On the Utilisation and Pricing of Ride-splitting Vehicles in Bus Lanes

Fundamentals of Traffic Operations and Control

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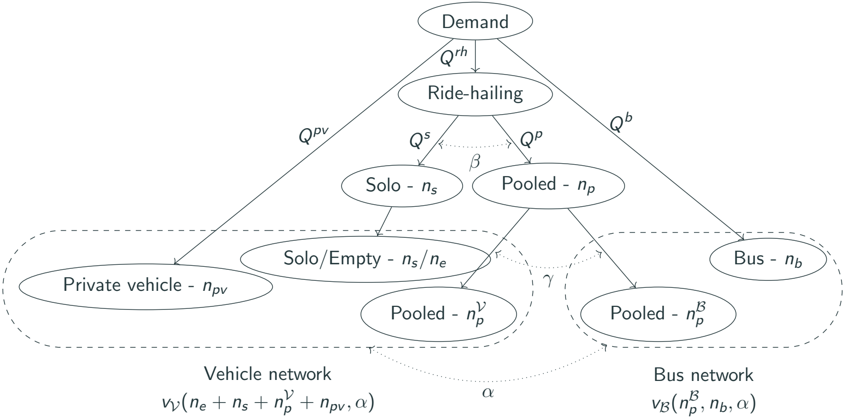
Introduction – Ride-Hailing Services and Congestion

Multi-Modal Networks and Ride-Hailing

- **Ride-hailing** (RH) generally increases **congestion** in urban areas.
- Despite the **low engagement** levels, **pooling** is one potential solution.
- Uneven distribution of **constrained network space** on available transport modes.
- **Underutilized capacity** due to existing spatial allocation strategies.

Can we, by allowing pool services on bus lanes, reduce delays for all commuters in the network?

Model



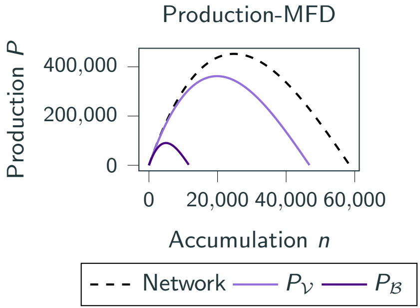
Demand: Q^{pv} – private vehicle, Q^{rh} – ride-hailing, Q^b – bus

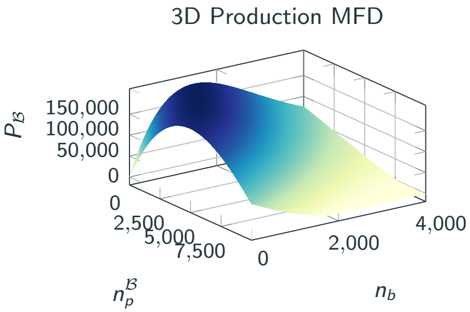
Control variables: α – split between networks, β – split between solo and pool rides, γ – split between pool vehicles

Speed: $v_{\mathcal{V}}$ – running speed in vehicle network, $v_{\mathcal{B}}$ – running speed in bus network

Aggregate Traffic Flow Dynamics - Production MFD

- MFD functions to compute the relationship between speed and accumulation.
- Partition network MFD into vehicle and bus networks based on α.

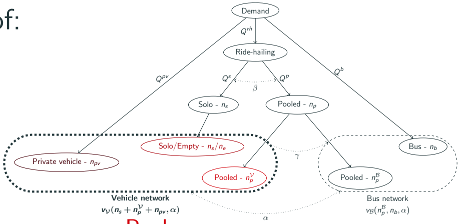




Vehicle network $\mathcal{V}$ – private vehicles n_{pv}, idle RH vehicles n_e, solo RH vehicles n_s, pooled RH vehicles $n_p^{\mathcal{V}}$
Bus network $\mathcal{B}$ – buses n_b with occupancy o_b, pooled RH vehicles $n_p^{\mathcal{B}}$

