



Prof. Anastasios P. Vassilopoulos

Fatigue of Composites

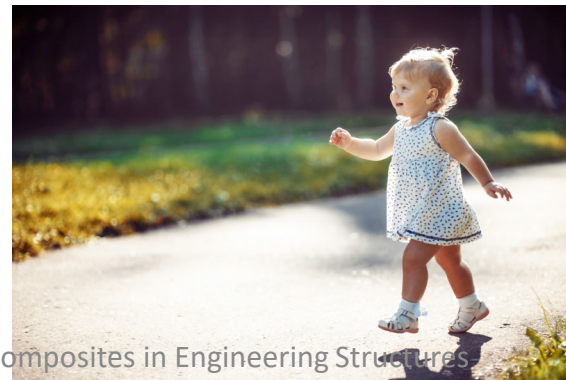
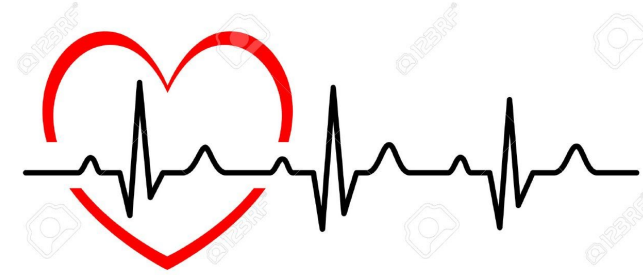
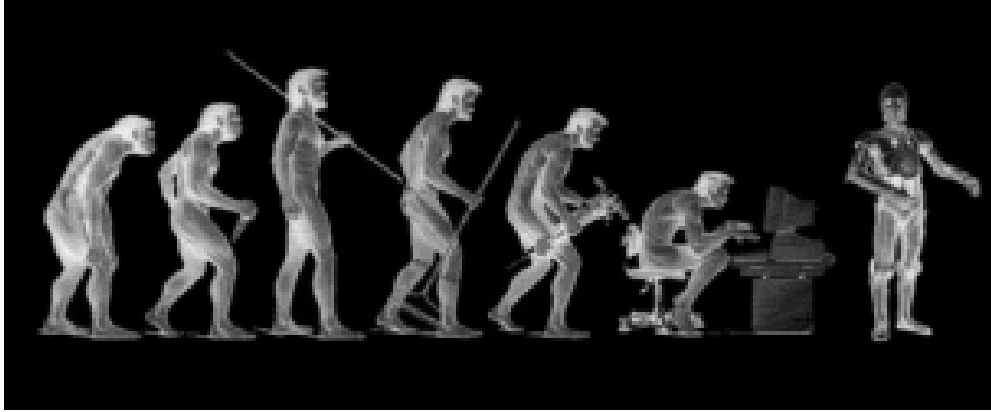
Lecture VIII – Part A: Introduction



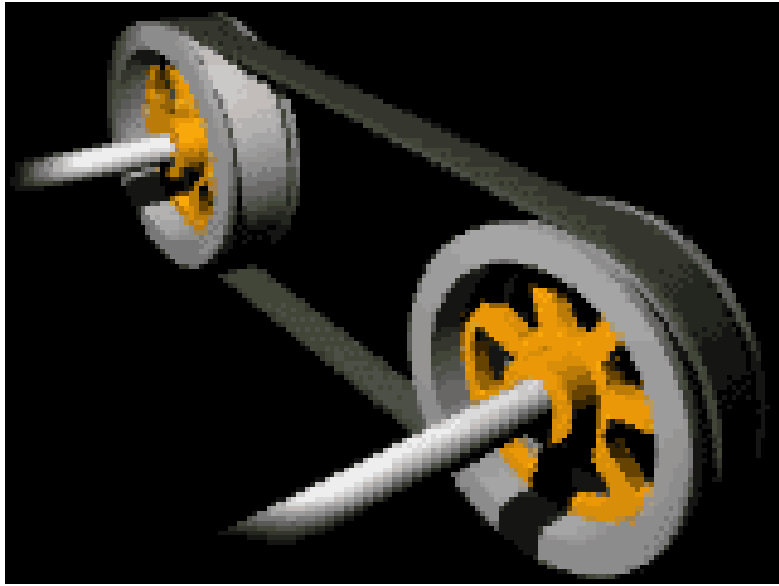
What is fatigue?

