FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT
ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes
RESSLAB

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

In-class Exercise – Week #8: Zero length elements for material nonlinearity

Exercise #1:

Derive the stiffness matrix for an element that is comprised of a zero-length rotational element and an elastic beam-column element as shown in Figure 1.1 in the basic reference system. Assume that the rotational stiffness of the zero-length element is $n\frac{3EI_e}{L}$ where $I_e = \frac{n+1}{n}I$.

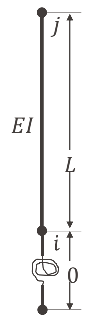

Figure 1.1. Zero length rotational element and elastic beam-column element in series

Hint: By using static condensation, determine the coefficients S_{22}, S_{23}, S_{32} and S_{33} of the stiffness matrix $\hat{\mathbf{k}}_{mod}$ of the elastic beam-column element given by

$$\hat{\mathbf{k}}_{mod} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{S_{22}EI_e}{L} & \frac{S_{23}EI_e}{L} \\ 0 & \frac{S_{32}EI_e}{L} & \frac{S_{33}EI_e}{L} \end{bmatrix}$$

such that a beam-column member can be modeled with the derived elastic beam-column element and a rotational spring.

Recall that the elastic stiffness matrix, $\hat{\mathbf{k}}$ of an elastic beam-column element in the basic reference system is as follows:

$$\hat{\mathbf{k}} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{2EI}{L} \\ 0 & \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$$

Exercise #2:

Write a script for a zero-length rotational element that utilizes the moment-rotation constitutive relation shown in Figure 2.1. Consider both the loading and unload paths in your model. Validate your implementation for the following input parameters:

- $k_e = 400000 kN.mm/rad$
- $\theta_p = \theta_c - \theta_y = 0{,}02\ rad$
- $\theta_{pc} = \theta_u - \theta_c = 0{,}05\ rad$
- $M_y^* = 4000 kN.mm$ and $M_u = 4500 kN.mm$

1. Load case #1: Rotational monotonic loading, $\theta = \{0, 0{,}08\}^T rad$
2. Load case #2: Rotational cyclic loading, $\theta = \{0, 0{,}06, -0{,}06, 0\}^T rad$

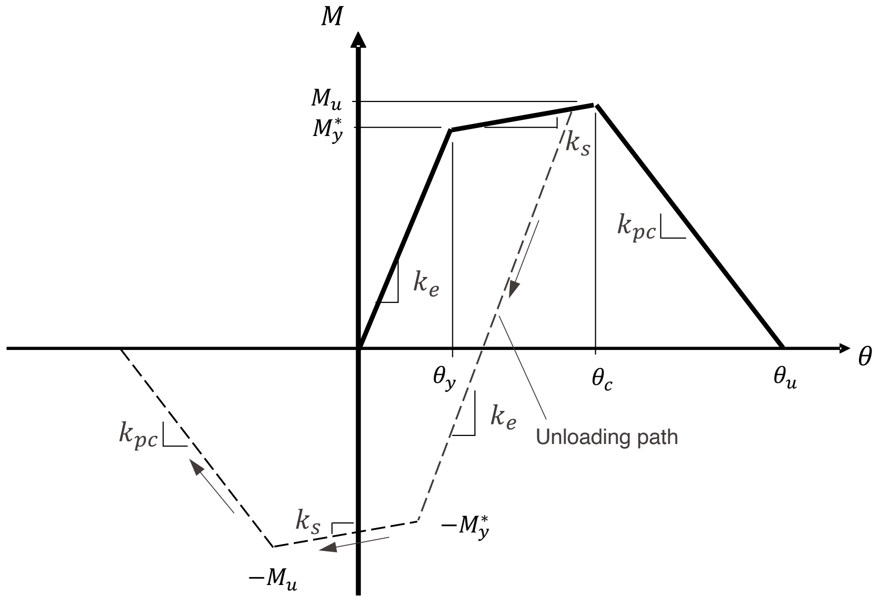

Figure 2.1. Moment-rotation relation for zero length element

Exercise #3:

Consider the following column from Week #5:

$$A = 1.27 \cdot 10^4 mm^2, I = 3.66 \cdot 10^7 mm^2, E = 200{,}000MPa$$

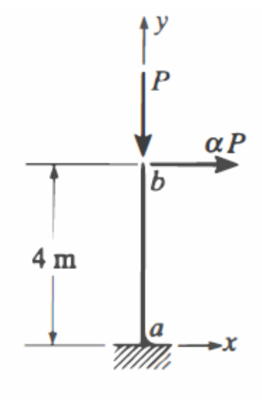

Figure 1. Column under axial and lateral loading

Consider material nonlinearity with the element model you developed in Exercise #2 and the element stiffness matrix you derived in Exercise #1. For a=0.05, determine the load-displacement relationship (secondary equilibrium path) of the cantilever member when:

- $\theta_p = \theta_c - \theta_y = 0{,}02\, rad$
- $\theta_{pc} = \theta_u - \theta_c = 0{,}05\, rad$
- $M_y^* = 4000 kN.mm$ and $M_u = 4500 kN.mm$

1. Would you use a displacement or load-control scheme for your solution? Explain your answer.

2. Compare the computed secondary equilibrium path for the following cases:

    a. Case #1: Linear elastic analysis (from Week #5)
    b. Case #2: Nonlinear geometric analysis and linear material (from Week #5)
    c. Case #3: Nonlinear analysis with material nonlinearity and linear geometric transformation
    d. Case #4: Nonlinear analysis with both material and geometric nonlinearities

3. Calculate the displacement at which the cantilever member reaches zero lateral strength (i.e., collapse) with your program.

Notes:

- The spring should be considered to be n times stiffer than the flexural stiffness of the elastic element:,
    - $k_e^{spring} = \frac{n3EI_e}{L}, I_e = \frac{n+1}{n} I$
- The post-yield stiffness of the spring should be adjusted accordingly,
    - $k_s^{spring} = a_s \cdot k_e^{spring} = \frac{a_s^{member}}{1+n\cdot(1-a_s^{member})} \cdot k_e^{spring}$

- Strain hardening ratio of the member: $a_s^{member} = \frac{M_u - M_y^*}{K_e^{member} \cdot \theta_p}$

- The post-capping stiffness of the spring should be adjusted accordingly,
    - $k_{pc}^{spring} = a_{pc} \cdot k_e^{spring} = \frac{a_{pc}^{member}}{1+n \cdot \left(1-a_{pc}^{member}\right)} \cdot k_e^{spring}$
    - Post-capping hardening ratio of the member: $a_{pc}^{member} = -\frac{M_u}{K_e^{member} \cdot \theta_{pc}}$