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In-class Exercise Week #1: Matrix structural analysis – Suggested solutions

Exercise #1:

What is the magnitude and direction of the force P at point a required to displace that point a vertically downward 5mm without any horizontal displacement? Assume that $E = 200\ GPa$.

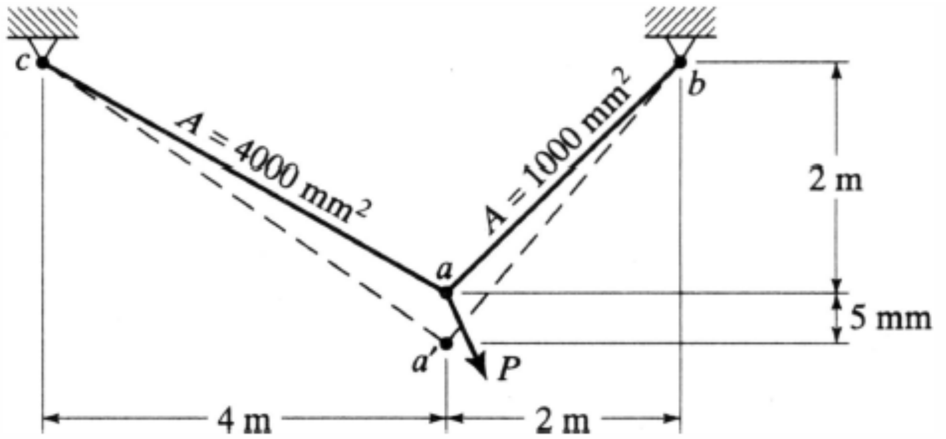
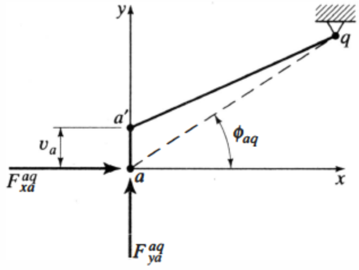


Figure 1. Schematic representation of the truss frame when subjected to the force P

Solution

All degrees of freedom, except v_a, equal zero. $v_a = -5$ mm. From the first and second parts of Equation 2.5, using the nomenclature indicated, with q a typical support point



$$F^{aq}_{xa} = \left(\frac{EA}{L}\right)_{aq} \cdot (\sin\phi_{aq}\cos\phi_{aq}) \cdot v_a$$

$$F^{aq}_{ya} = \left(\frac{EA}{L}\right)_{aq} \cdot (\sin^2\phi_{aq}) \cdot v_a$$

For ab, $\phi_{ab} = 45°$:

$$F^{ab}_{xa} = \frac{200 \times 1000}{2\sqrt{2} \times 10^3}(\sin 45° \cos 45°)(-5) = -176.8 \text{ kN}$$

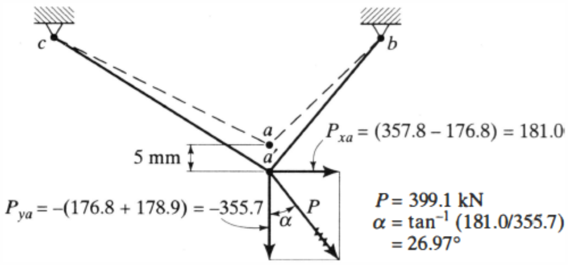
$$F^{ab}_{ya} = 70.71(\sin^2 45°)(-5) = -176.8 \text{ kN}$$

For ac, $\phi_{ac} = \tan^{-1}(2/-4) = 153.43°$:

$$F^{ac}_{xa} = \frac{200 \times 4000}{\sqrt{2^2+4^2} \times 10^3}(\sin 153.43° \cos 153.43°)(-5) = 357.8 \text{ kN}$$

$$F^{ac}_{ya} = 178.89(\sin^2 153.43°)(-5) = -178.9 \text{ kN}$$

By equilibrium, the required force is the vector sum of these components:



Exercise #2:

Two bars ab and bc are pinned together as shown. Bar bc is cooled 40^oC. Determine the displacement of b and the force in the bars. The coefficient of thermal expansion $a = 1.17\ x\ 10^{-5}\ mm/mm^oC$. Assume that $E = 200GPa$.

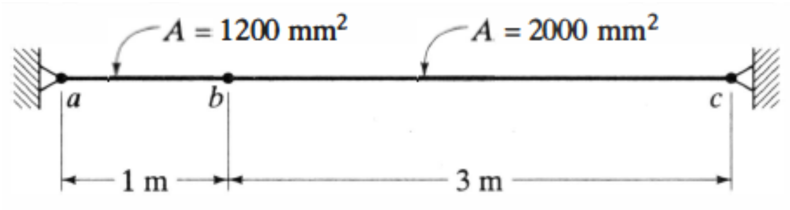
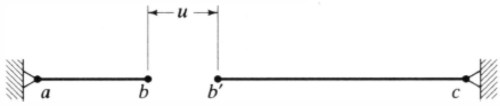


Figure 2. Bar system under temperature loading

Solution

Assume the pin at b is disconnected and calculate the gap due to free thermal contraction of bar bc due to a temperature change, T = 40°C:

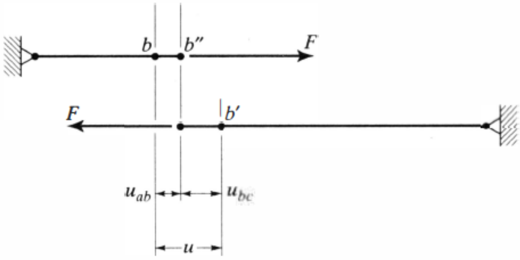


$$u = \alpha L_{bc} T$$
$$= 1.17 \times 10^{-5} \times 3 \times 10^3 \times 40 = 1.404 \text{ mm}$$

Apply equal tensile forces in each bar to close the gap:

$$u = u_{ab} + u_{bc}$$

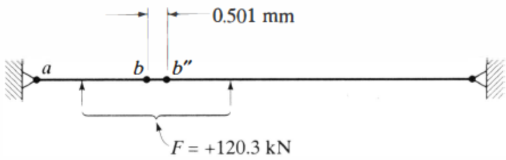
Using flexibility relationships for u_{ab} and u_{bc},



$$u = F\left[\left(\frac{L}{EA}\right)_{ab} + \left(\frac{L}{EA}\right)_{bc}\right]$$

$$1.404 = F\left[\frac{1 \times 10^3}{200 \times 1.2 \times 10^3} + \frac{3 \times 10^3}{200 \times 2 \times 10^3}\right]$$

Solving for F and using the flexibility relationships for u_{ab} and u_{bc},



$$F = 120.3 \text{ kN}$$
$$u_{ab} = 0.501 \text{ mm}$$
$$u_{bc} = 0.903 \text{ mm}$$