- Fatigue is an interdisciplinary subject as depending on who asks the question and in what context a reasonably answer exists!
 - Fatigue in life of humans or animals is connected to continuous activity—“loading”
 - Fatigue of materials is connected to continuous loading—usually under fluctuating loads
 - **A typical definition:** The phenomenon under which properties of a material are degraded due to cyclic loads. Fatigue failure may result after a number of reversals, even if the applied loads are (way much) lower than the material strength.

Physical activity - fatigue



Structures in operation - fatigue



What happens when a structure operates?

Fatigue (failures)

- Fatigue has been the subject of intensive investigations for almost 200 years and, despite the progress made, fatigue failures continue to occur* !
- The first attempts to analyse fatigue behaviour of materials and structures were led by the experience with constructions that were operated under real loading conditions. Everything that is loaded, even if the loads are low, has a finite life.
- It is well documented that **80-95% of structural failures** are due to fatigue or fatigue related phenomena** - Most of them are unexpected failures – fatigue can be considered “...as an essential statistical effect of happenings on a submicroscopic level”***

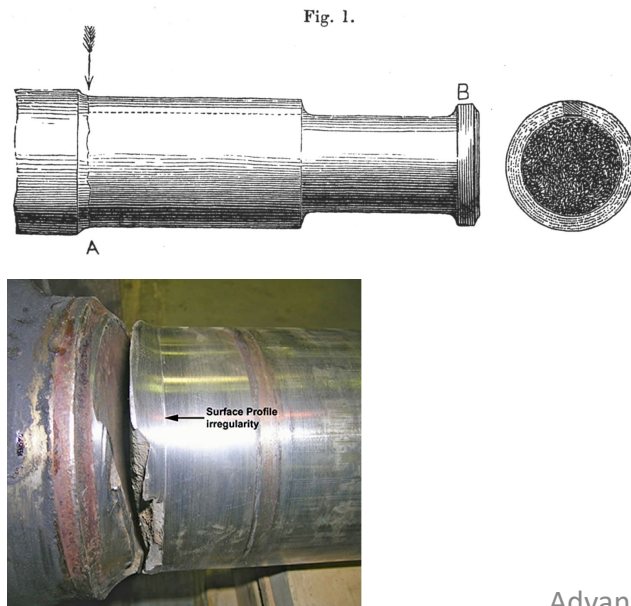
*A. J. McEvily. Failure by fatigue. National Bureau of standards special publication 423, May 1976.G.

** A. M. Freudenthal. Fatigue mechanisms, fatigue performance and structural integrity. Airforce conference on fatigue and fracture of aircraft structures and materials, Miami beach, Florida, Dec. 1969.

