EPFL

CIVIL 449: Nonlinear Analysis of Structures

School of Architecture, Civil & Environmental Engineering
Civil Engineering Institute

Distributed plasticity – Fiber-based elements
Constitutive formulations for construction materials

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Objectives of today's lecture

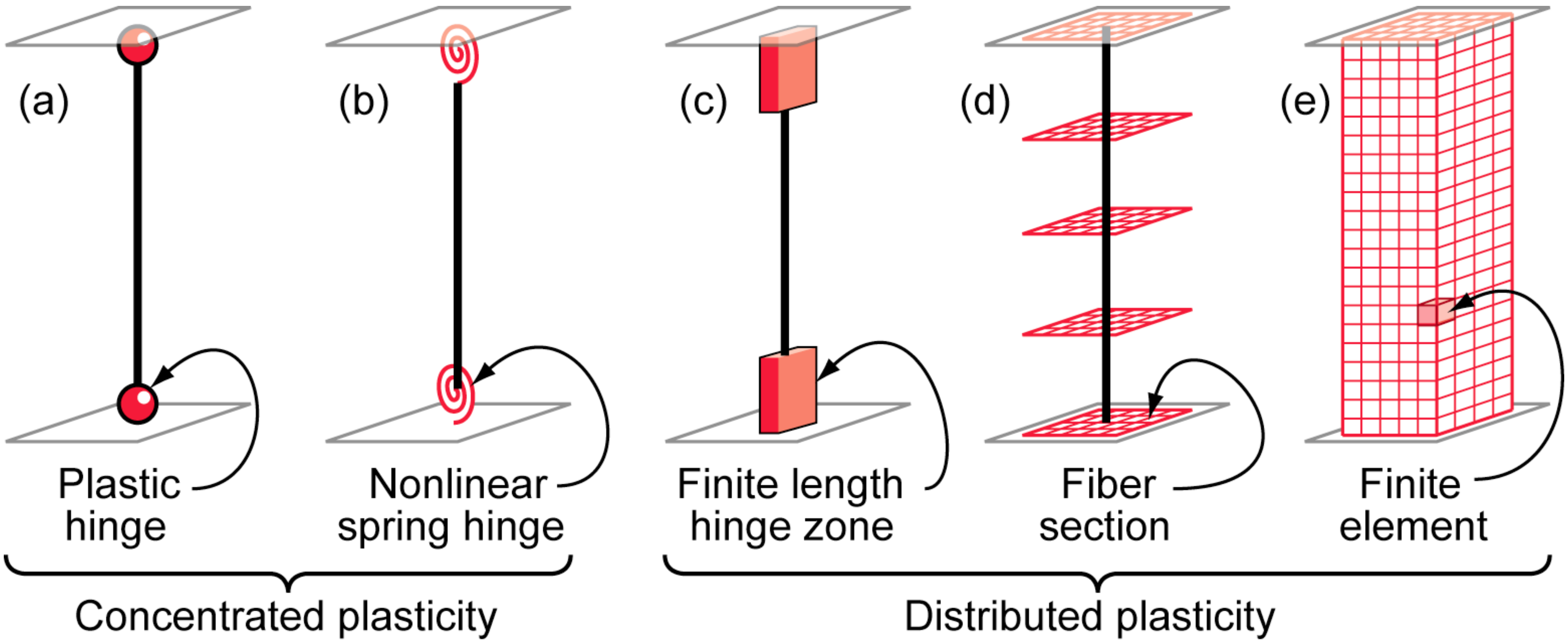
To introduce:

- Fiber-based beam-column elements
- Fiber discretization of cross sections
- Constitutive models for fiber-based elements

This week's material

- Computation of input strains
- Section analysis
- Type of element formulations
  - Displacement-based beam-column elements
  - Force-based beam-column elements
- Integration methods for member forces and member stiffness

Frame element formulations



Source: NIST GSR 10-917-5

Limitations of concentrated plasticity models

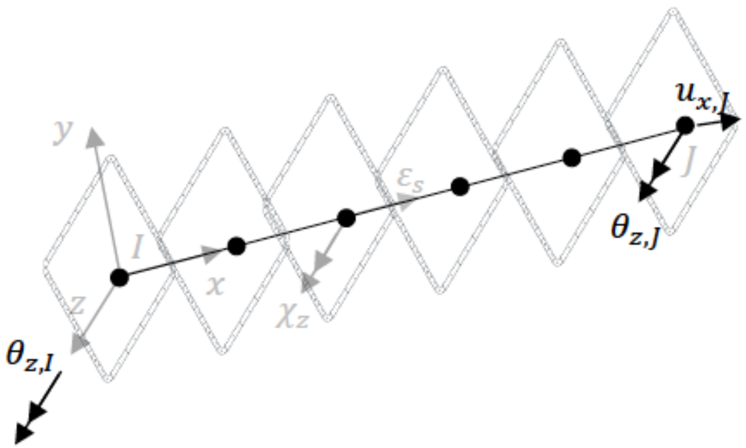
- Typically, axial-flexural interaction (coupling) is neglected
  - Important for columns: axial load demands fluctuate during seismic events
- Rate effects are typically neglected
- Spread of plasticity is not captured

- Input model parameters for zero-length elements rely on datasets
  - Sensitive to loading history especially at large inelastic deformations (incipient collapse)
  - Damage predictions at the local level (if strains are of interest) may be grossly inaccurate

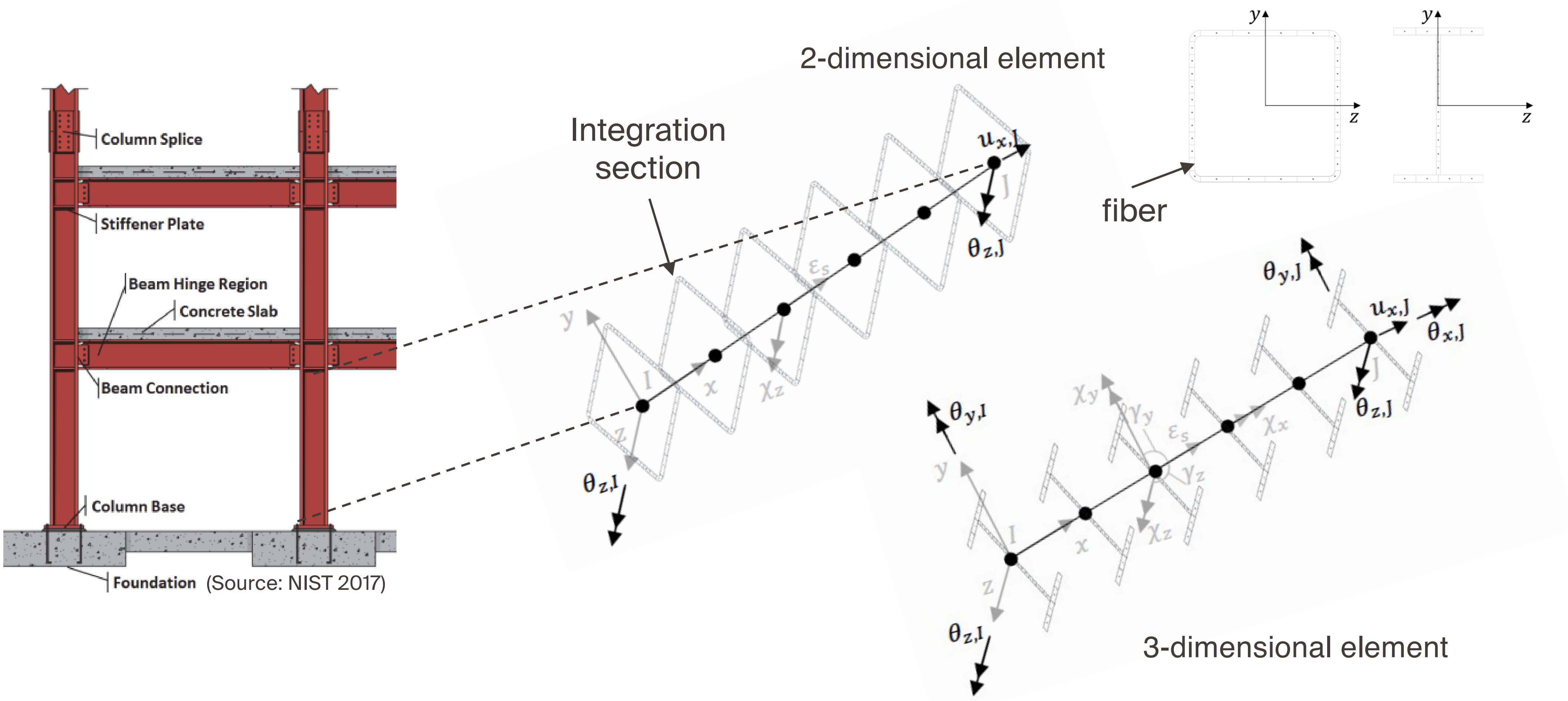
- Softening (deterioration) is a challenge for combined actions (axial load and flexure) for RC and steel members
- Member instabilities (lateral torsional buckling) cannot be explicitly captured

Fiber-based beam-column elements

- Material nonlinearity can take place at any section through stress-strain constitutive laws
- The element behavior is derived by weighted integration of “selected” section responses
- Either the element deformations or the element forces are the model unknowns
- The constitutive behavior of the cross section is derived by discretization of the cross section into fibers
- Plane sections remain plane (strains are linearly distributed over the cross section)

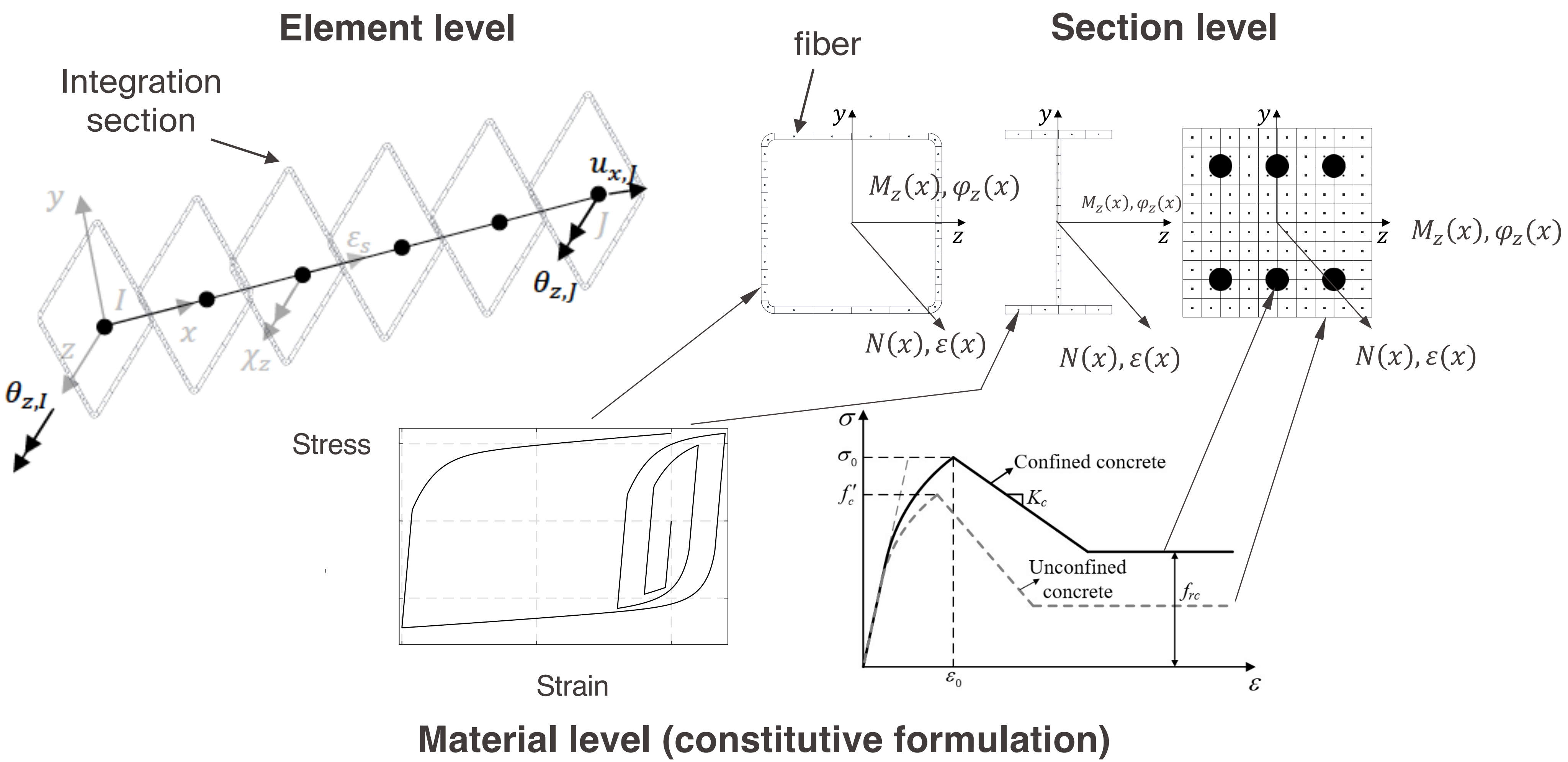


Fiber-based beam-column elements (2)



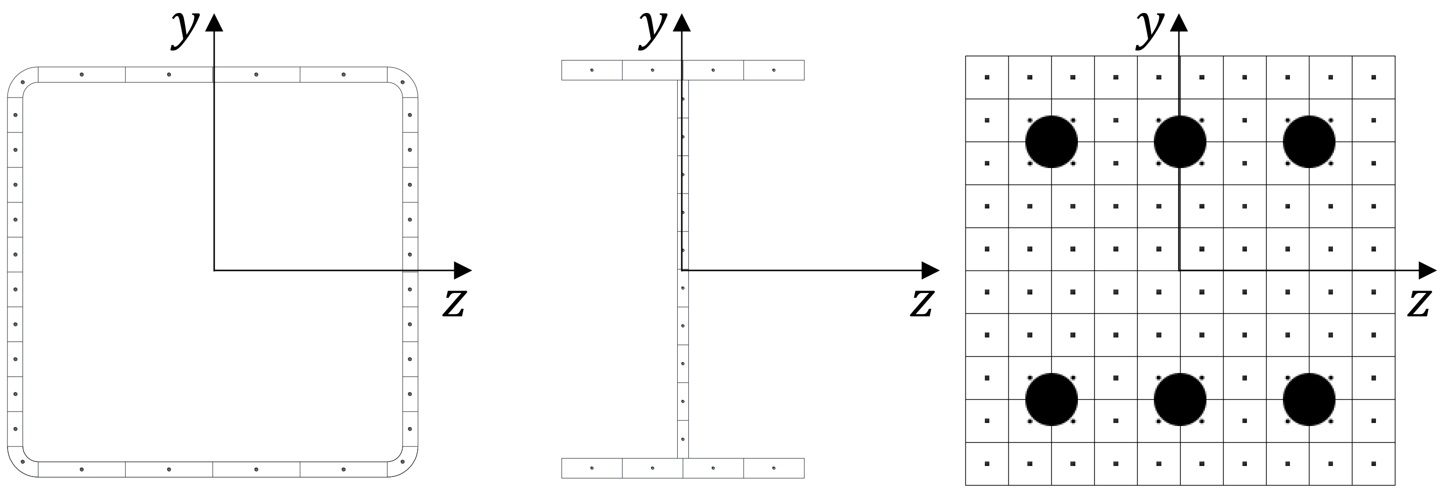
Source Heredia, de Castro e Sousa and Lignos (2024)

Fiber-based beam-column elements: basic workflow



Fiber section discretization

- A cross section is meshed into smaller blocks called fibers.
- The strain of each fiber is determined by the section deformation vector $\mathbf{d}^s(x)$ and its coordinates (y_{fiber}, z_{fiber}).