Accumulation in Vehicle Network

- Accumulation in the vehicle network $n_{\mathcal{V}}$ consists of:
  1. Idle/dispatching empty vehicles (n_e)
  2. Solo ride vehicles
  3. Private vehicles
  4. Pool ride vehicles



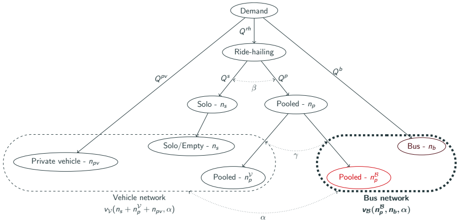
$$n_{\mathcal{V}} = \overbrace{I(d)}^{\text{Idle}} + \overbrace{\left(Q^s + \frac{1}{2}Q^p\right)\tau}^{\text{Dispatching}} + \overbrace{Q^s \frac{\bar{l}}{v_{\mathcal{V}}}}^{\text{Solo}} + \overbrace{Q^{pv}\frac{\bar{l}}{v_{\mathcal{V}}}}^{\text{Private}} + \overbrace{\frac{1}{2}(1-\gamma)Q^p\left(\frac{\bar{l} + \Delta l_d(Q^p)}{v_{\mathcal{V}}}\right)}^{\text{Pool}}$$

$\bar{l}$ – average trip length, τ – target waiting time, $d = \tau v_{\mathcal{V}}$ – dispatched distance

I – number of idling vehicles

- The higher the number of idling vehicles, the lower the d.
- The fleet size is endogenous and is function of Q^s, Q^p, and τ.

Accumulation in Bus Network



- Accumulation in the bus network $n_{\mathcal{B}}$ consists of:
  1. Pool ride vehicles

$$n_p^{\mathcal{B}} = \frac{1}{2}\gamma Q^p \left(\frac{\bar{l} + \Delta l_d(Q^p)}{v_{\mathcal{B}}} \right)$$

  2. Buses

$$n_b = \frac{Q^b \bar{l}_b}{o_b v_b}$$

- Average trip lengths:
  - $\bar{l}_b$ – average bus trip generally greater than $\bar{l}$
  - Δl_d – driver detour, Δl_p – passenger detour

$\Longrightarrow$ Half as many drivers required for pooling but trip lengths are at least equal to $\bar{l}$.

Defining the Driver and Passenger Detours

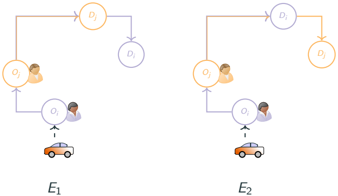
- The detour is a **decreasing** function of the pooling demand Q^p.
- Even with a trip of two passengers, deriving an analytical function is difficult.

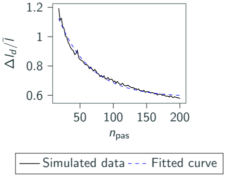
Batch pooling framework

$$\underset{z \in \{0,1\}^{n_{pas} \times n_{pas}}}{\text{minimize}} \quad \sum_i \sum_j c_{ij} z_{ij}$$

$$\text{subject to} \quad \sum_k (z_{ik} + z_{ki}) = 1 \quad \forall i \in n_{pas}$$

$$c_{ij} = \begin{cases} \min(E_1, E_2) & \text{if } i \neq j \\ +\infty & \text{if } i = j. \end{cases}$$





ω – batching time window, n_{pas} – number of passengers per batch where $n_{pas} = Q^p \omega$

z_{ij} – binary decision variable indicating if i is pooling with j, c – distance cost matrix

Solution Existence

For simplicity, assume $n_e = 0$, $n_p^{\mathcal{V}} = 0$, and $n_p = n_p^{\mathcal{B}}$. Also note that $v(n) = v_{\mathcal{V}}(\alpha n)$.