*** A. M. Freudenthal. The statistical aspect of fatigue of materials. Proc. Royal Society London A, 187;1946

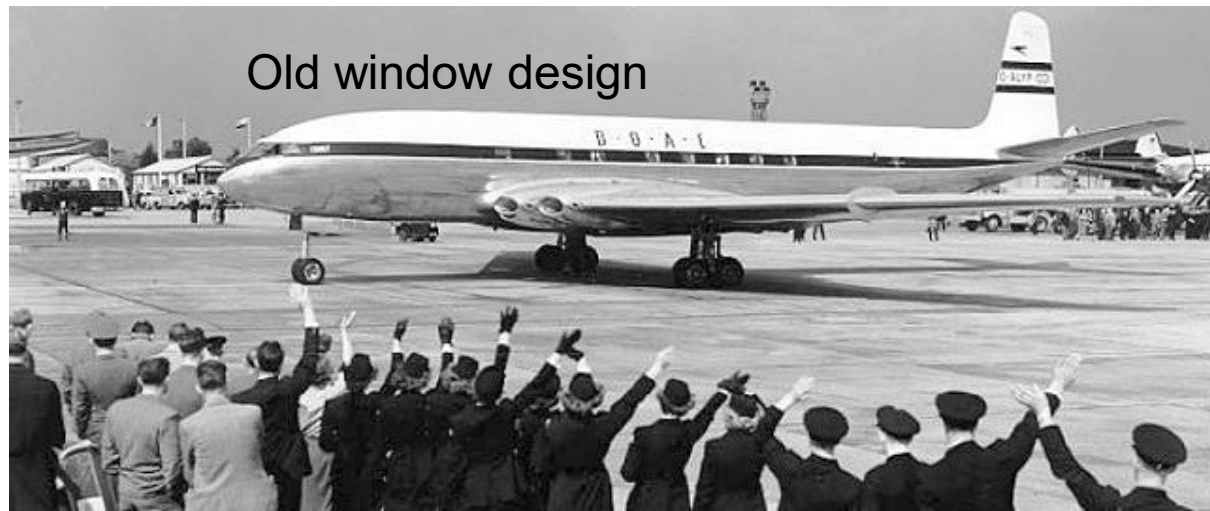
“Early” fatigue failures

- On **May 8, 1842**, one of the trains carrying revellers on their return from Versailles to Paris, having witnessed the celebrations of the birthday of Louis Philippe, derailed and caught fire. The derailment had been the result of a broken **locomotive axle**. **The failure has been attributed to fatigue loading.**

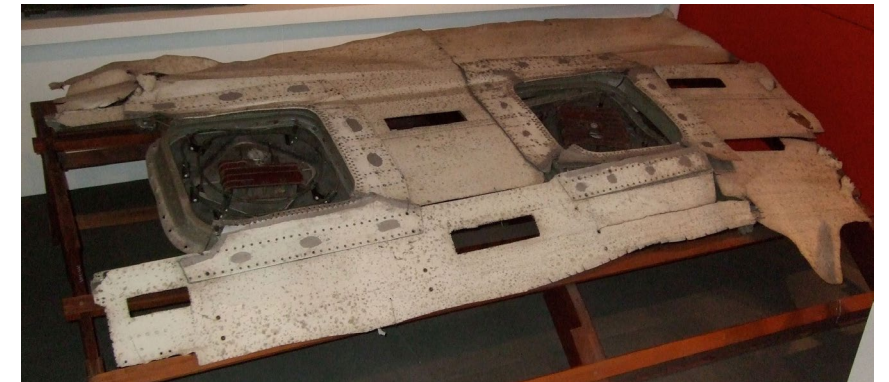


“Famous” fatigue failures

- In **1954** three **de Havilland Comet** passenger jets had broken up in mid-air and crashed within a single year. Investigators found out that **sharp corners** around the plane's window openings acted as initiation sites for cracks. **All aircraft windows were immediately redesigned with rounded corners.**