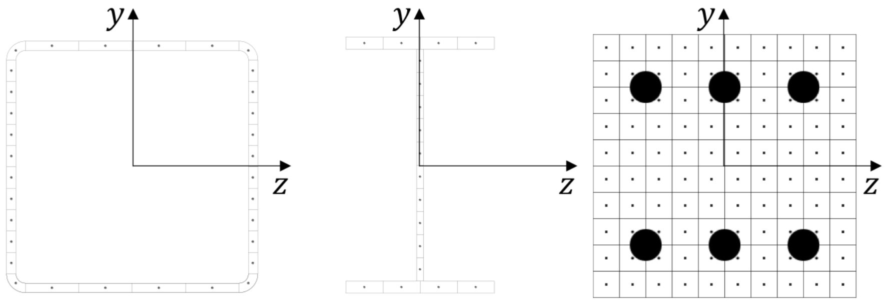


State determination of fiber section

- The section deformations at the corresponding integration point of an element,

$$\mathbf{d}^s(x) = \{\varepsilon(x) \quad \varphi_z(x)\}^T$$

- A section is meshed into smaller blocks called fibers.



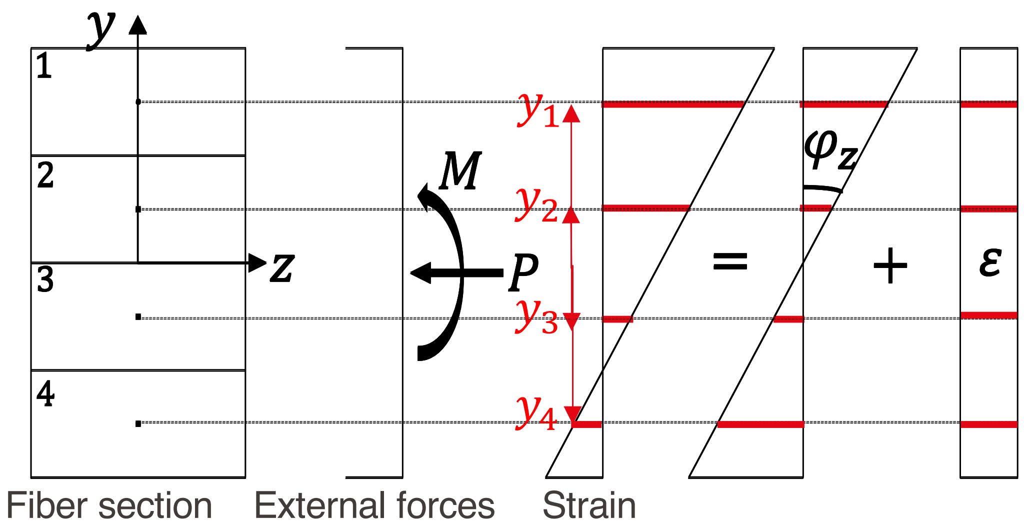
- The strain of each fiber is determined by section deformation vector $\mathbf{d}^s(x)$ and its coordinates (y_{fiber}, z_{fiber}) along the section.
- The strain of the *k*-th fiber in a cross section,

$$\varepsilon_{k.fiber} = \varepsilon(x) + \varphi_z(x) y_{k.fiber} = \{1, y_{k.fiber}\} \cdot \begin{Bmatrix} \varepsilon(x) \\ \varphi_z(x) \end{Bmatrix} \quad \text{(2-d element)}$$

$$\varepsilon_{k.fiber} = \varepsilon(x) + \varphi_z(x) y_{k.fiber} + \varphi_y(x) z_{k.fiber} = \{1, y_{k.fiber}, z_{k.fiber}\} \cdot \begin{Bmatrix} \varepsilon(x) \\ \varphi_z(x) \\ \varphi_y(x) \end{Bmatrix} \quad \text{(3-d element)}$$

State determination of fiber section (2)

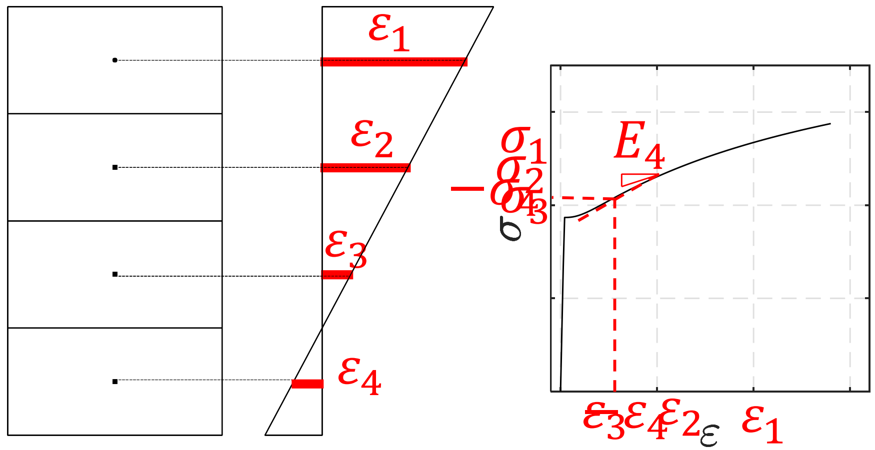
- Start by computing the strain of all fibers in the cross-section:



$$\varepsilon_{fiber1} = \varepsilon(x) + \varphi_z(x) y_1$$

State determination of fiber section (3)

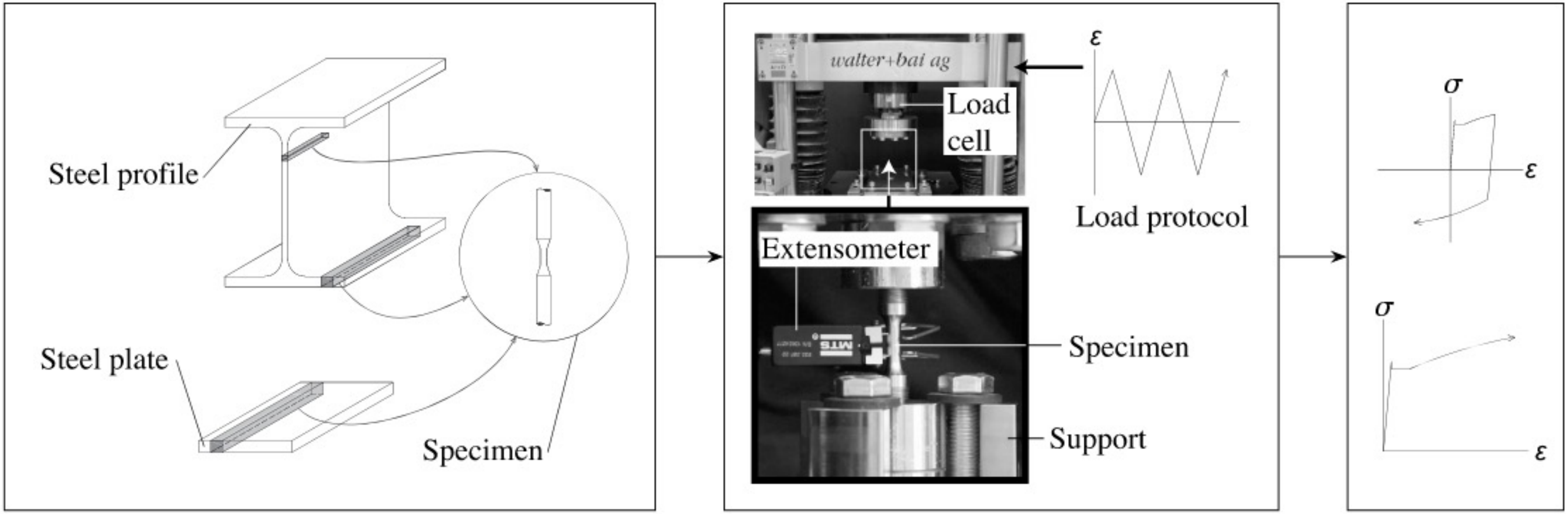
- Based on the material constitutive formulation and the fiber strain, the tangent modulus $E_{k.fiber}$ and stress, $\sigma_{k.fiber}$ of the *k*-th fiber can be determined.
- $E_{k.fiber}$ and $\sigma_{k.fiber}$ are used to compute the section stiffness and forces.



Material constitutive formulations

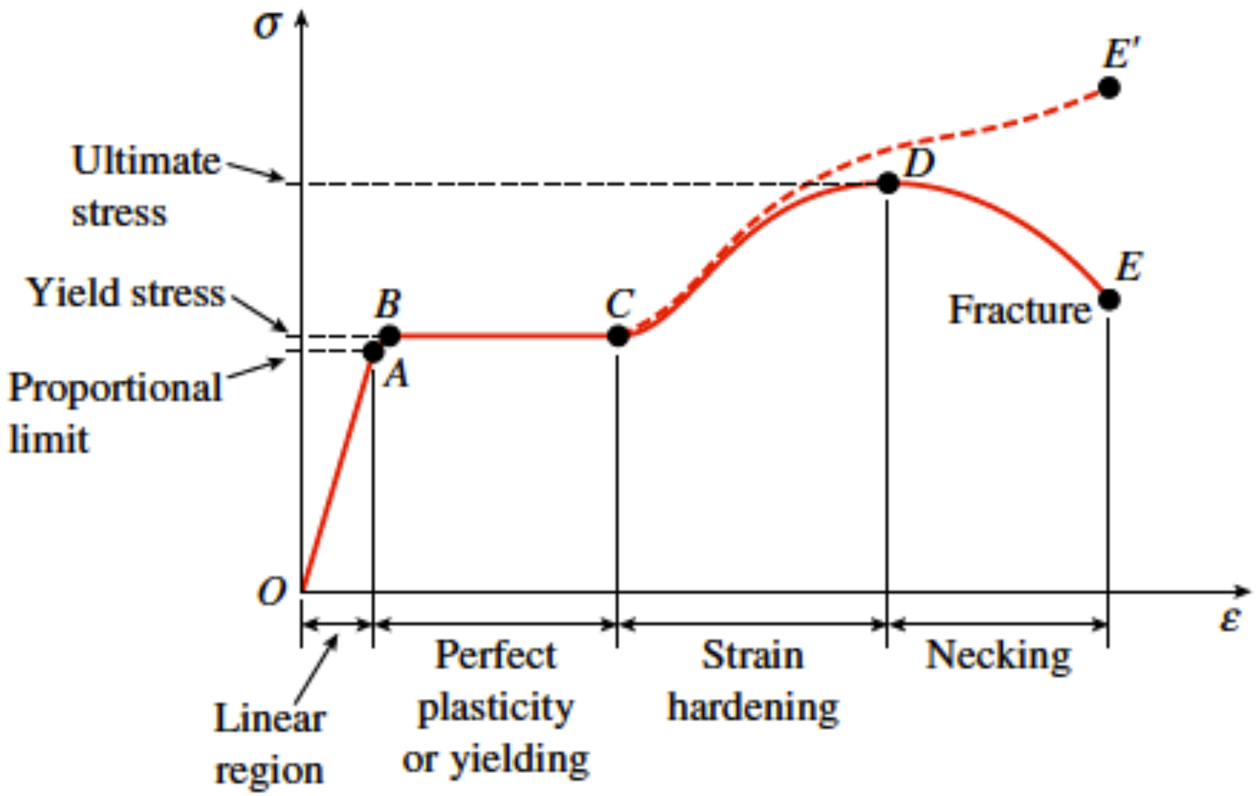
- They follow the basic material response we know from mechanics of materials
- “Usually” implies that we often use artifacts to mimic damaging phenomena (e.g., softening) through the material constitutive formulation
- We will focus on steel and concrete materials in this course
- We will be concerned with material response at room temperature (i.e., 21^{o}C)
- We will mostly discuss uniaxial material response though a multiaxial stress state is common

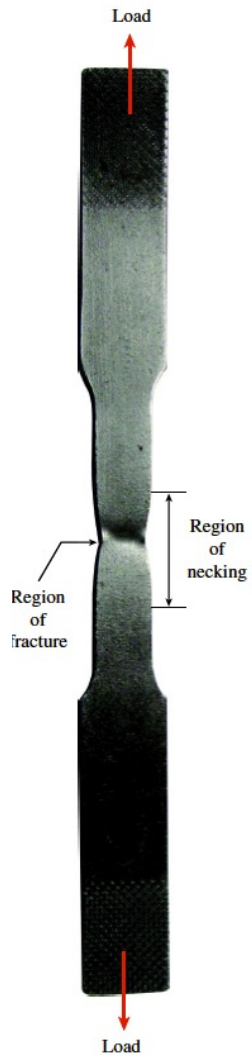
Material constitutive formulations
-Structural steel



Source: Hartloper, Ozden, de Castro e Sousa, Lignos (2023)

