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In-class Exercise – Week #10: Numerical integration in beam-column elements

The tapered beam shown in Figure 1 has a linear elastic material. The material modulus of elasticity E is constant. The beam depth changes linearly from $2d$ at the fixed support to d at the tip. The beam width, b, is constant. This beam is analyzed with a single displacement-based beam-column element with two nodes. The left note is *Node-i* and the right node is *Node-j* as shown in the figure.

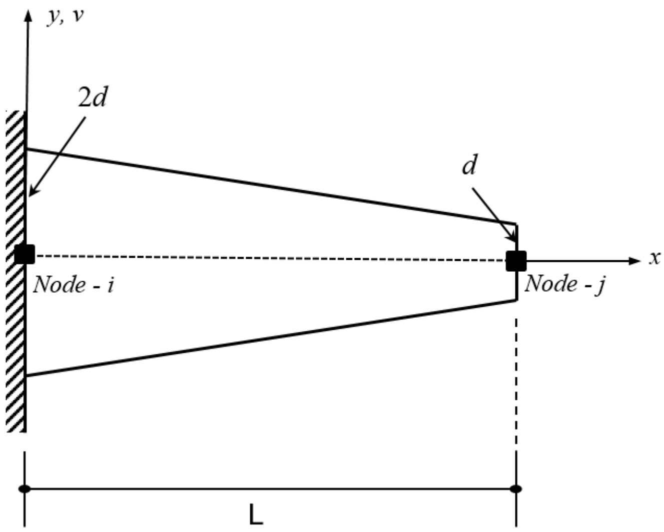

Figure 1. tapered element

The transverse displacement field $v(x)$ along the beam is approximated by the Euler-Bernoulli beam theory assumptions that we discussed in class and,

$$v(x) = \left[3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3\right] v_j + \left[\frac{x^3}{L^2} - \frac{x^2}{L}\right] \theta_j \tag{1}$$

From the above displacement field, the curvature field $k(x)$ can be calculated to get the following transformation matrix,

$$k(x) = \mathbf{B}(\mathbf{x}) \cdot \begin{Bmatrix} v_j \\ \theta_j \end{Bmatrix} \tag{2}$$

With the use of the principle of virtual displacement method, the resulting stiffness matrix of the element is 2x2 and can be calculated as:

$$\mathbf{k} = \int_0^L [\mathbf{B}(\mathbf{x})]^T \mathbf{k}_s(x) \, [\mathbf{B}(\mathbf{x})] dx \qquad (3)$$

Where $\mathbf{k}_s(x) = EI(x)$ is the section stiffness matrix.

Answer to the following questions:

1. What type of numerical integration method do you propose in order to calculate the above stiffness matrix "numerically exact"? How many integration points should be used with this method and explain why?
2. Calculate the "numerically exact" stiffness matrix.
3. Is this stiffness matrix the "theoretically exact" stiffness matrix for this tapered beam? Explain your reasoning.