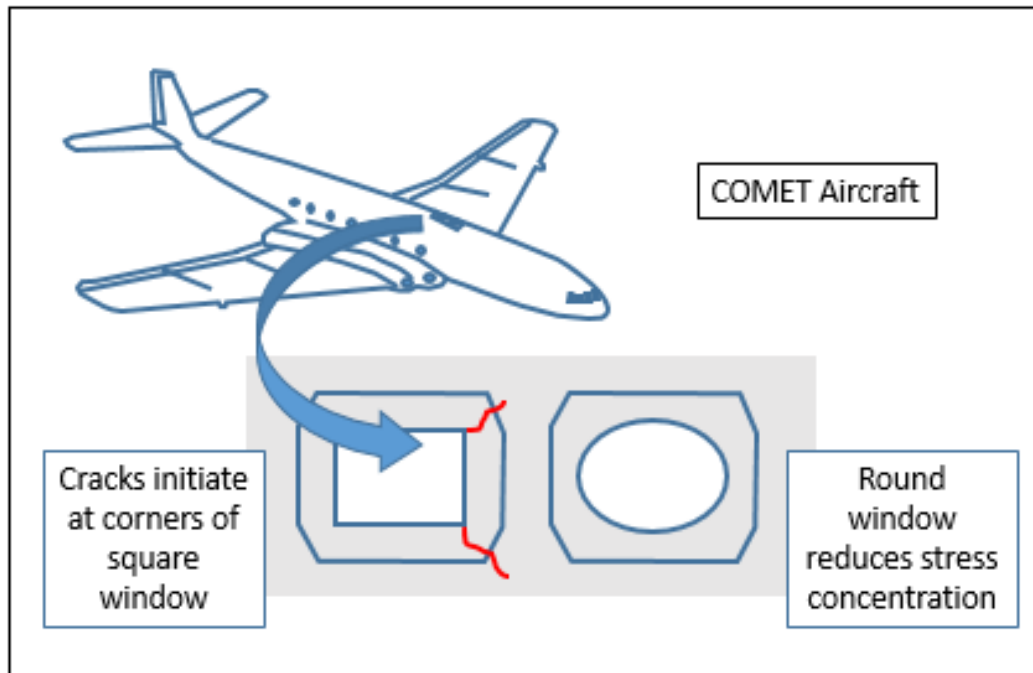


de Havilland Comet passenger jet



Fuselage fragment (Science museum London)

de Havilland Comet failure

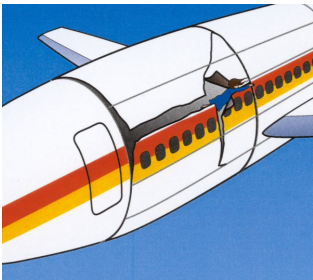


Stress concentrations created by geometry



New window design

The ALOHA Airlines flight 243 (1988)



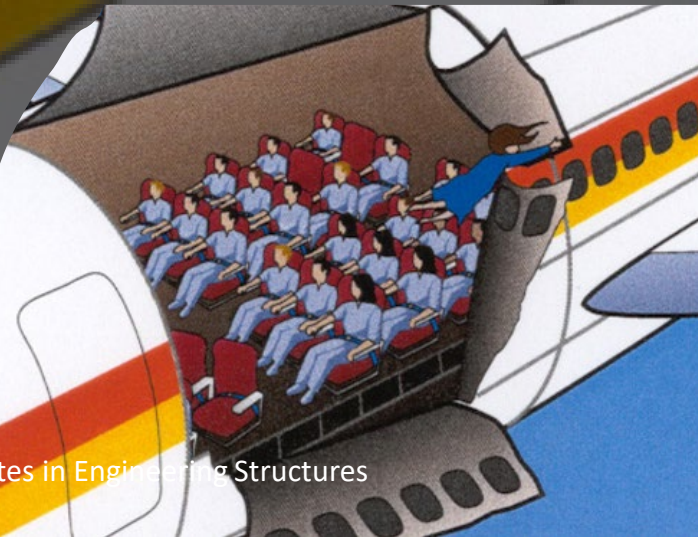
This was a scheduled flight between Hilo (USA) and Honolulu (Hawaii). Due to a weak Boeing's [B737](#)– 297 fuselage, the body of an aircraft, there was an explosive decompression in flight which ripped the top of the fuselage off leaving passengers in scarce air. One cabin crew member was ejected, but unfortunately, his body was never found. The pilot maneuvered the [plane](#) to a safe altitude for breathing and was able to land safely at Kahulu Airport on Maui. There was only one fatality in this airplane crash and another 65 passengers and crew were just injured.

The National Transportation Safety Board (NTSB) blamed a combination of corrosion and widespread fatigue damage, the result of repeated pressurization cycles during the plane's **89,680**-plus flights after of 19 years of service!

The plane was designed for 20 years lifetime and **75000** flights

<http://www.newfoxy.com/2016/10/23/top-5-memorable-airplane-crashes/>

The Aloha 243 flight



Train 9T90 axle failure (Australia 2017)

The Australian Transport Safety Bureau says a fatigue crack on a failed axle has likely caused two derailments on the Mount Isa – Townsville railway line.