Basic mechanical behavior of structural steel

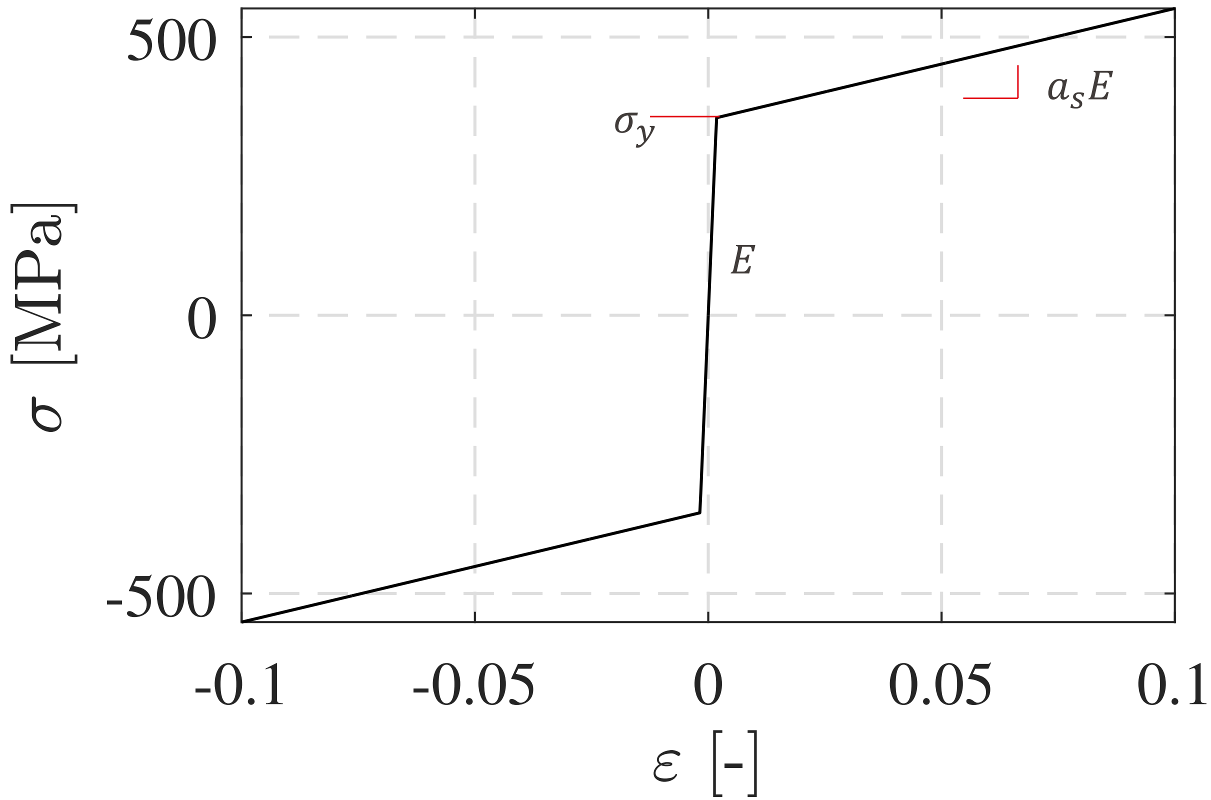




Source: Suzuki and Lignos (2018)

Modeling structural steel under monotonic loading

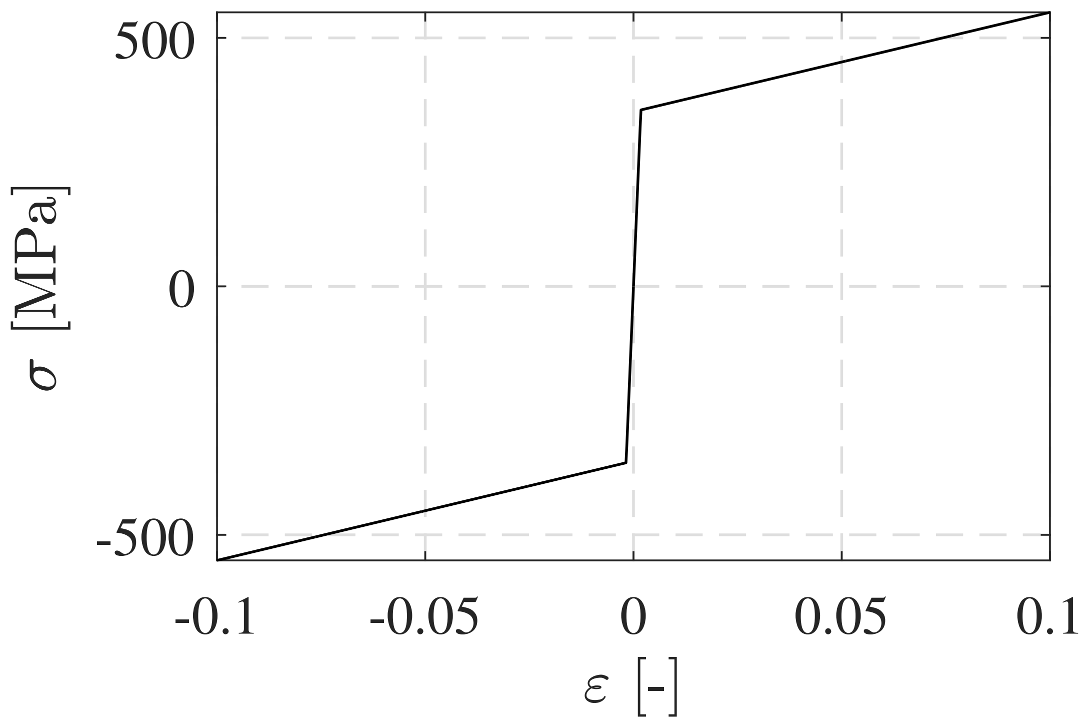
-Bilinear material model



Source: Heredia and Lignos (2023)

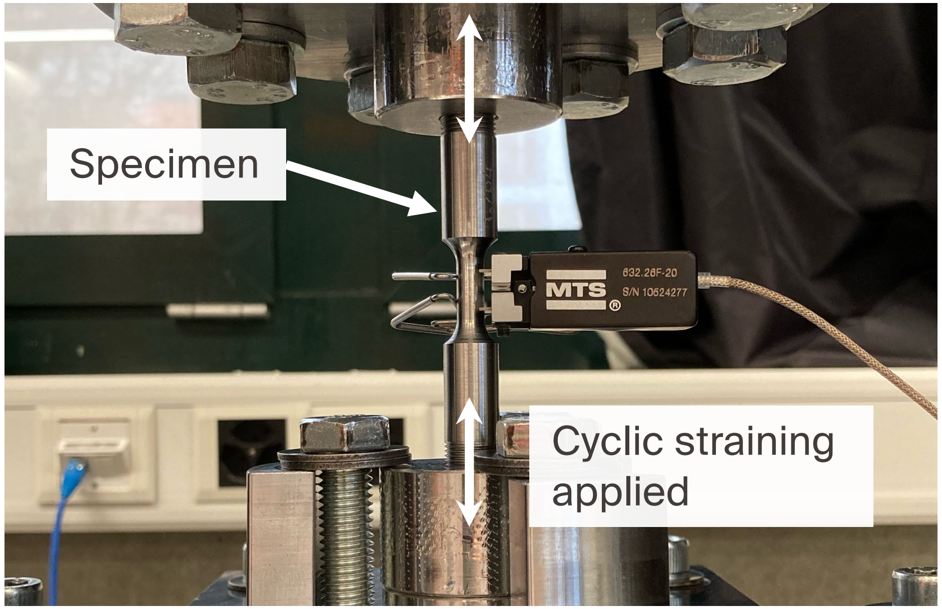
Modeling structural steel under monotonic loading

- In OpenSees, a simple bilinear material is known as Steel01

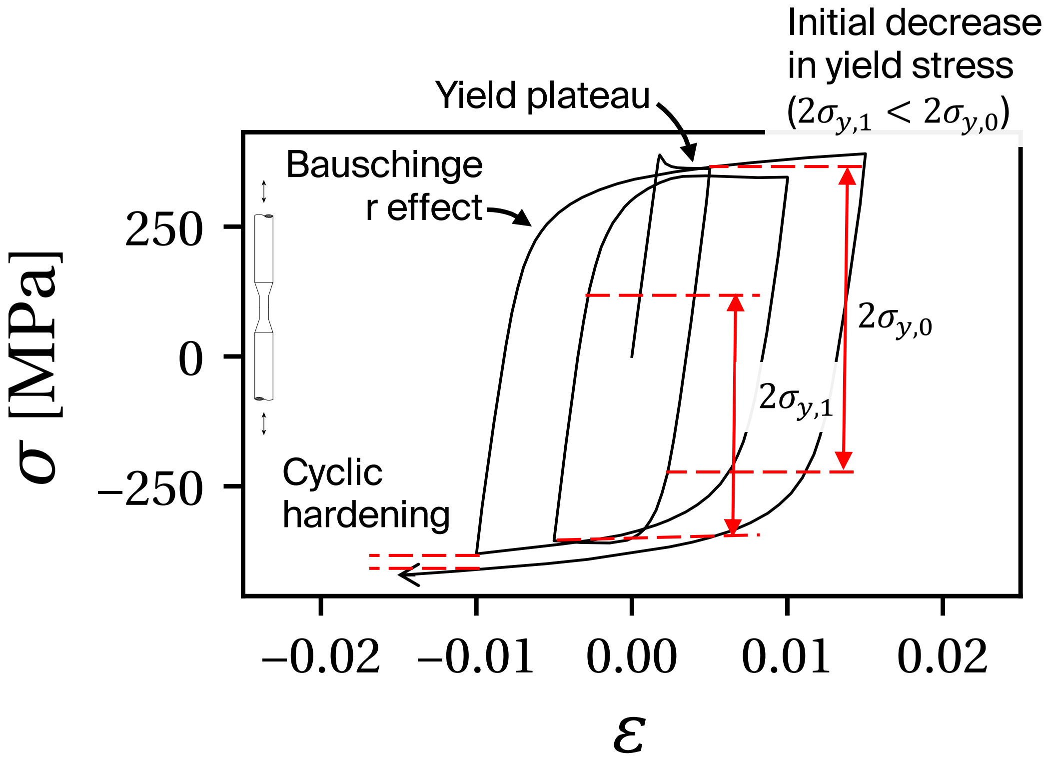


Behavior of structural steel under cyclic loading

Material testing

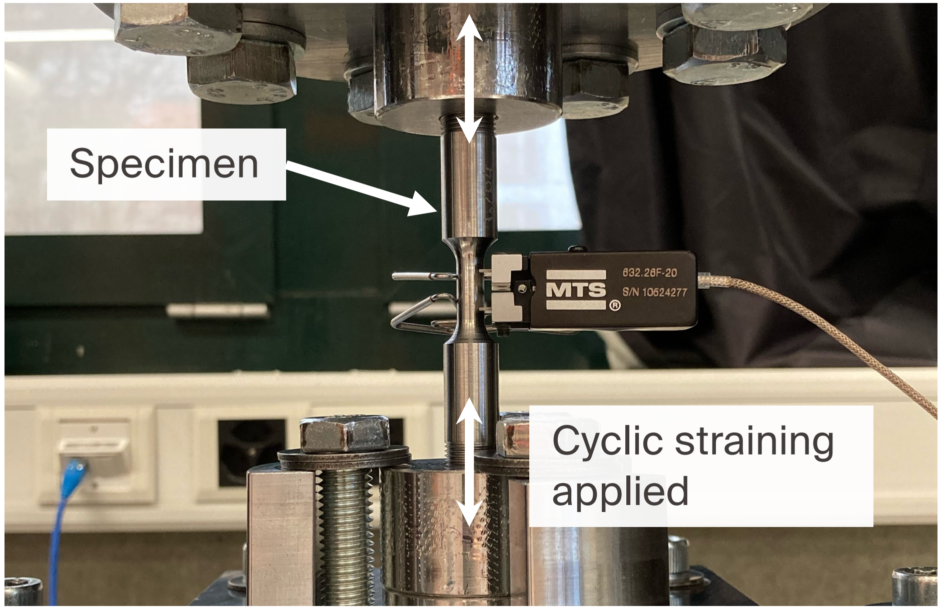


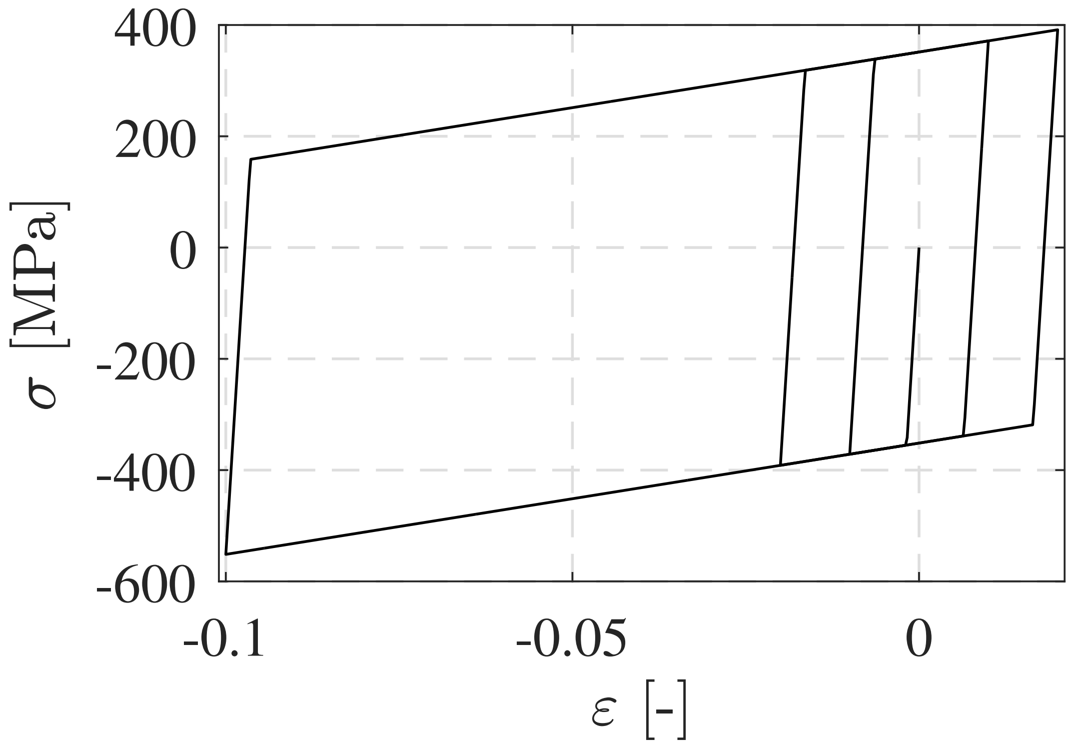
Resulting stress-strain response



Source: Hartloper, de Castro e Sousa and Lignos (2020)