Vehicle network demand

$$(Q^s + Q^{pv})\bar{l}$$

Bus network demand

$$\frac{1}{2} Q^p(\bar{l} + \Delta l_d(Q^p))$$

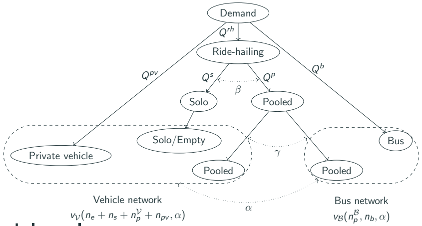
Finding an equilibrium for $n_{\mathcal{V}}$ and n_p for a given Q^s and Q^p where $Q^{rh} = Q^s + Q^p$ requires to simultaneously find solutions for:

- $P_{\mathcal{V}} = n_{\mathcal{V}} v_{\mathcal{V}}(n_{\mathcal{V}}) = (Q^s + Q^{pv})\bar{l}$
- $P_{\mathcal{B}} = n_p v_{\mathcal{B}}(n_p, n_b) = \frac{1}{2} Q^p(\bar{l} + \Delta l_d(Q^p))$

Proposition

For a concave network MFD, the vehicle accumulation $n_{\mathcal{V}}$ in the vehicle network and the pool vehicle accumulation n_p have at most two solutions at equilibrium depending on the MFD shapes $P_{\mathcal{V}}$ and $P_{\mathcal{B}}$.

Pros and Cons of Allowing Pool Vehicles in Bus Lanes



Pros

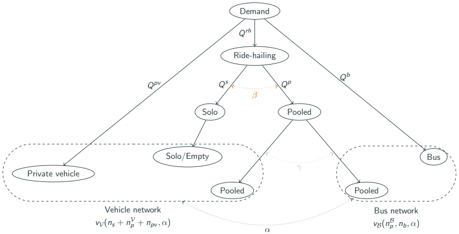
- Allows pool passengers to travel faster.
- Reduces driver and passenger detours.
- Requires a smaller fleet size to serve the same demand level.
- Leads to a more efficient distribution of network space.

Cons

- Buses become slower.

$\Longrightarrow$ Hence the need to optimize network space for multi-modal transport users.

System Optimum



Objective: Minimize network delays for all commuters

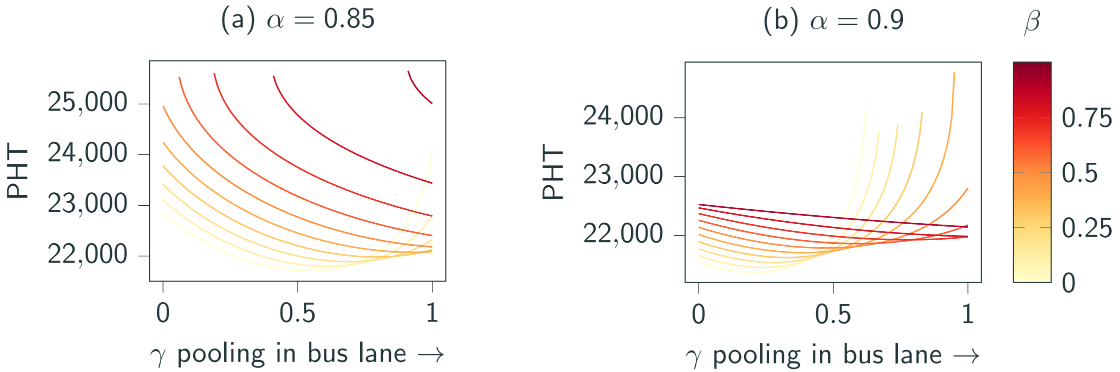
- Decision variables: β and γ
- α is an exogenous variable
- Speeds depend on $n_{\mathcal{V}}$ and $n_{\mathcal{B}}$
- Delays represented here by Passenger Hours Traveled (PHT)