The ATSB are drafting a final report into the Derailment of Aurizon train 9T90 at Kimburra, near Pentland, on the night of September 28, 2017.

The train was travelling from Townsville to Phosphate Hill with 1.67 million litres of sulphuric acid.

The ATSB said the leading wheel set of the trailing bogie of a wagon failed and derailed at 72 km/h. The train crew were initially unaware of the axle failure and derailment as it had no noticeable effect on the performance of the train and the train travelled a further 1.3km.

Later, the driver confirmed to the NCC an axle on the train had fractured, allowing one wheel set on the fourteenth wagon to derail.

An examination of the failed axle found it fractured due to **“a fatigue crack that propagated until it reached a critical size resulting in an overstress fracture”** which separated of the axle halves and caused the derailment.

The ATSB said the crack was big enough to be detected at the previous inspection but wasn't and as a result Aurizon increased the number of inspections from April 2018

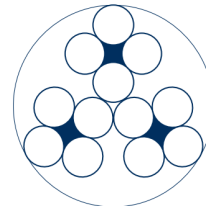


<https://www.northweststar.com.au/story/5878054/atsb-puts-two-derailments-blame-on-cracked-axles/>

History of fatigue – the starting point

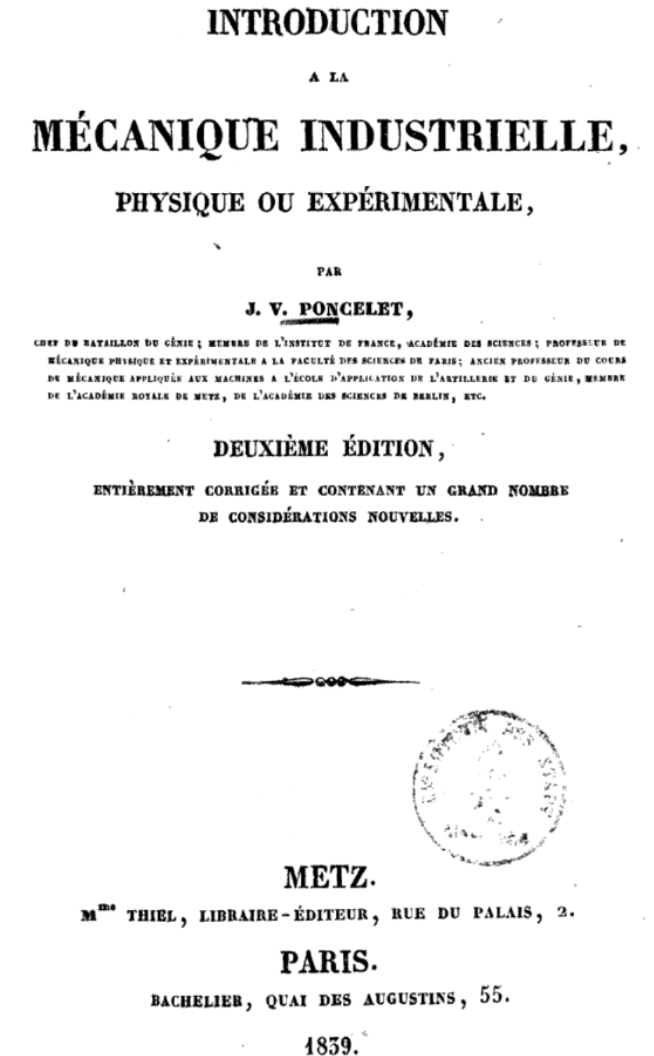
- Fatigue has been identified as a critical loading pattern very early by the scientific community.
- In **1829** the German mine engineer W. A. S. Albert was the first that realized fatigue tests on metallic conveyor chains and later reported his observations.
- In **1837** Wilhelm Albert published the first article on fatigue, establishing a correlation between applied loads and durability.
 - Also invented the wire rope to replace the expensive imported hemp ropes!