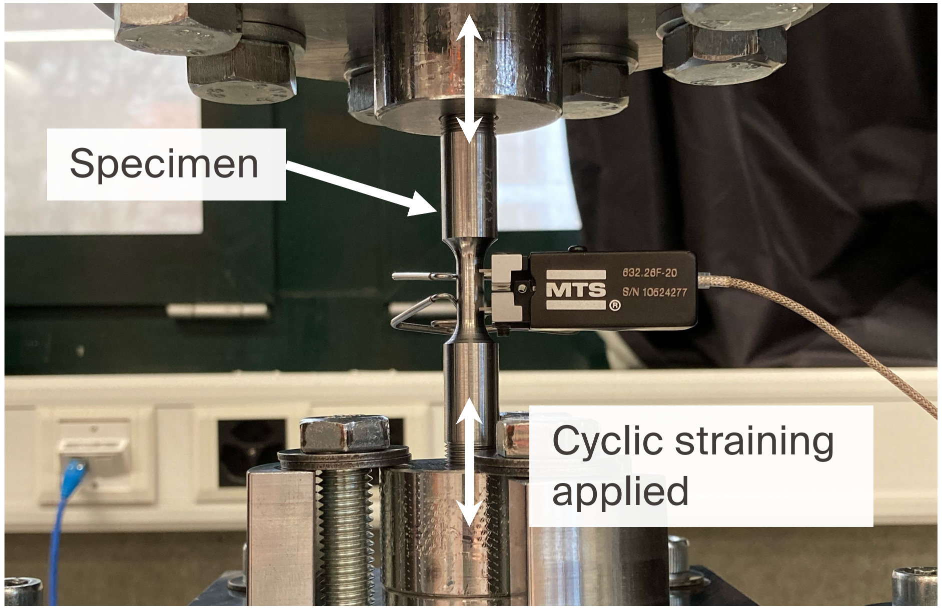
Modeling structural steel under cyclic loading

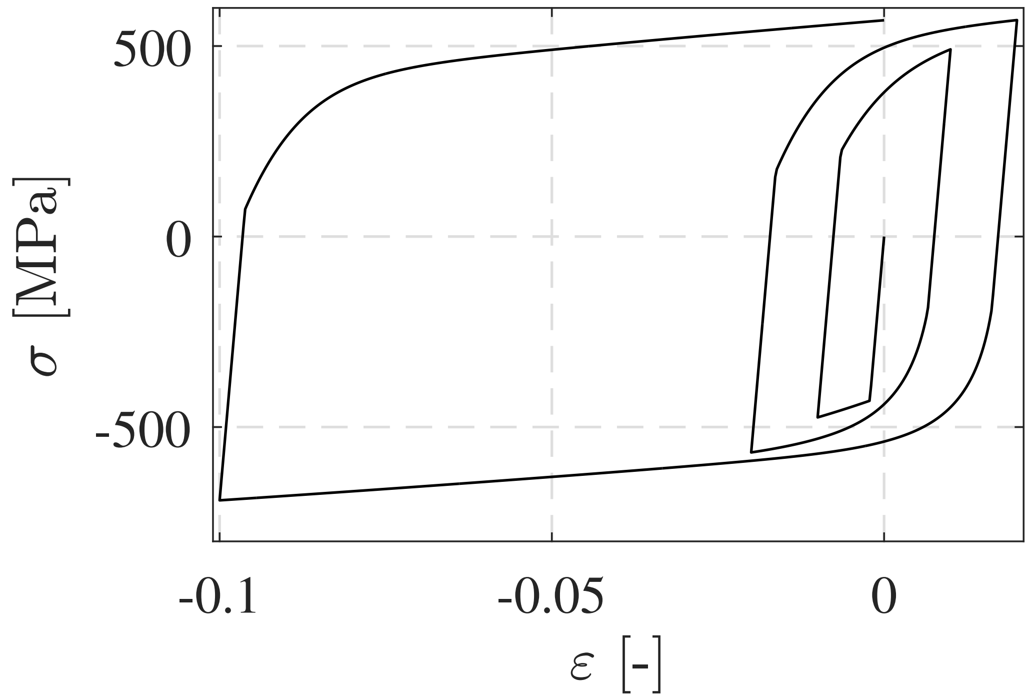




- Steel01 neglects cyclic hardening (combined kinematic/isotropic component)

Modeling structural steel under cyclic loading

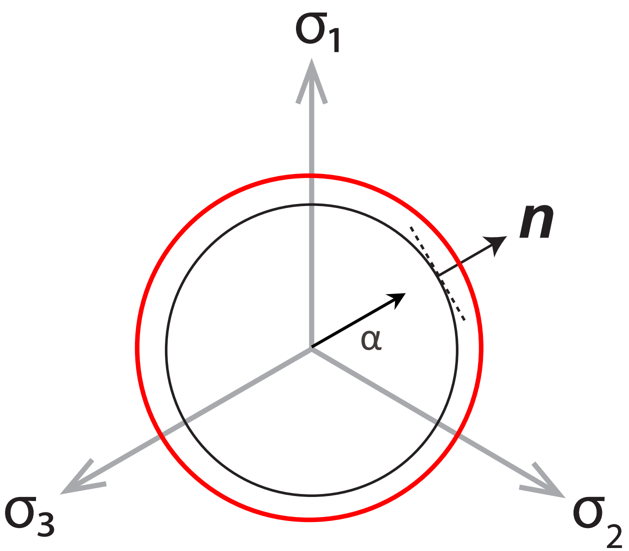




- Updated Voce and Chaboche (UVC)

Source: Hartloper, de Castro e Sousa and Lignos (2020)

Classic metal plasticity: Voce-Chaboche model



Yield surface
in π-plane

von Mises yield criterion

$$f := \|\boldsymbol{\sigma}' - \boldsymbol{\alpha}'\| - \sqrt{2/3}\,\sigma_y \leq 0$$

Associated flow rule

$$\dot{\boldsymbol{\varepsilon}}^p = \lambda\,\boldsymbol{n}$$

Isotropic hardening (Voce, 1947)

$$\sigma_y = \sigma_{y,0} + Q_\infty\left(1 - \exp\left[-b\,\varepsilon_{eq}^p\right]\right)$$

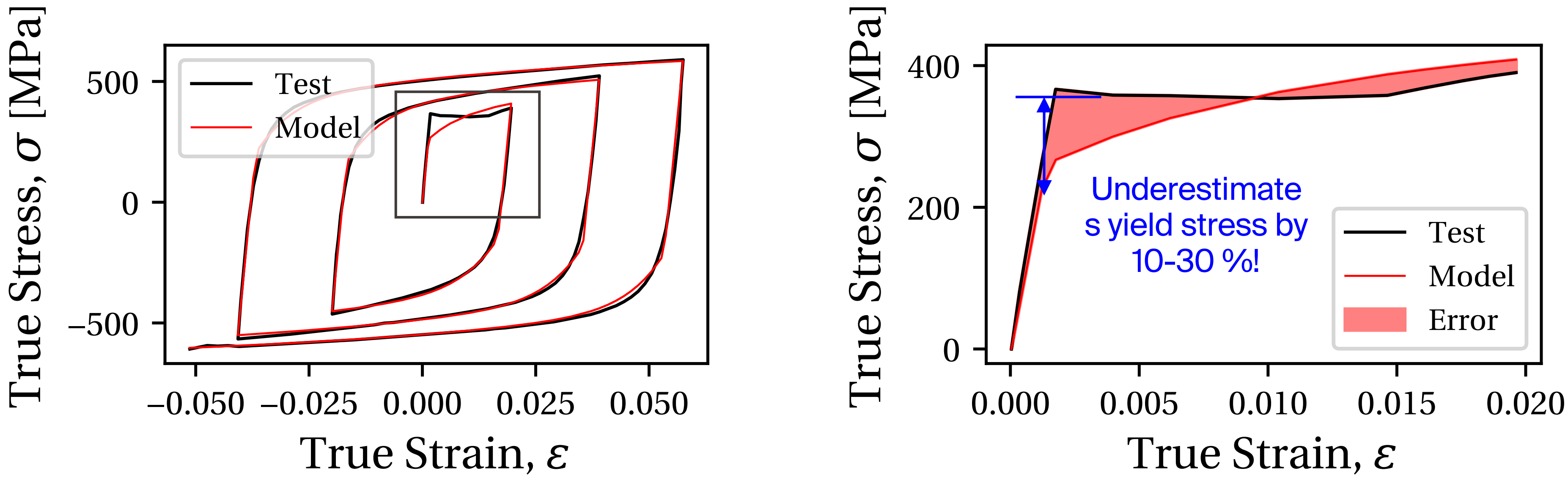
Kinematic hardening
(Chaboche et al., 1979)

$$\boldsymbol{\alpha} = \sum_k \boldsymbol{\alpha}_k\,, \quad \dot{\boldsymbol{\alpha}}_k = \sqrt{\frac{2}{3}} C_k \dot{\varepsilon}_{eq}^p \boldsymbol{n} - \gamma_k \boldsymbol{\alpha}_k \dot{\varepsilon}_{eq}^p$$

Does not account for
discontinuous yielding!

Source: Hartloper, de Castro e Sousa and Lignos (2020)

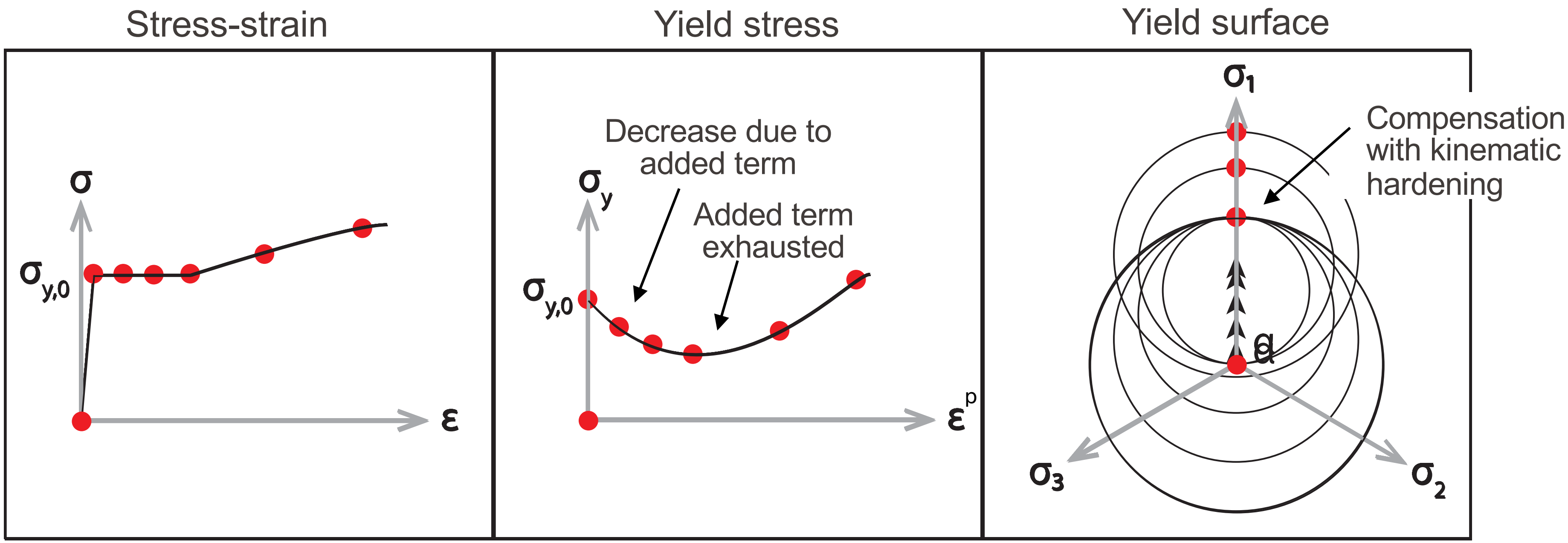
Limitations of Voce-Chaboche model



Source: Hartloper, de Castro e Sousa and Lignos (2020)

Proposed constitutive model for structural steels

-Updated Voce and Chaboche (UVC)

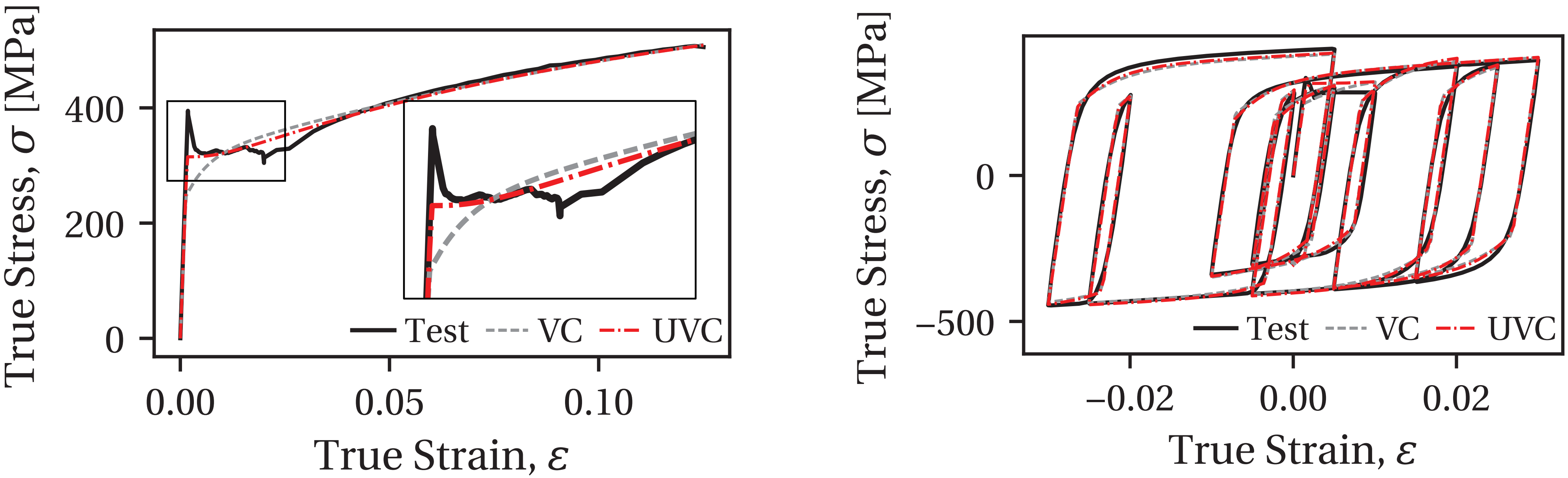


Proposed isotropic hardening rule: $\sigma_y = \sigma_{y,0} + Q_\infty(1 - \exp[-b\ \varepsilon_{eq}^p]) - D_\infty(1 - \exp[-a\ \varepsilon_{eq}^p])$