$$\underset{\beta\in[0,1],\gamma\in[0,1]}{\text{minimize}} \quad Q^{pv}\frac{\bar{l}}{v_{\mathcal{V}}} + Q^{s}\frac{\bar{l}}{v_{\mathcal{V}}} + (1-\gamma)Q^{p}\left(\frac{\bar{l}+\Delta l_p(Q^p)}{v_{\mathcal{V}}}\right) + \gamma Q^{p}\left(\frac{\bar{l}+\Delta l_p(Q^p)}{v_{\mathcal{B}}}\right) + Q^{b}\frac{\bar{l}_b}{v_b},$$

where $Q^s = \beta Q^{rh}$ and $Q^p = (1-\beta)Q^{rh}$.

Results for Different Network Spatial Splits α

PHT for $\gamma \in [\mathbf{0}, \mathbf{1}]$



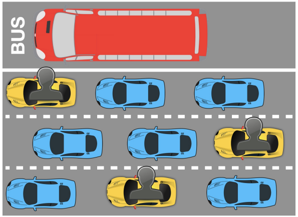
- For high values of β, the value of γ that minimizes the PHTs is equal to 1.

Static Modelling Outcomes

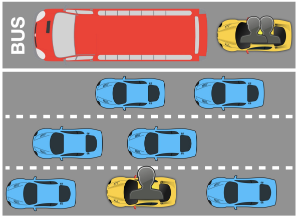
So far, we have seen through the **static macroscopic model** that:

- Allowing **pooled vehicles in bus lanes** improves network delays.
- Without any pricing schemes, this policy can potentially **slow down bus users**.

No pooling on bus lanes – **Congestion**

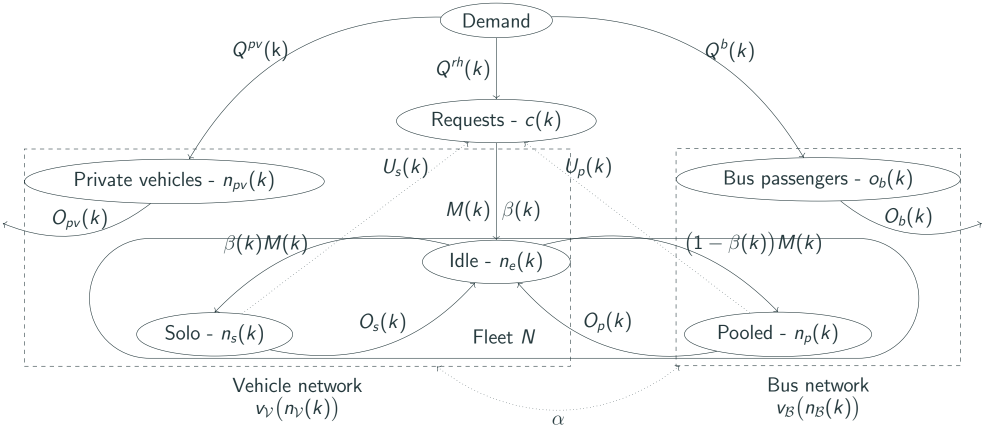


Pooling on bus lanes – **Bus delays**



How to develop and test this pricing scheme? For that we move to the **non-equilibrium dynamic** realm.

Dynamic Non-equilibrium Model Summary



O_i – trip completion rate of category i, M – matching rate, β – solo fraction

U_s – utility for solo trips, U_p – utility for pooled trips, α – spatial split

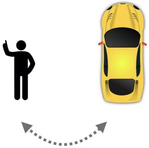
N – fleet size, k – time step

Mode Choice and Matching

Next, we define the **fractional split** between solo and pool $\boldsymbol{\beta}$ and the **matching rate** $\boldsymbol{M}$.