[The Albert rope](#)



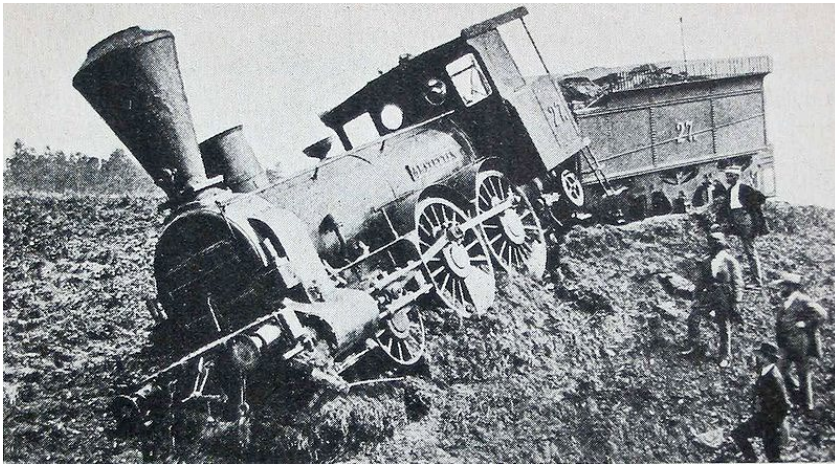
History of fatigue – the starting point

- In **1839** Jean-Victor Poncelet described metals as being tired (fatigue) in his lectures at the military school at Metz.
- The first manuscript where fatigue was referred to is probably the book written in 1841 by Jean-Victor Poncelet, a French mathematician and mechanical engineer. In that book, entitled **“Introduction à la mécanique industrielle physique ou expérimentale”** Poncelet mentioned that any spring subjected to push-pull force will, eventually, break under a load which is far smaller than the static breaking load.



History of fatigue – observations...

- Later, **failure of axles of post coaches** was attributed to fatigue loading. With the development of railways, in the middle of 19th century, failure of wagon's axles was so frequent that attracted the attendance of the engineers.



Still happens today!



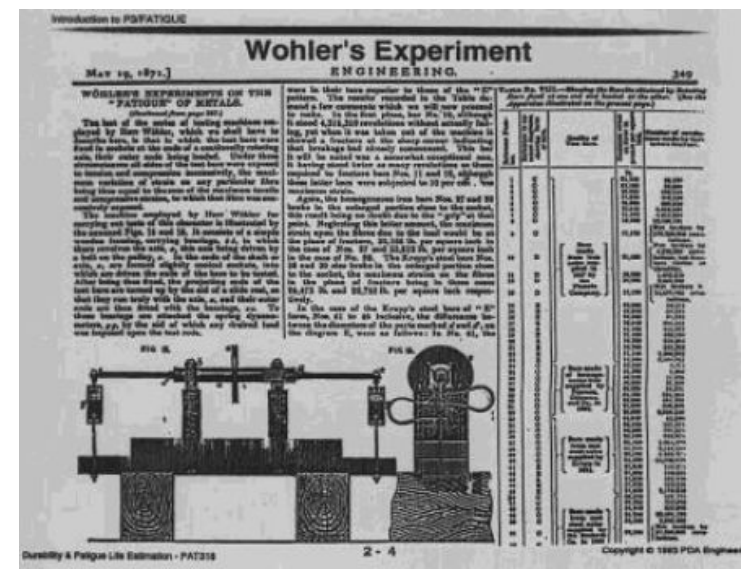
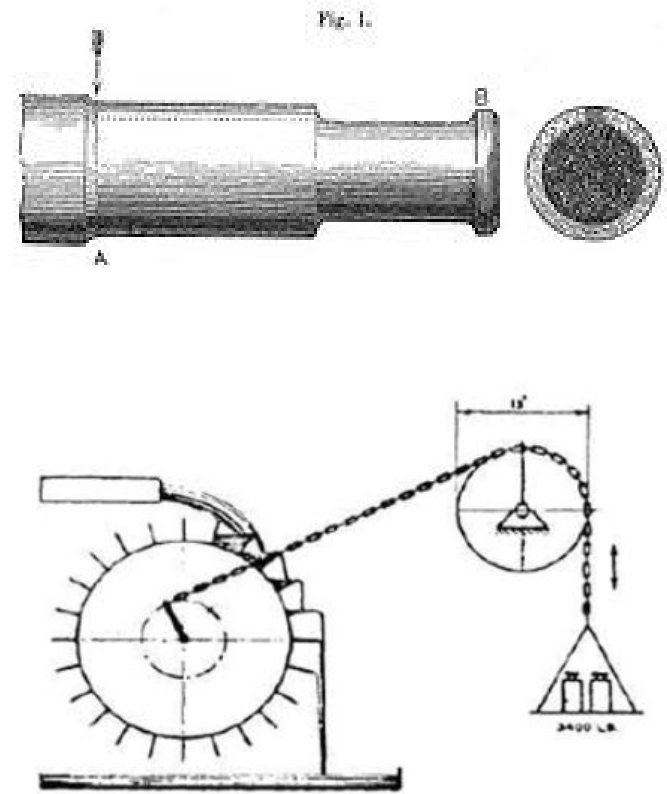
ATSB – Derailment of Aurizon train 9T90 at Kimburra, near Pentland, on the night of **September 28, 2017** – due to a fatigue crack.