Source: Hartloper, de Castro e Sousa and Lignos (2020)

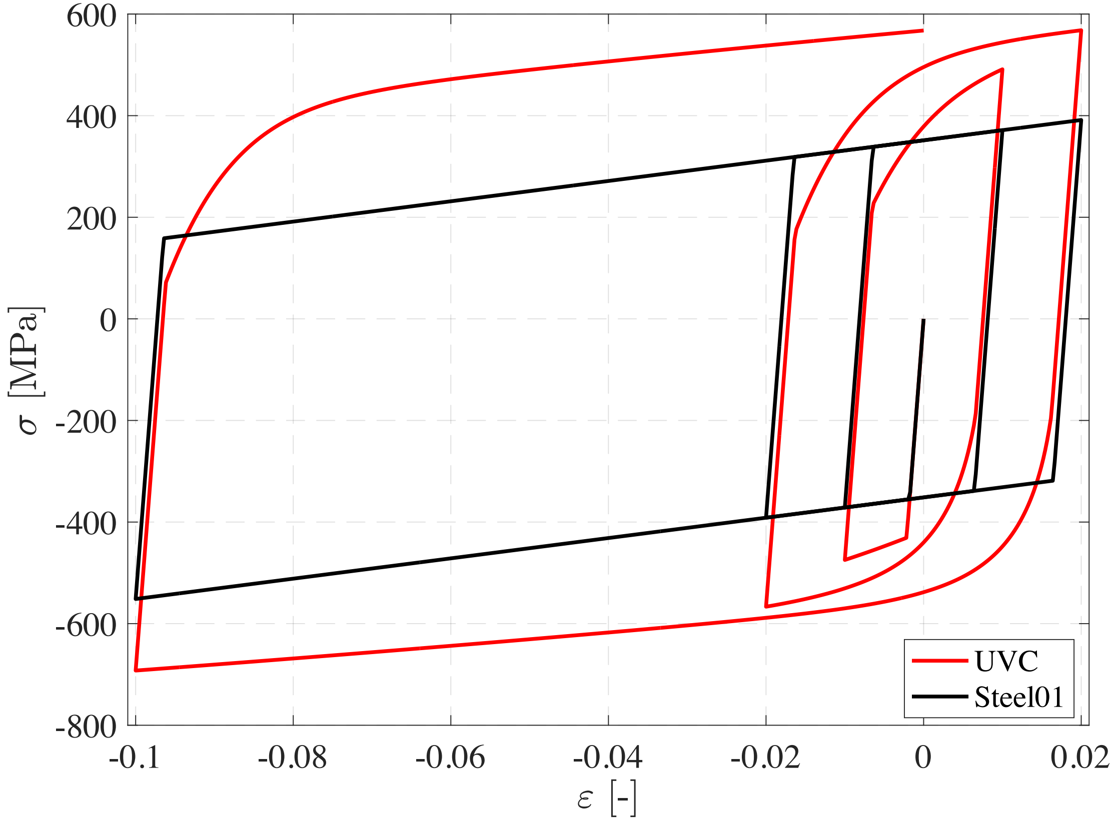
Updated Voce and Chaboche model

-Comparisons with experimental data

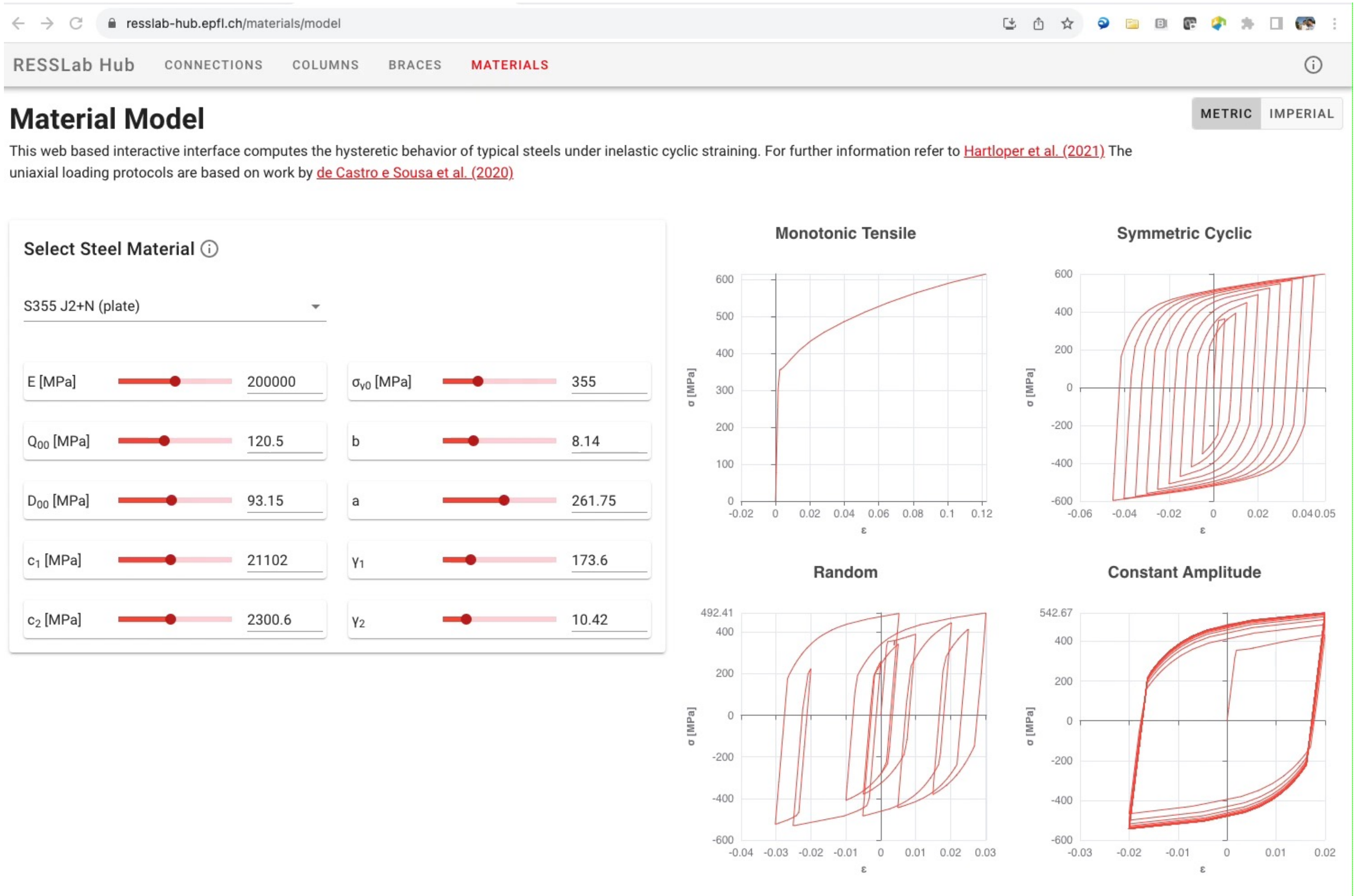


Source: Hartloper, de Castro e Sousa and Lignos (2020)

Comparisons with other constitutive formulations



Input parameter model calibrations for mild steels



Input parameter model calibrations for mild steels

Measured value

Mild structural steels

ID	Material	$f_{y,n}$ (MPa)	$\sigma_{y,0}$ (MPa)	Q_∞ (MPa)	b	D_∞ (MPa)	a	C_1 (MPa)	γ_1	C_2 (MPa)	γ_2
1	S355J2+N 50 mm plate	355	332.18	120.48	8.14	93.15	261.75	21,102.00	173.60	2,300.60	10.42
2	S355J2+N 25 mm plate	355	338.80	134.34	14.71	133.75	229.25	26,242.00	199.04	2,445.30	11.66
3	S355J2 HEB500 flange	355	315.04	138.01	11.36	96.16	223.66	18,587.84	257.31	1,351.98	6.52
4	S355J2 HEB500 web	355	334.94	139.32	14.07	120.33	274.73	28,528.03	315.17	2,569.45	24.68
5	S460NL 25 mm plate	460	439.20	97.35	14.02	136.64	226.40	26,691.00	188.75	2,892.40	10.44
6	S690QL 25 mm plate	690	685.39	0.11	0.11	132.30	285.15	34,575.00	185.16	3,154.20	20.14
7	A992 Gr.50 W14X82 web	345	378.83	122.63	19.74	143.49	248.14	31,638.00	277.32	1,548.60	9.04
8	A992 Gr.50 W14X82 flange	345	373.72	141.47	15.20	135.95	211.16	25,621.00	235.12	942.18	3.16
9	A500 Gr.B HSS305X16	315	324.09	228.02	0.11	50.41	270.40	17,707.00	207.18	1,526.20	6.22
10	BCP325 22 mm plate	325	368.03	112.25	10.78	105.95	221.92	20,104.00	200.43	2,203.00	11.76
11	BCR295 HSS350X22	295	412.21	0.09	0.09	103.30	212.83	20,750.59	225.26	1,245.04	2.09
12	HYP400 27 mm plate	400	454.46	62.63	16.57	109.28	145.74	13,860.00	141.61	1,031.10	3.53

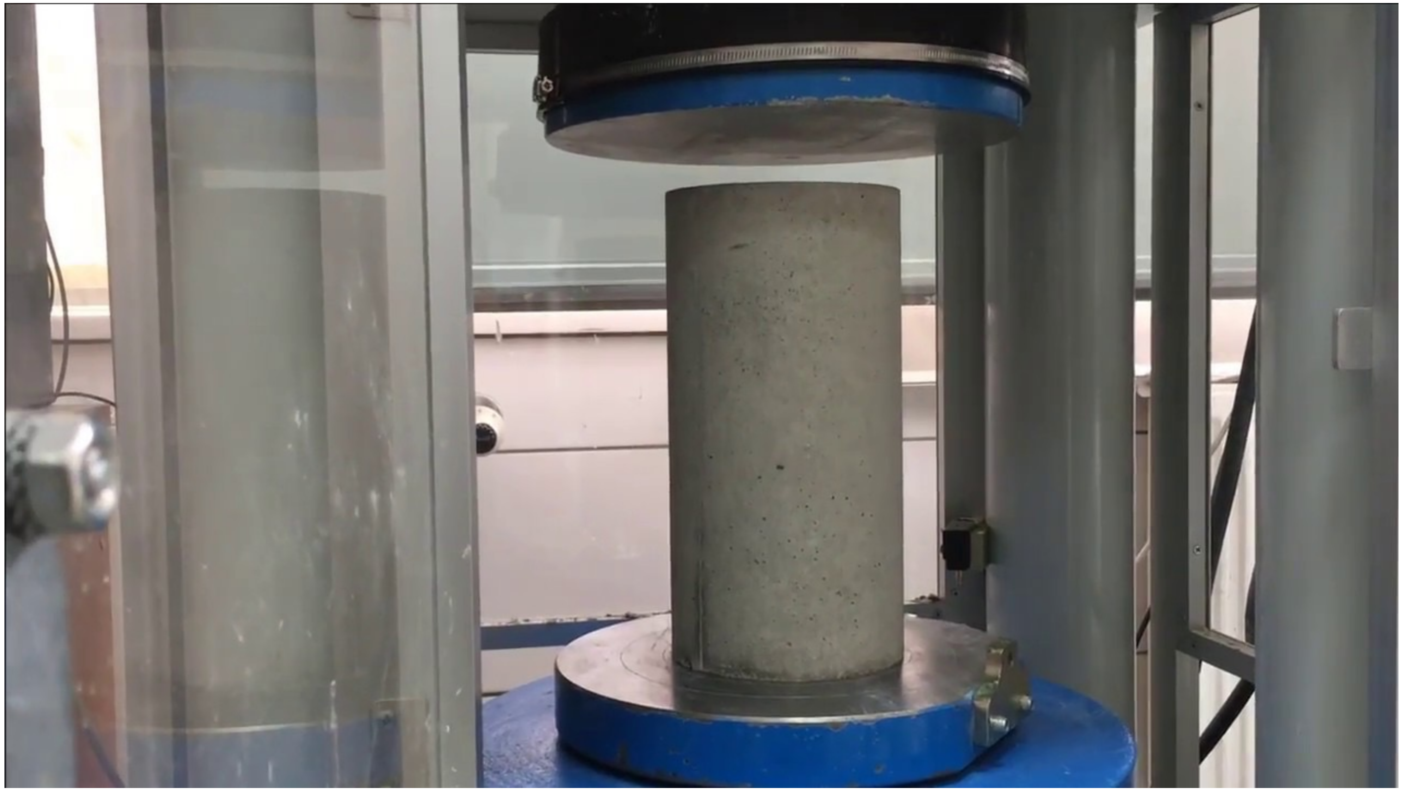
Source: Hartloper, de Castro e Sousa and Lignos (2021)

Reinforcing rebars B500B (same values for other diameters hold true)

ID	$\sigma_{y,0}$ [MPa]	Q_∞ [MPa]	b	D_∞ [MPa]	a	C_1 [MPa]	γ_1	C_1 [MPa]	γ
M8	505.82	123.62	42.87	173.02	146.40	24454.62	142.18	1492.05	2.
M12	518.75	240.28	90.24	273.26	131.35	21826.43	126.18	1188.21	0.0

Source: El Jisr and Lignos (2021)

