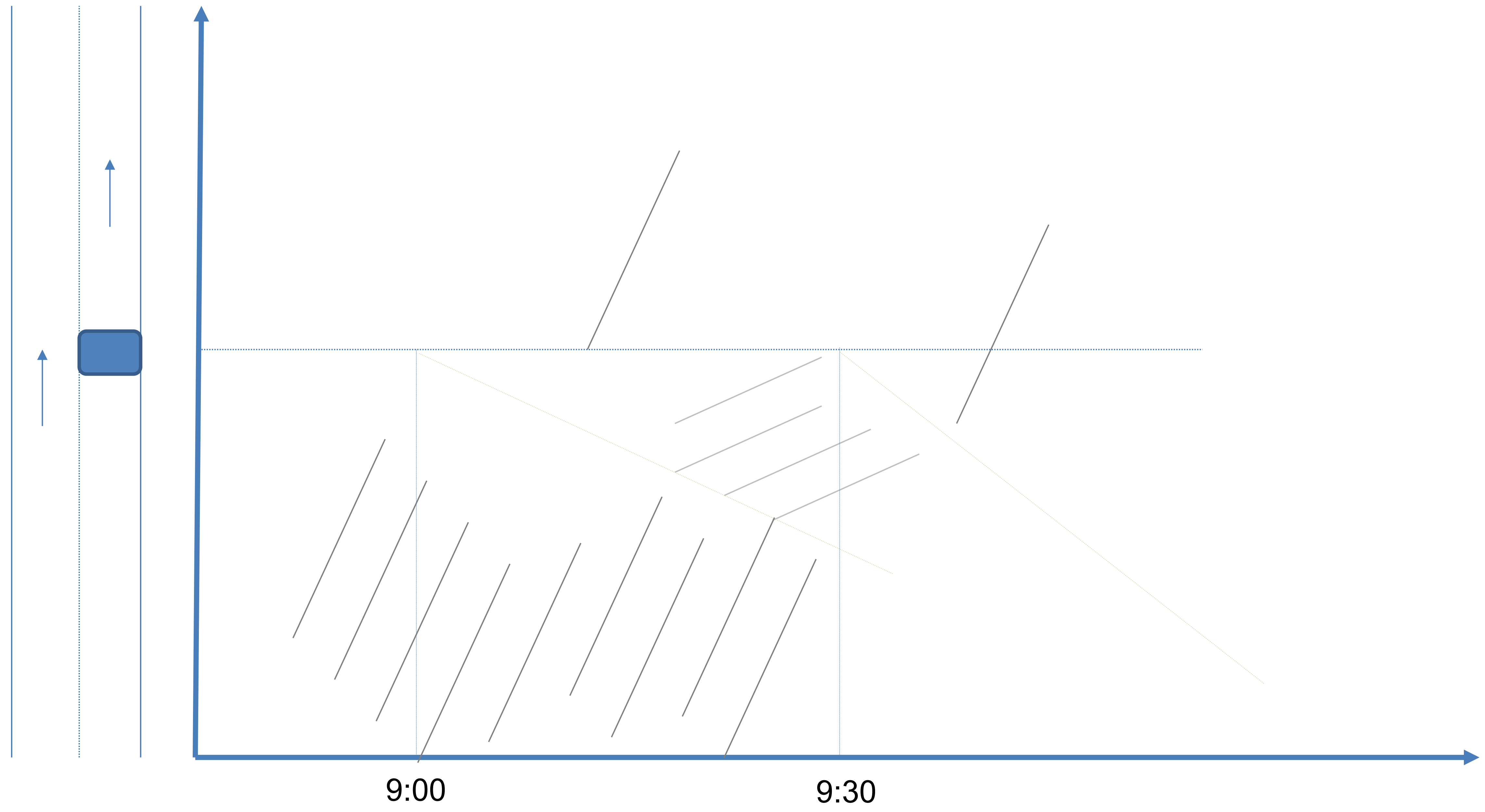
Example I: A freeway accident

Consider a two-lane freeway with a triangular FD with the characteristics below. Traffic is in the uncongested regime with flow q=0.8c. At 9am an accident happens that blocks one lane for 30min.

Estimate the maximum queue length.

What is the clearance time of the accident?