<https://www.northweststar.com.au/story/5878054/atsb-puts-two-derailments-blame-on-cracked-axles/>

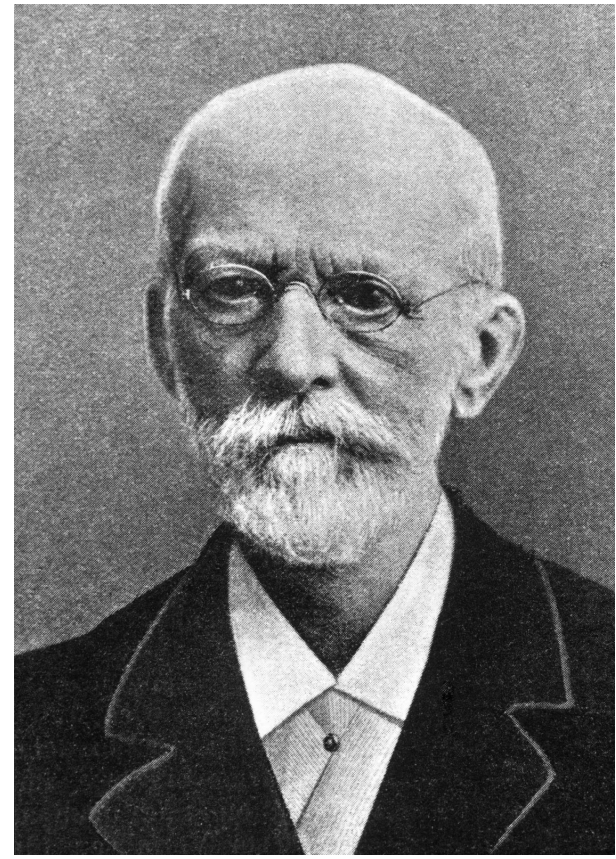
History of fatigue – Implementation...

- **Between 1852 and 1870** German engineer **August Wöhler** realized the first **extended experimental program** on fatigue of metallic materials
 - The program was comprised of full-scale fatigue tests on wagon axles but also of coupon tests under cyclic loading patterns of tensile, bending and torsional loads.
 - **Wöhler** constructed a test rig on which he could test wagon axles under bending moments which were developed by loads that were suspended by the ends of the axles.
 - The developed stresses were recorded along with the number of rotations up to failure. The results were drawn on the σ -N plane **and the first S-N curve (?) was a fact.**