Mechanical properties of cementitious materials

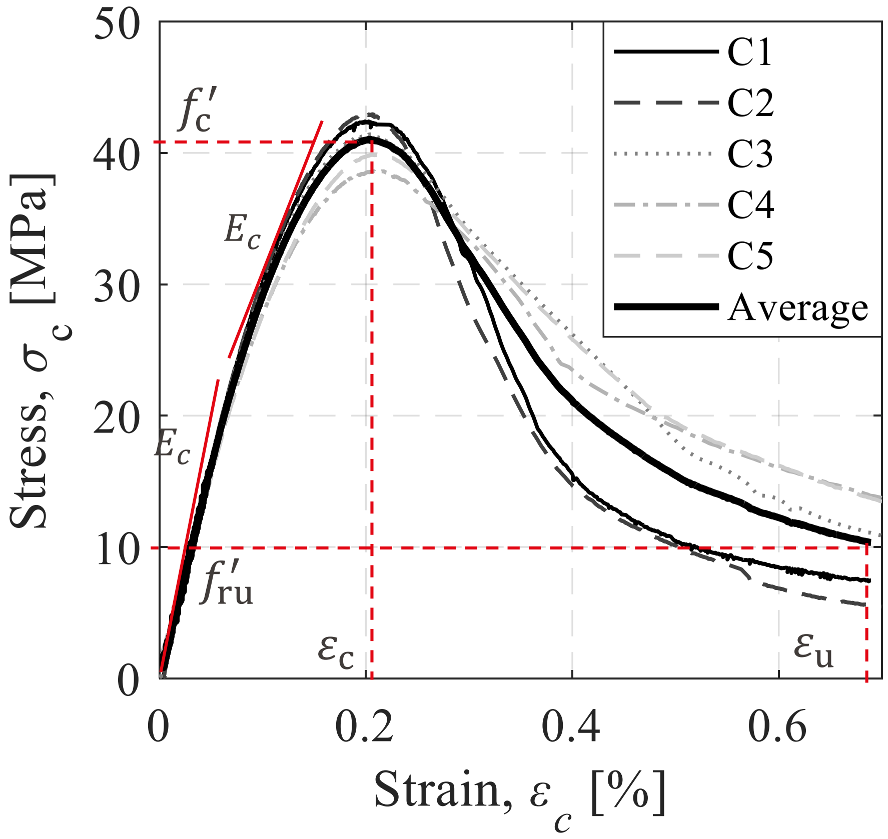


Source: MTS

Mechanical properties of cementitious materials (2)

Mechanical properties of cementitious materials

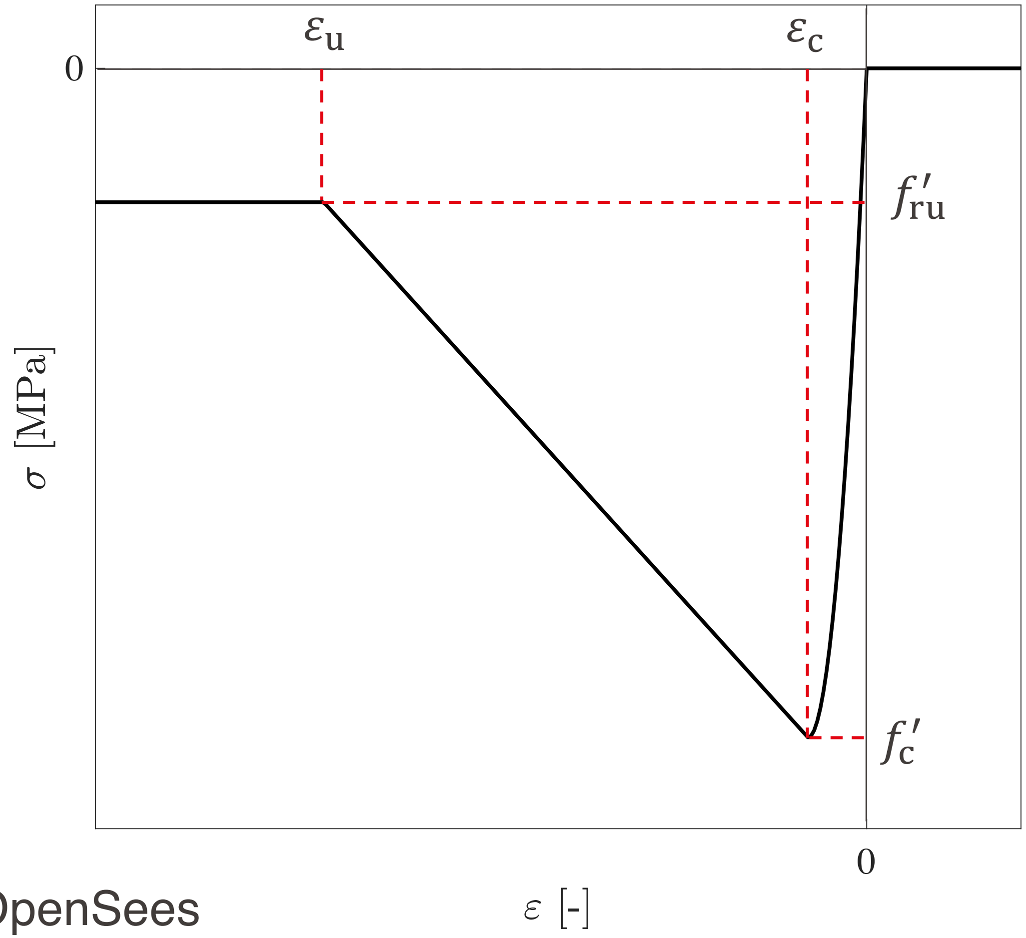
-Concrete under uniaxial compression



Source: El Jisr and Lignos (2021)

Modeling concrete under uniaxial monotonic loading

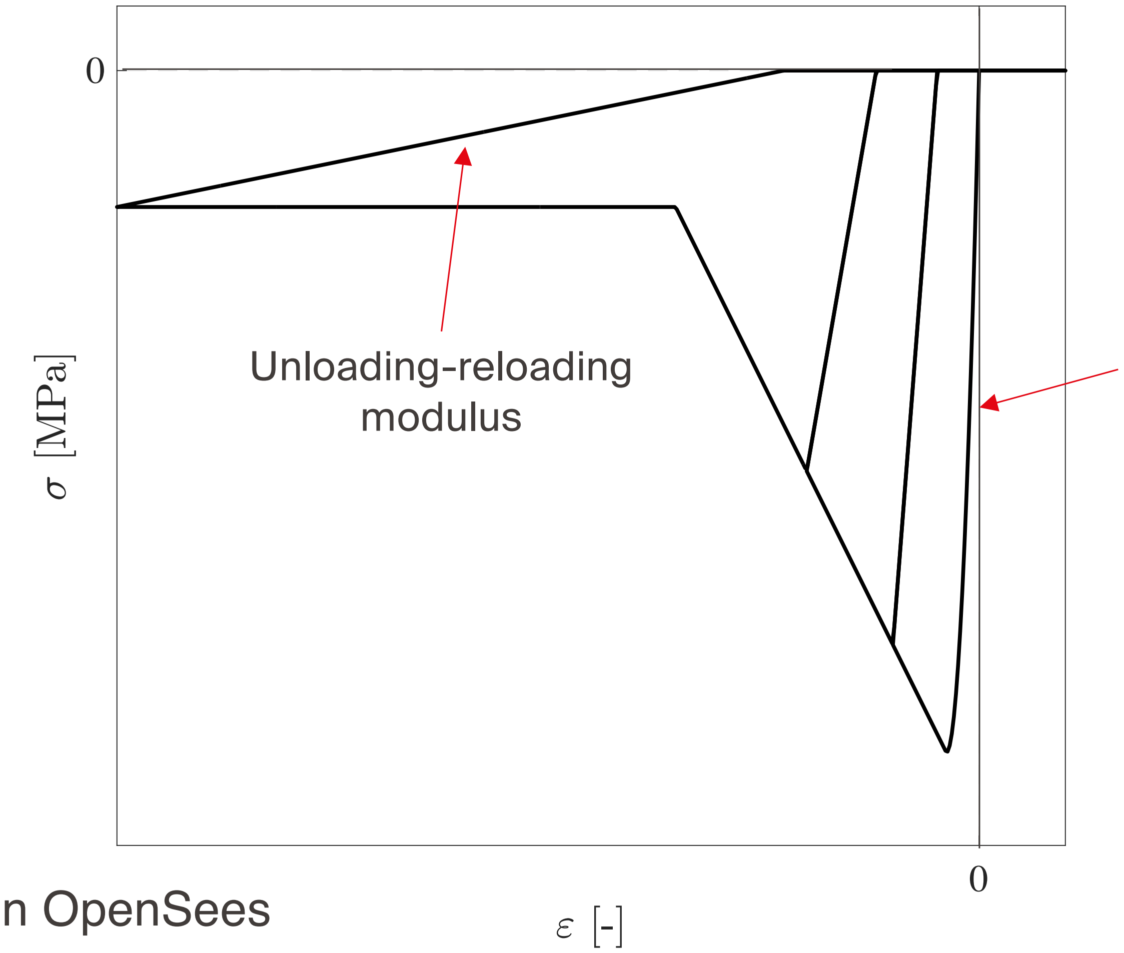
-Kent Scott Park uniaxial material model



- Concrete01 in OpenSees

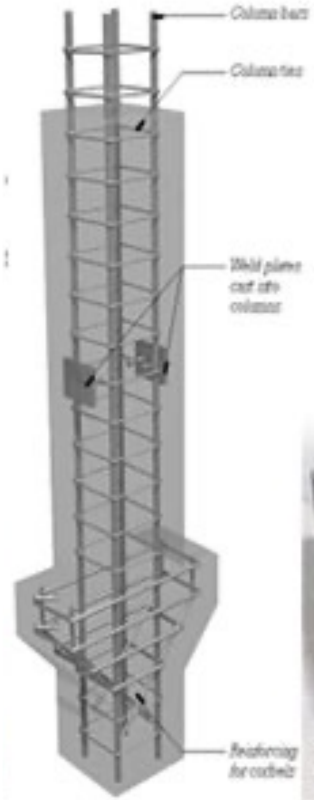
Modeling concrete under uniaxial cyclic loading

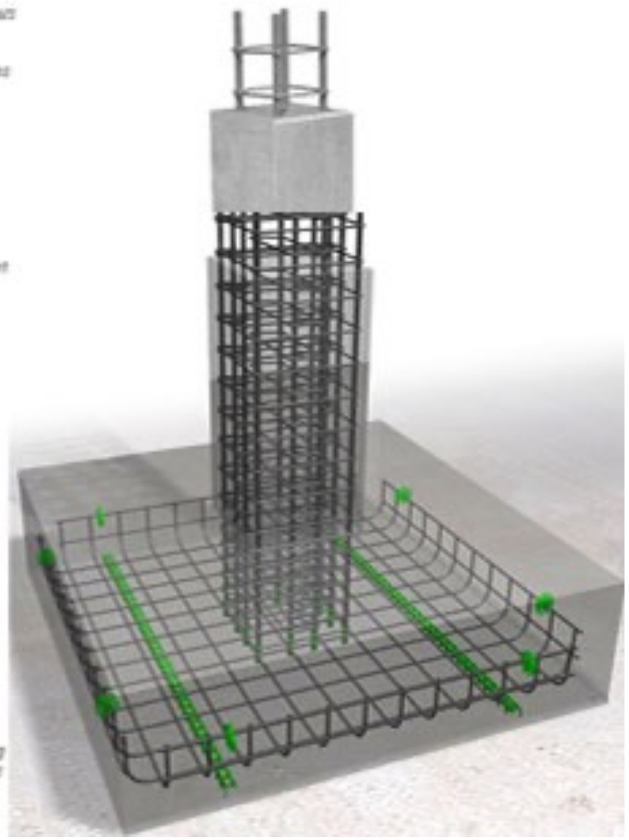
-Kent Scott Park uniaxial material model

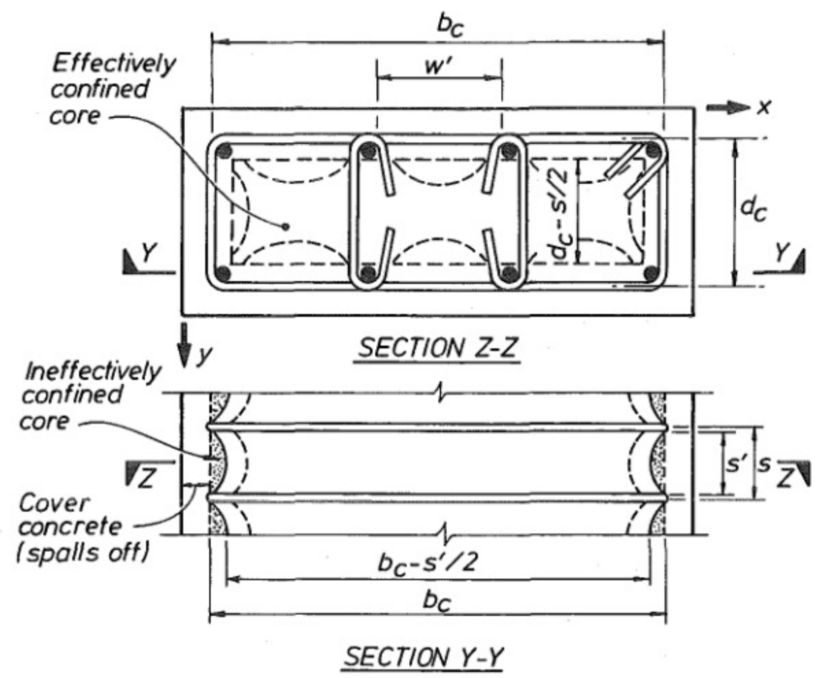


- Concrete01 in OpenSees

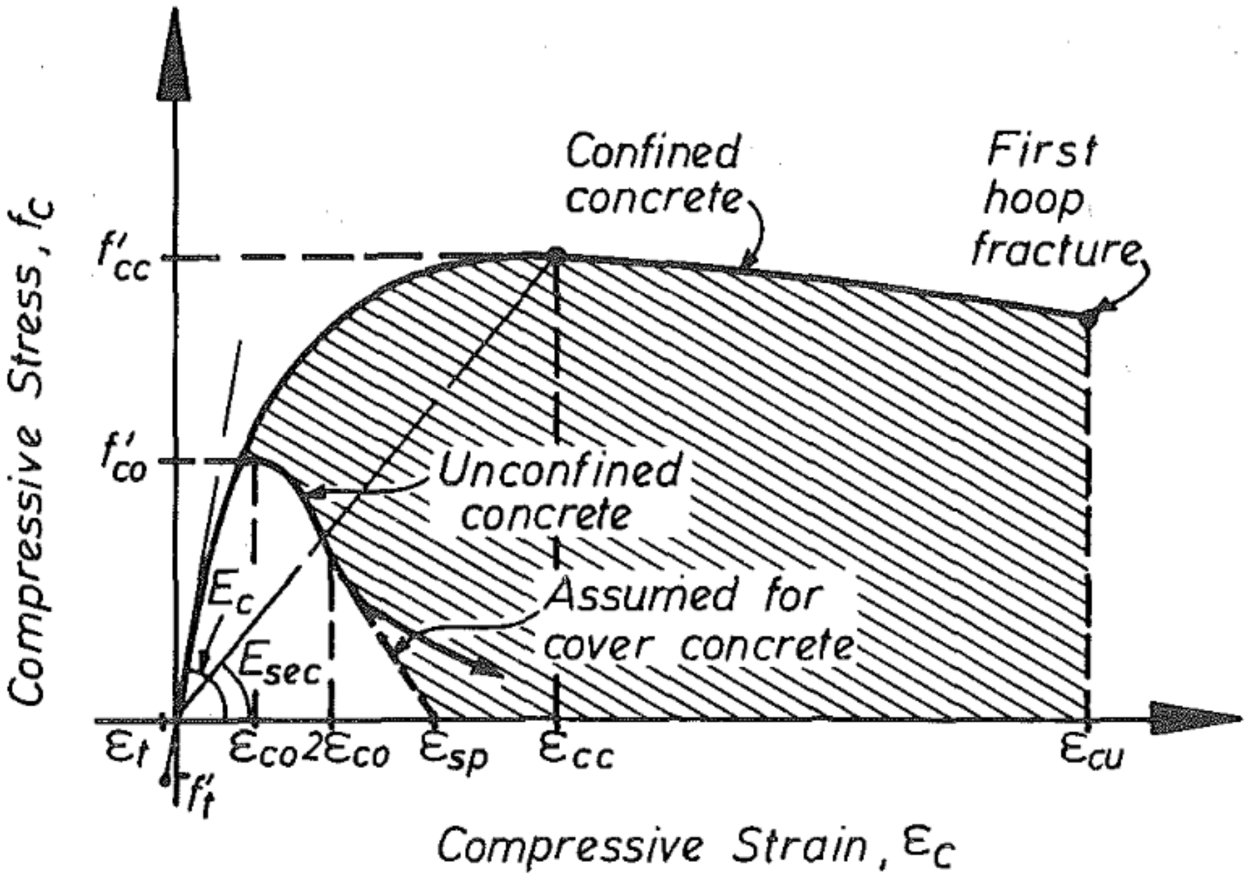
Modeling reinforced concrete structural elements