- Matching function – Meeting rate between passengers and vehicles

$$M(k) = a_0 n_e(k)^{\alpha_e} \left(c_s(k) + \frac{1}{2} c_p(k) \right)^{\alpha_c}$$



a_0, α_e, α_c – parameters for Cobb Douglas meeting function

c_s, c_p – number of solo and pool passengers respectively both function of c

- Queue – Number of passengers waiting to be assigned

$$c(k) = c(k-1) + \Delta[Q^{rh}(k) + (\beta(k-1) - 2) M(k-1)]$$

Therefore, $c_s(k) = \beta(k)c(k)$ and $c_p(k) = (1 - \beta(k))c(k)$

Δ – discretized time step length

Next, we define the **fractional split** between solo and pool β and the **matching rate** $\boldsymbol{M}$.

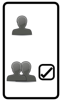
- Disutilities – Consisting of service fare and travel time

$$U_s(k) = F_s + \kappa \frac{\bar{l}_s}{v_{\mathcal{V}}(k)} \qquad U_p(k) = F_p + \phi(k) + \kappa \frac{\bar{l}_s + \Delta l_p}{v_{\mathcal{B}}(k)}$$

$F_s > 0$, $F_p > 0$ – platform fare for solo and pool, ϕ – pooling discount/toll

Δl_p – passenger detour, v_p – pool vehicles speed

- Logit model – Determining the fraction of solo passengers



$$\beta(k) = \frac{e^{-\mu U_s(k)}}{e^{-\mu U_s(k)} + e^{-\mu U_p(k)}}$$

$\mu > 0$ – scale parameter

Network Dynamics - Private Vehicles and Buses

Private vehicle and bus dynamics: We display below the discretized dynamics using a time step Δ.

- **Private vehicles**

$$n_{pv}(k) = n_{pv}(k-1) + \Delta\left[\frac{Q^{pv}(k)}{\bar{o}_{pv}} - \underbrace{\frac{n_{pv}(k-1)}{n_{\mathcal{V}}(k-1)}\frac{P_{\mathcal{V}}(n_{\mathcal{V}}(k-1))}{\bar{l}_{pv}}}_{O_{pv}(k)}\right]$$

$$Q^{pv} \longrightarrow \boxed{\text{Private vehicles}} \longrightarrow O_{pv}$$

$\bar{o}_{pv}, \bar{l}_{pv}$ – average private vehicle occupancy and trip length respectively, $n_{\mathcal{V}}$ – total vehicle accumulation in $\mathcal{V}$.

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- **Average bus occupancy**

$$o_b(k) = o_b(k-1) + \Delta \frac{1}{n_b} \left[Q^b(k) - \underbrace{\frac{P_b(n_p(k-1), n_b)}{\bar{l}_b} o_b(k-1)}_{O_b(k)} \right]$$

$$Q^b \longrightarrow \boxed{\text{Bus passengers}} \longrightarrow O_b$$

P_b – bus production, $\bar{l}_b$ – average bus trip length

Network Dynamics - Ride-Hailing Vehicles

Ride-hailing dynamics: Ride-hailing dictates the traffic dynamics for all categories, including buses and private vehicles.

- **Solo vehicles**

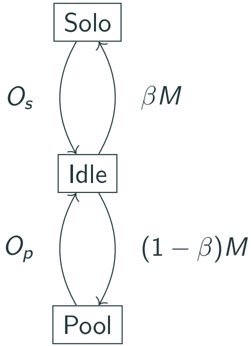
$$n_s(k) = n_s(k-1) + \Delta \left[\beta(k-1)M(k-1) - \underbrace{\frac{n_s(k-1)}{n_{\mathcal{V}}(k-1)} \frac{P_{\mathcal{V}}\left(n_{\mathcal{V}}(k-1)\right)}{\bar{l}_s}}_{O_s(k)} \right]$$

