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Assignment #1: Analysis of elastic structures

Problem 1

Q1 (70 points): Write a program at any programming language of your preference to determine the joint displacements, member axial forces and support reactions for planar trusses by linear analysis. Assume that that truss members are all prismatic (their axial rigidity is constant along the truss length).

Q2 (30 points): Use your program from Q1 to compute the joint displacements, member axial forces and support reactions for the example shown in Figure 1. Assume the following properties:

- Prismatic members ($EA = constant$)
- $E = 70\ GPa$
- $A = 645.2mm^2$

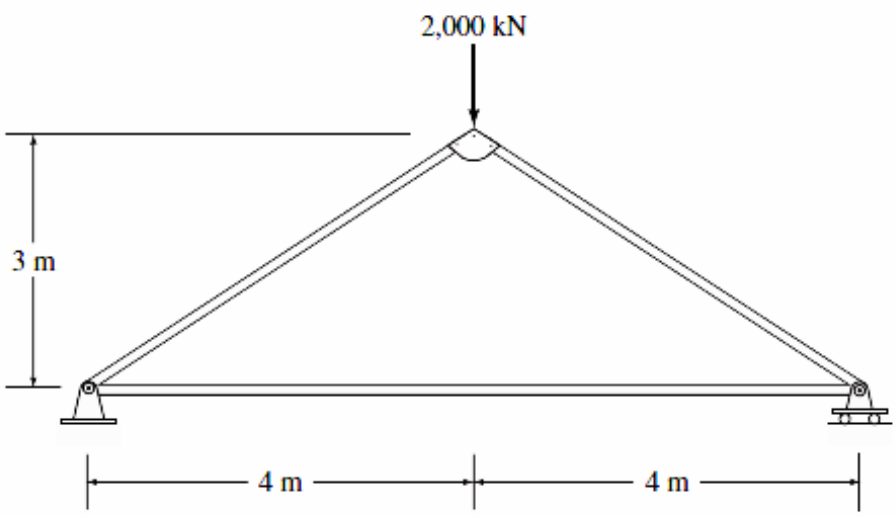

Figure 1. Planar truss

Solution:

The following steps can be used:

1) Define the member properties (E, A and l)

2) Define the connectivity matrix and the mapping matrix between local and global degrees of freedom. The figure below shows the global degrees of freedom used for the structure

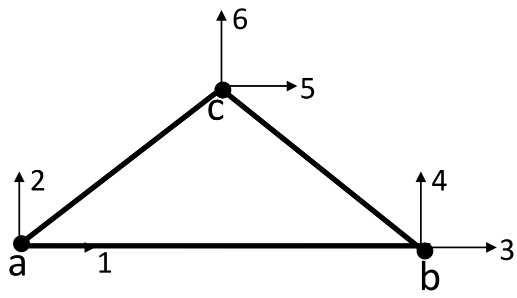

Figure 2. Global degrees of freedom

The mapping matrix $\boldsymbol{numEq}$ is therefore given by

$$\boldsymbol{numEq} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 6 \\ 3 & 4 & 5 & 6 \end{bmatrix} \tag{1}$$

3) For each member, determine the transformation matrix $\boldsymbol{T}$ between local and global coordinates. In this exercise, the local x-axis is defined in the axial direction of the element

4) Assemble the initial structure stiffness matrix $\boldsymbol{K}_{structure}$. The following matrix is obtained (the units are N and mm):

$$\boldsymbol{K}_{structure} = 10^4 \cdot \begin{bmatrix} 1.14 & 4.34 & -5.65 & 0 & -5.78 & -4.34 \\ 4.34 & 3.25 & 0 & 0 & -4.34 & -3.25 \\ -5.65 & 0 & 1.14 & -4.34 & -5.78 & 4.34 \\ 0 & 0 & -4.34 & 3.25 & 4.34 & -3.25 \\ -5.78 & -4.34 & -5.78 & 4.34 & 1.16 & 0 \\ -4.34 & -3.25 & 4.34 & -3.25 & 0 & 6.50 \end{bmatrix} \tag{2}$$

5) Define the boundary conditions, the external loads $\boldsymbol{F}_{ext}$ the fixed and the free degrees of freedom of the problem. The external load vector is given by

$$\boldsymbol{F}_{ext} = [0,0,0,0,0,-2 \cdot 10^6] \tag{3}$$

6) Compute the structure displacements $\boldsymbol{v}$:

$$\boldsymbol{v}_f = \left(\boldsymbol{K}_{structure,ff}\right)^{-1} \boldsymbol{F}_{ext} \tag{4}$$

Where the subscript f denotes the free degrees of freedom of the system

The following displacement vector is obtained:

$$\boldsymbol{v} = \begin{pmatrix} 0 \\ 0 \\ 236.2 \\ 0 \\ 118.1 \\ -465.0 \end{pmatrix} \tag{5}$$

7) Compute the structure internal forces $\boldsymbol{F}_{int}$:

$$\boldsymbol{F}_{int} = \boldsymbol{K}_{structure}\boldsymbol{v} \tag{6}$$

The following vector of internal forces is obtained:

$$\boldsymbol{F}_{int} = 10^6 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -2 \end{pmatrix} \tag{7}$$

8) Compute the member internal forces. The following internal forces are obtained:

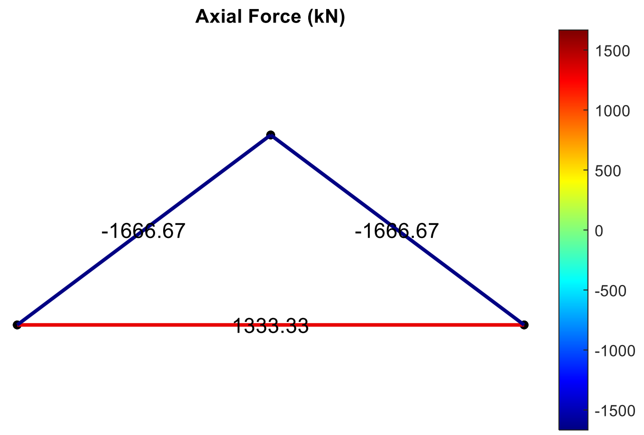

Figure 3. Member axial forces