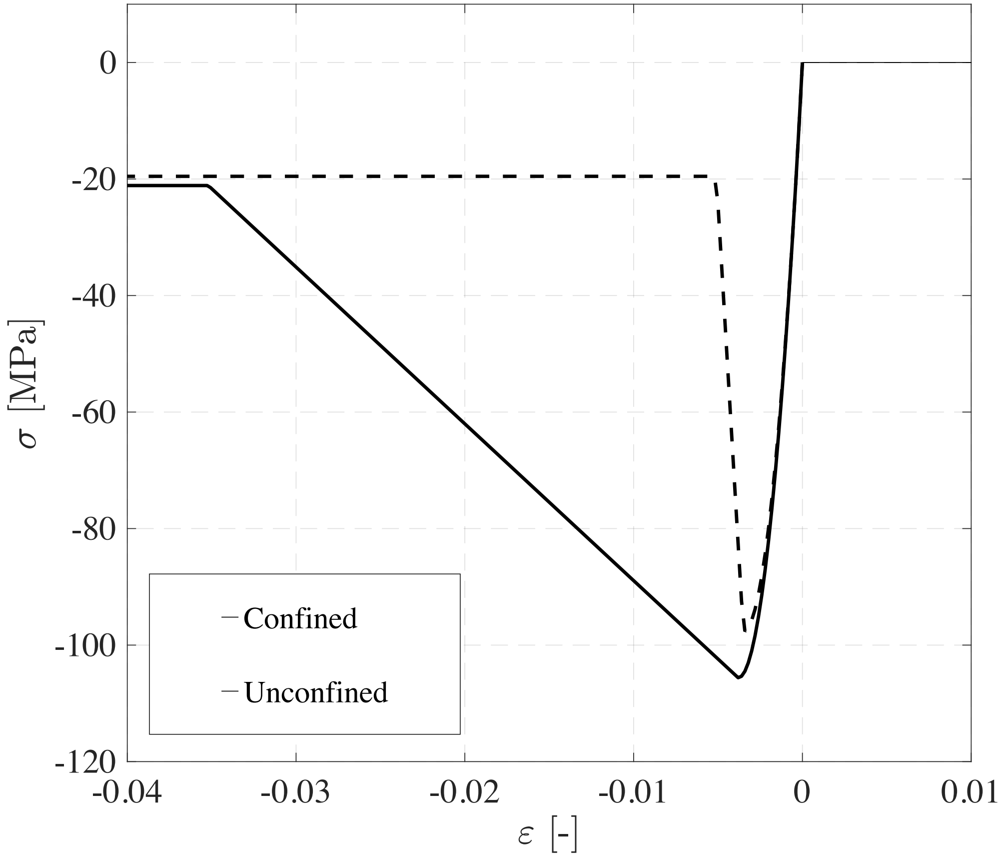


Modeling reinforced concrete structural elements



Source Mander (1988)

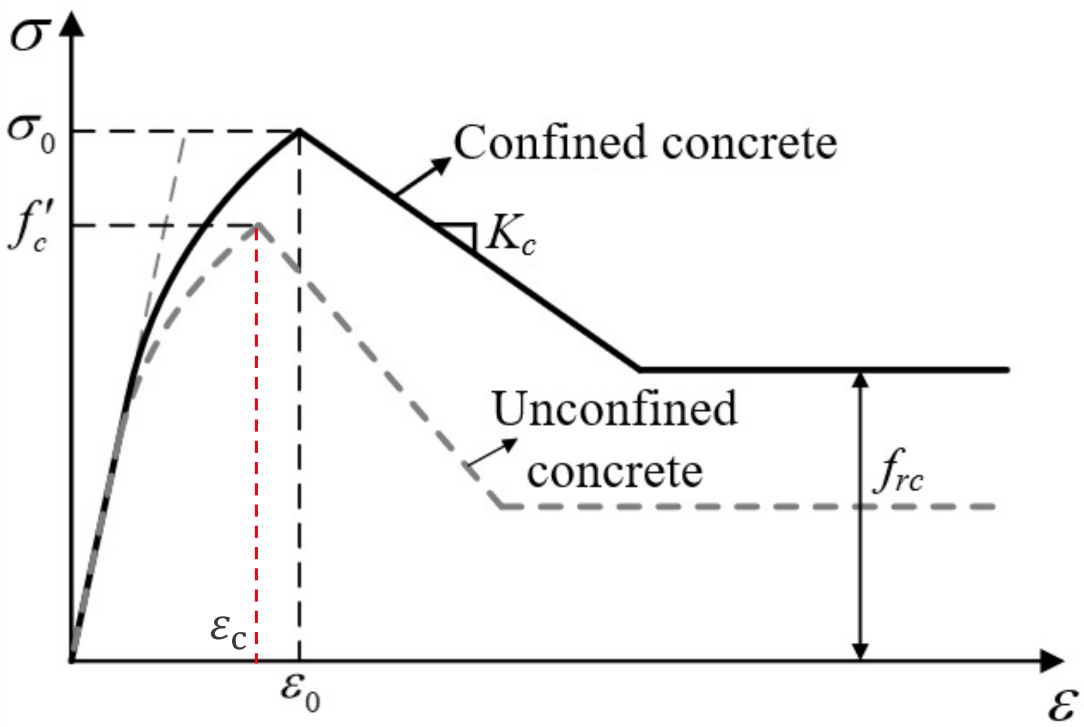
Modeling reinforced concrete structural elements



- Concrete01 in OpenSees

Modeling reinforced concrete structural elements

-Input model parameter identification



Source Mander (1988)

Modeling reinforced concrete structural elements
-Input model parameter identification

- Compressive strain at crushing of confined concrete:

$$\varepsilon_0 = \varepsilon_c \left[1 + 5\left(\frac{\sigma_0}{f_c'} - 1\right)\right]$$

- Compressive stress at crushing of confined concrete:

$$\sigma_0 = f_c'\left(-1.254 + 2.254\sqrt{1 + 7.94\frac{f_l'}{f_c'}} - 2\frac{f_l'}{f_c'}\right)$$

Source Mander (1988)

Modeling reinforced concrete structural elements

-Input model parameter identification

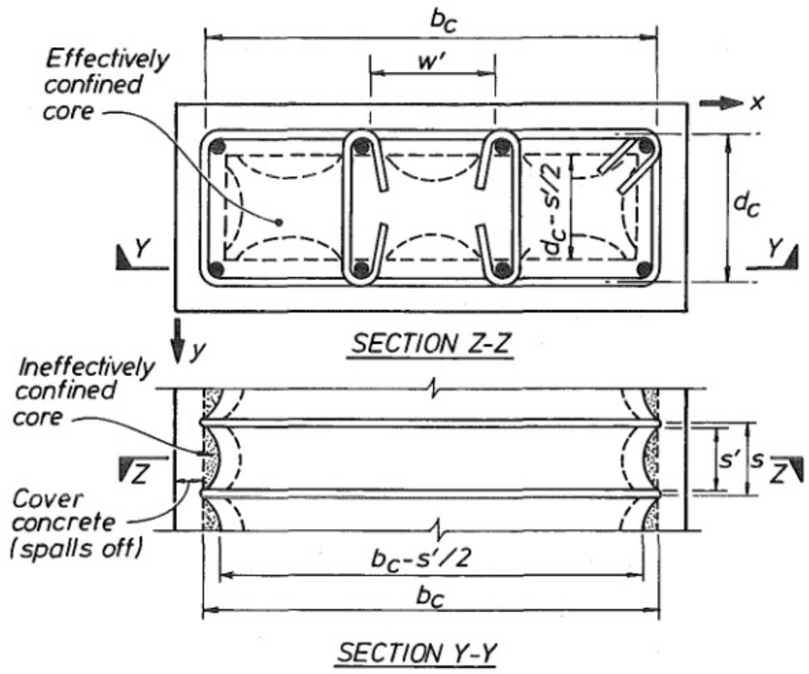
- Effective lateral confinement stress for the concrete in x and y loading direction:

$$f'_{lx} = k_e \frac{A_{sx}}{s d_c} f_{yh} = k_e \rho_{sx} f_{yh} \qquad f'_{ly} = k_e \frac{A_{sy}}{s b_c} f_{yh} = k_e \rho_{sy} f_{yh}$$

- f_{yh}: yield strength of transverse reinforcement

- ρ_{sx} ρ_{sy}: ratio of the volume of transverse confining steel in the x and y direction, respectively, to the volume of confined concrete core

$$\rho_{sx} = \frac{A_{sx}}{s d_c} \qquad \rho_{sy} = \frac{A_{sy}}{s b_c}$$

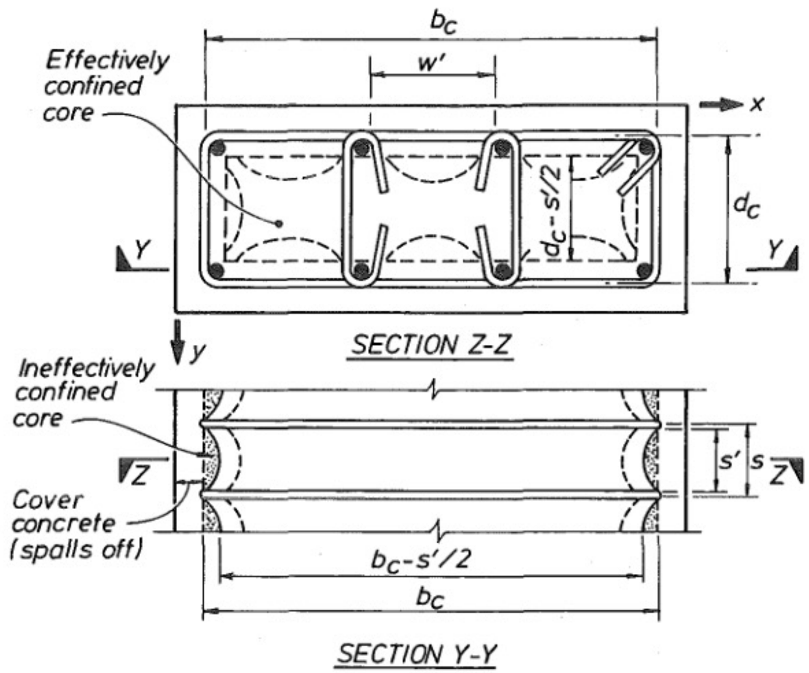


Source Mander (1988)

Modeling reinforced concrete structural elements
-Input model parameter identification

- The confinement effective coefficient:

$$k_e = \frac{A_e}{A_{cc}} = \frac{\left(b_c d_c - \sum_{i=1}^{n} \frac{(w_i')^2}{6}\right)\left(1 - \frac{s'}{2b_c}\right)\left(1 - \frac{s'}{2d_c}\right)}{1 - \rho_{cc}}$$

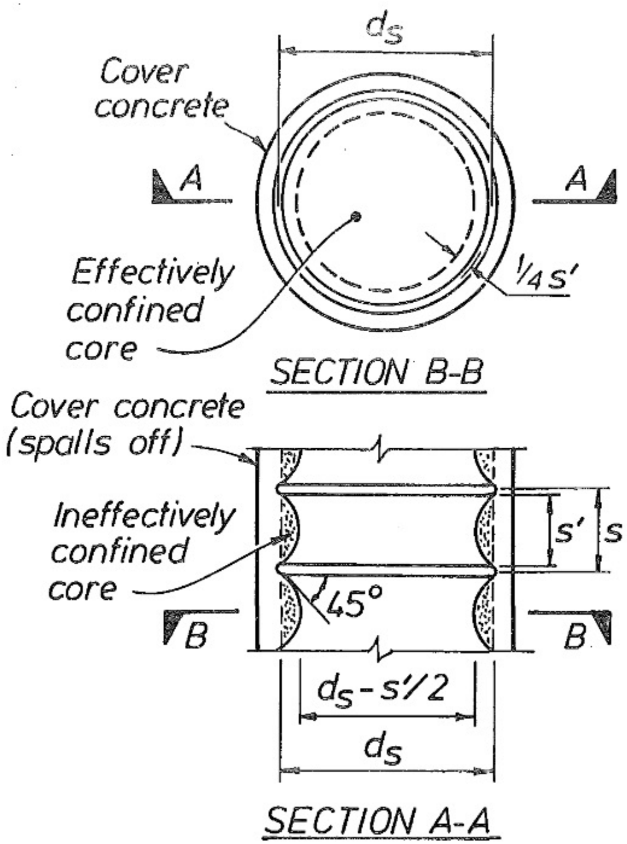


- n: number of longitudinal rebars
- A_e: area of effectively confined concrete core
- A_{cc}: area of the concrete within the centerlines of the perimeter hoop
- ρ_{cc} : ratio of area of longitudinal reinforcement to area of core of cross section

Source Mander (1988)