- **Idle vehicles**

$$n_e(k) = n_e(k-1) + \Delta \left[O_s(k) + O_p(k) - M(k-1) \right]$$

- **Pooled vehicles**

$$n_p(k) = n_p(k-1) + \Delta \left[\left(1-\beta(k-1)\right)M(k-1) - \underbrace{\frac{P_{\mathcal{B}}\left(n_p(k-1), n_b\right)}{\bar{l}_s + \Delta l_d}}_{O_p(k)} \right]$$



$P_{\mathcal{B}}$ – pooling vehicles production, $\bar{l}_s$ – average solo trip length, Δl_d – driver detour

Controllers – PI Controller

The aim of implementing a PI controller is to limit bus delays due to pooling vehicles.

PI Controller

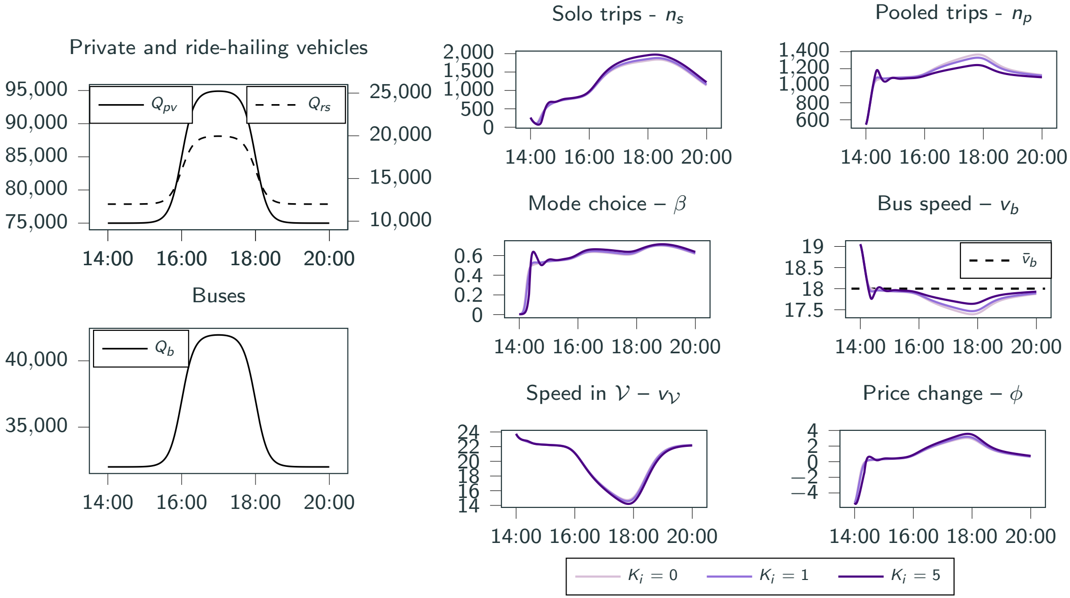
- **Objective:** Minimize bus speed gap δ between target and actual bus speed such that $\delta(k) = \bar{v}_b(k) - v_b(k)$
- **Control variable**: $\phi(k)$

$$\phi(k) = K_p \delta(k) + \frac{K_i}{N_e} \sum_{\tilde{k}=\max(k-(N_e+1),0)}^{k-1} \delta(k)$$

$K_p > 0$, $K_i > 0$ – proportional and integral gains, $N_e \in \mathcal{N}$ – accumulation history for the integral

ϕ here dictates the additional discount/toll that pool users will benefit from/incur to maintain the bus speed at $\bar{v}_b$.

Results – PI Controller



In this work,

- We analyze how, by giving **pooled ride-hailing vehicles** access to dedicated bus lanes, we can improve the performance of the transportation network.
- We set forward a **pricing scheme** that we test in an non-equilibrium dynamic model for the purpose of limiting delays in bus lanes.

Related area of research: An occupancy-based differential pricing scheme based on ride-hailing vehicles occupancies for occupancy greater than 2.