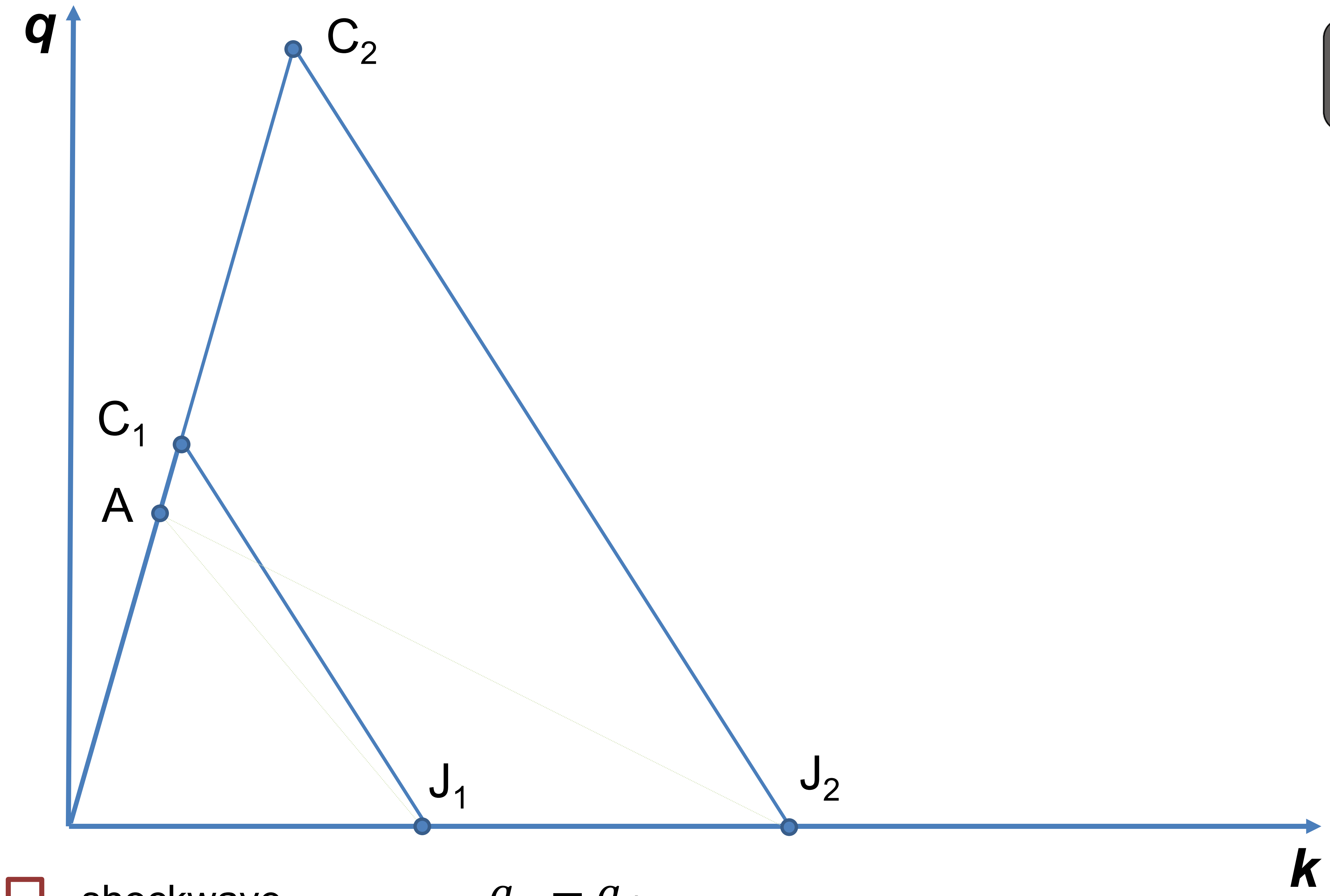


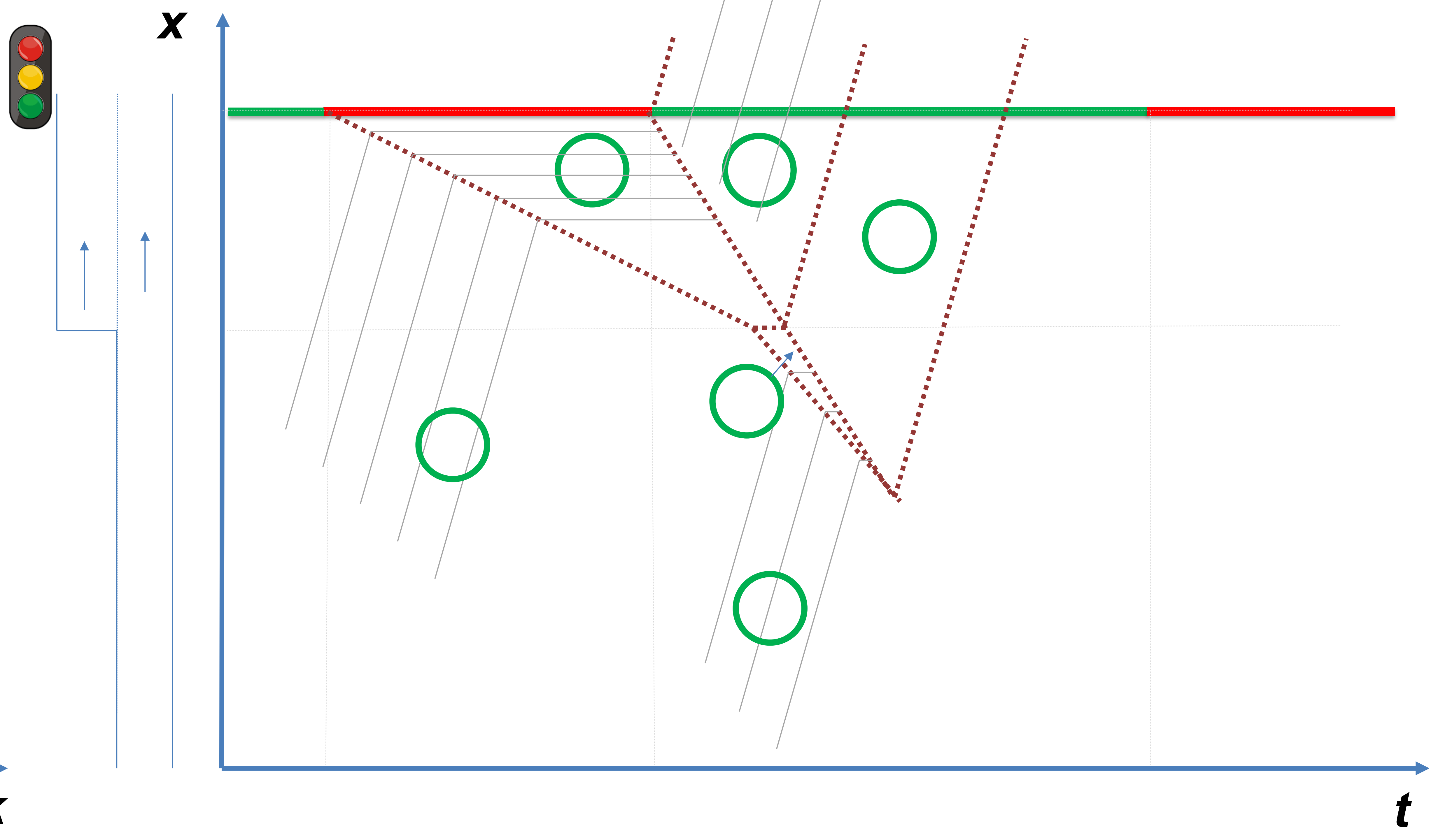


Example II: A complicated traffic light

Consider an arterial road with a traffic light (*R*=30sec, *G*=45sec). The road is one-lane but at distance L=150m upstream of the stop-line widens to two lanes. FDs are shown below. Vehicles approach the intersection from upstream at free-flow speed u_f and flow *q*=0.85*c* (*c* is the road capacity of one lane).

1. What is the length of the queue?
2. What is the maximum number of cars that can be served over one cycle for a larger *q* by choosing a proper L*?





shockwave

FD state

$$U_{AB} = \frac{q_B - q_A}{k_B - k_A}$$

LWR Theory - Summary