Modeling reinforced concrete structural elements

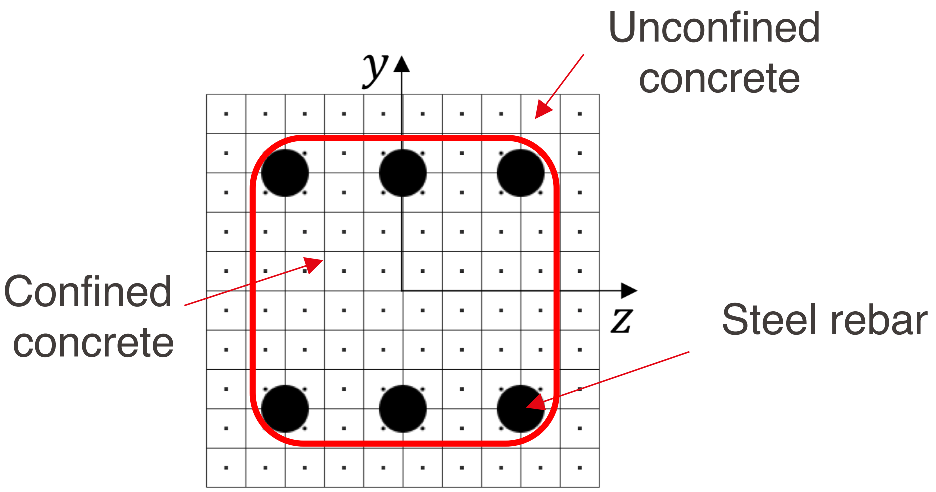
-Input model parameter identification



- Similar expressions hold true for circular cross sections

Source Mander (1988)

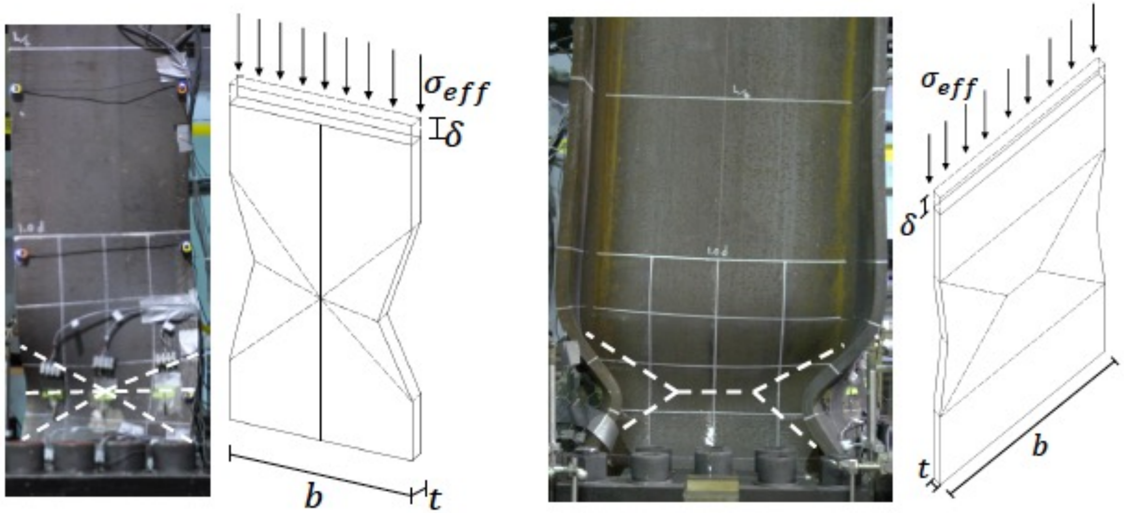
Modeling reinforced concrete structural elements



- Your fiber discretization shall be such that a fiber can line up with steel rebar to assign the proper material constitutive law accordingly

Rebar and cross-sectional steel plate buckling

Source: Giamundo et al. (2014)

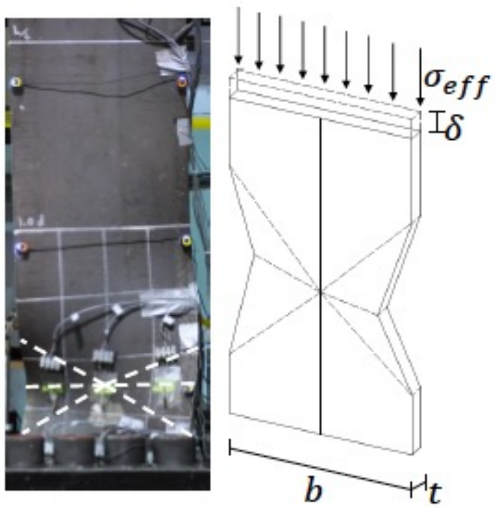


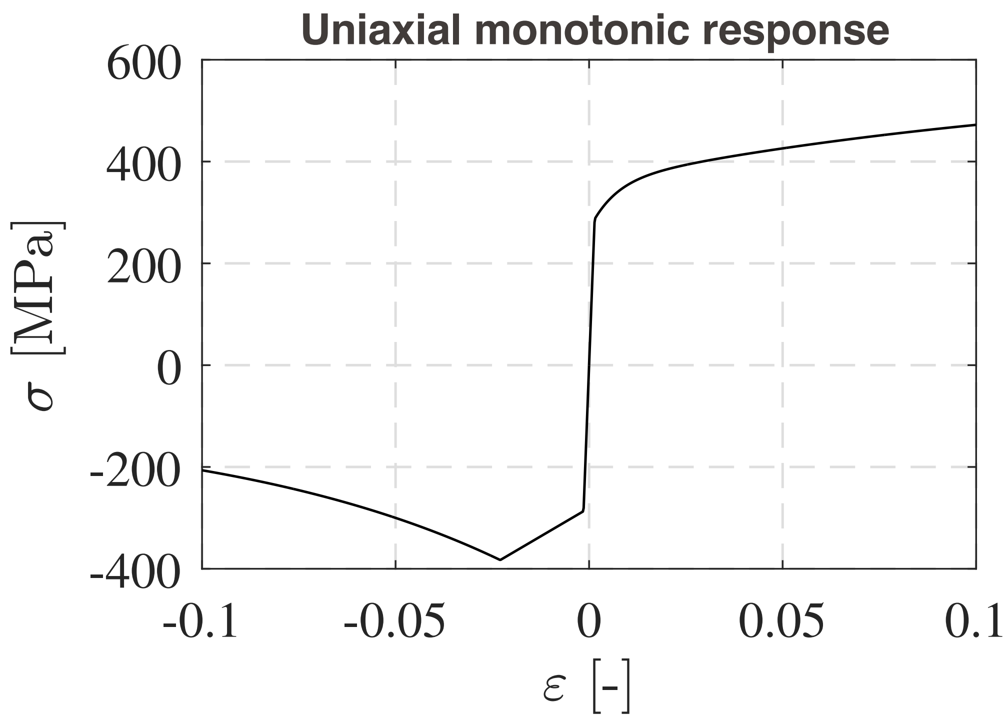
Source: Heredia, de Castro e Sousa and Lignos (2023)

- Geometric nonlinearities cannot be modelled explicitly within frame beam-column elements

Rebar and cross-sectional steel plate buckling

- Implicit modelling by “hacking” the stress-strain constitutive formulation

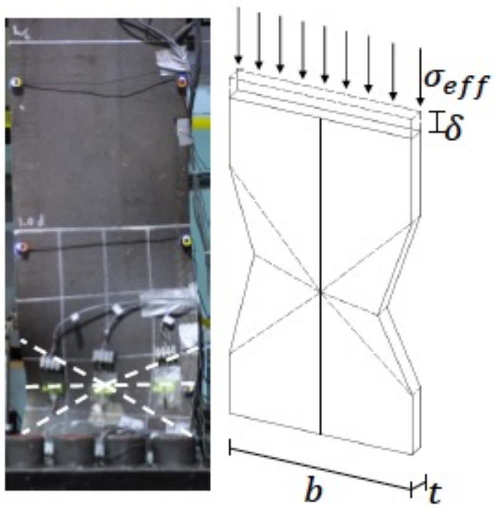


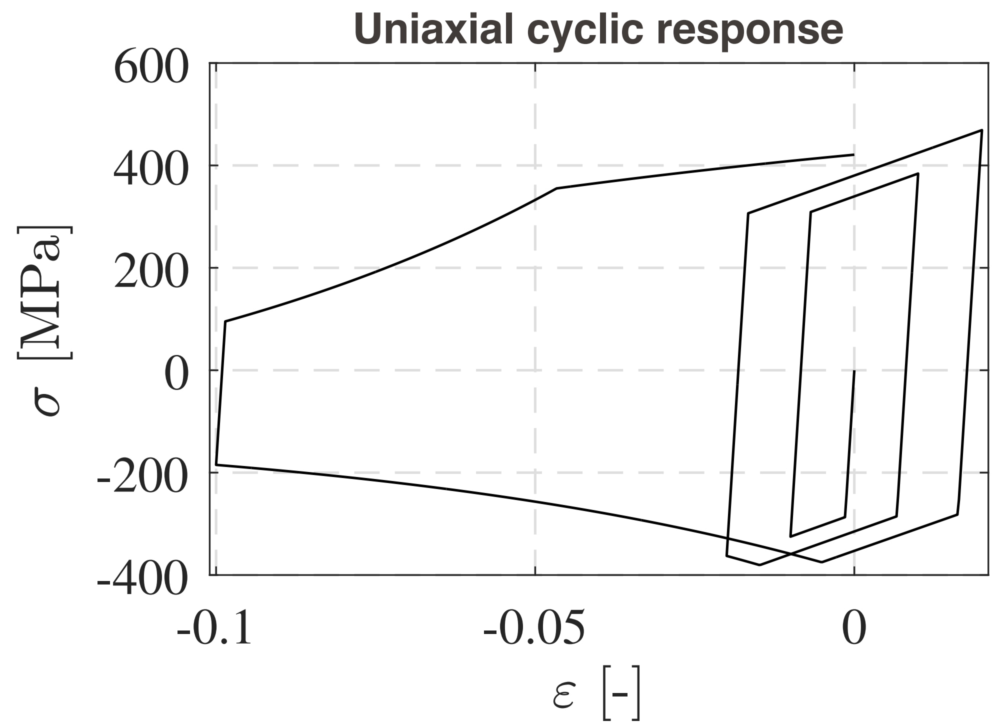


Source: Suzuki and Lignos (2021)

Rebar and cross-sectional steel plate buckling

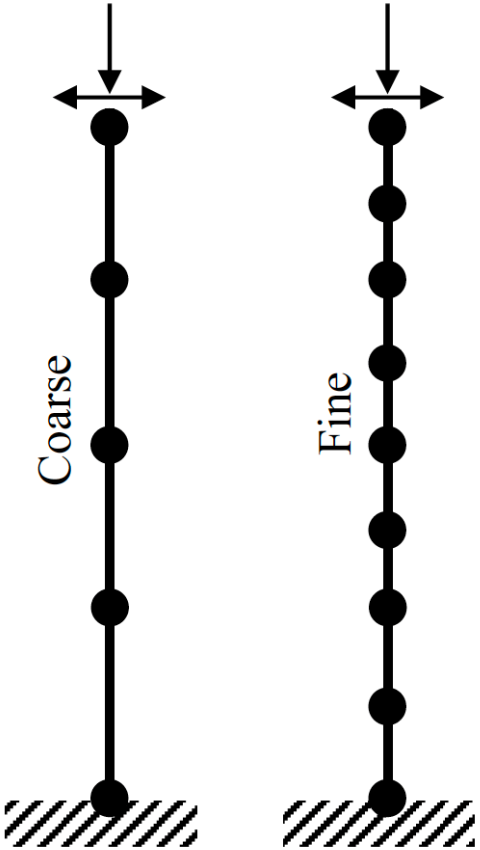
- Implicit modelling by "hacking" the stress-strain constitutive formulation

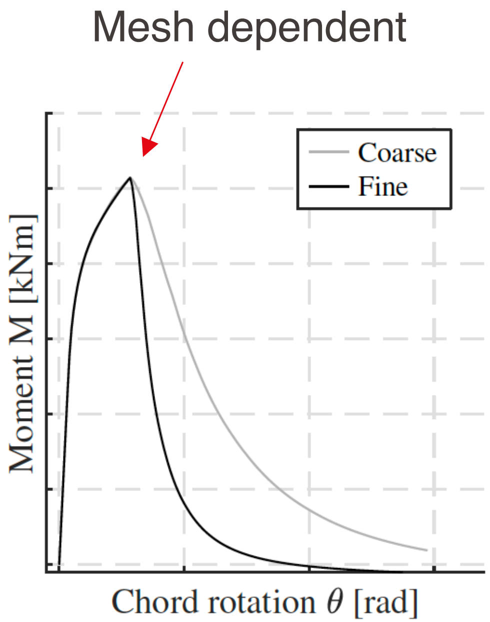


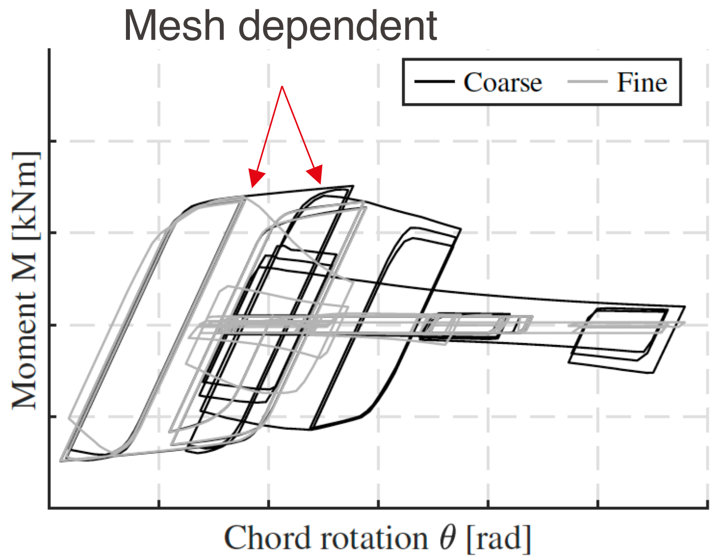


Source: Suzuki and Lignos (2021)

Well-known problems in constitutive laws with softening







Source: Heredia, de Castro e Sousa and Lignos (2023)

Well-known problems in constitutive laws with softening

- In the post-peak regime (softening), the solution is mesh-dependent
- Strain localization
- Loss of objectivity
- Convergence instabilities of the numerical solution algorithms

Some techniques to alleviate loss of objectivity

- Regularization (see Dr. Savvas Saloustros)
- Nonlocal mechanics / damage theory (Bazant and Zdenek 1987)