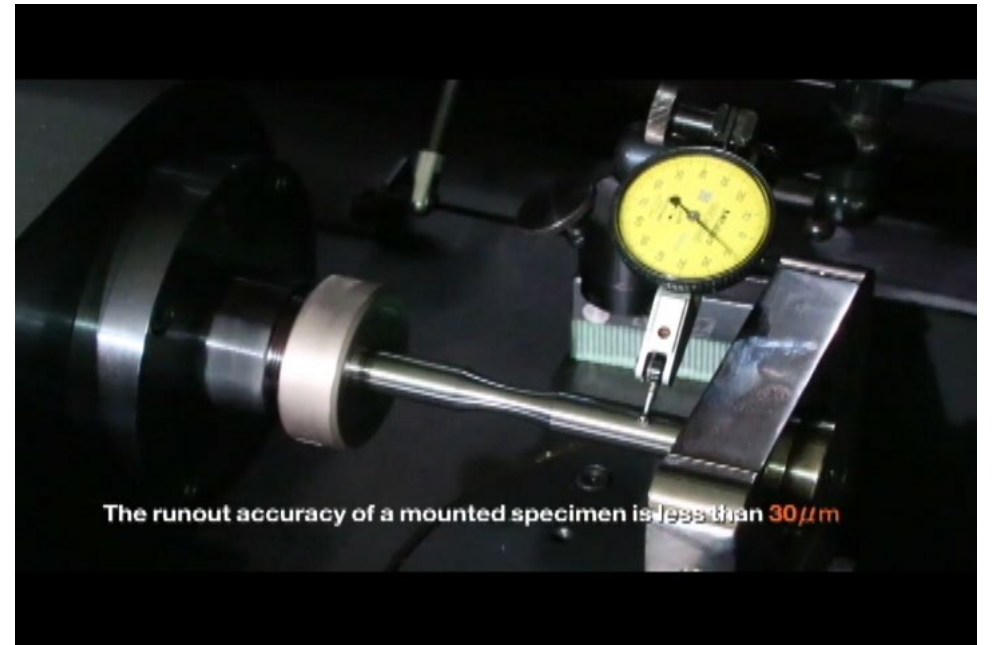
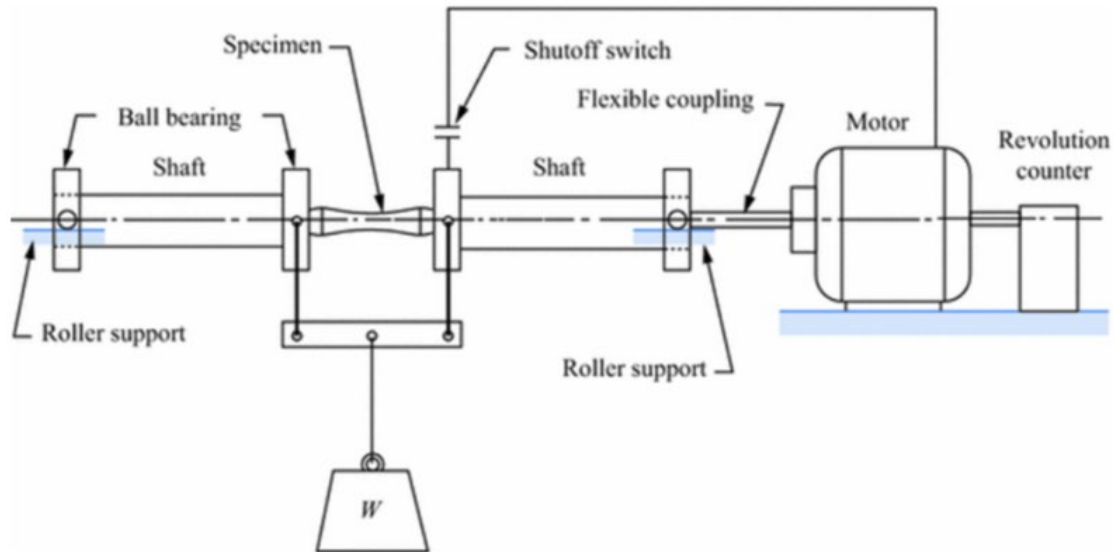
The Wöhler experiment (and publication)



Publication of Wöhler's fatigue experience, 1871



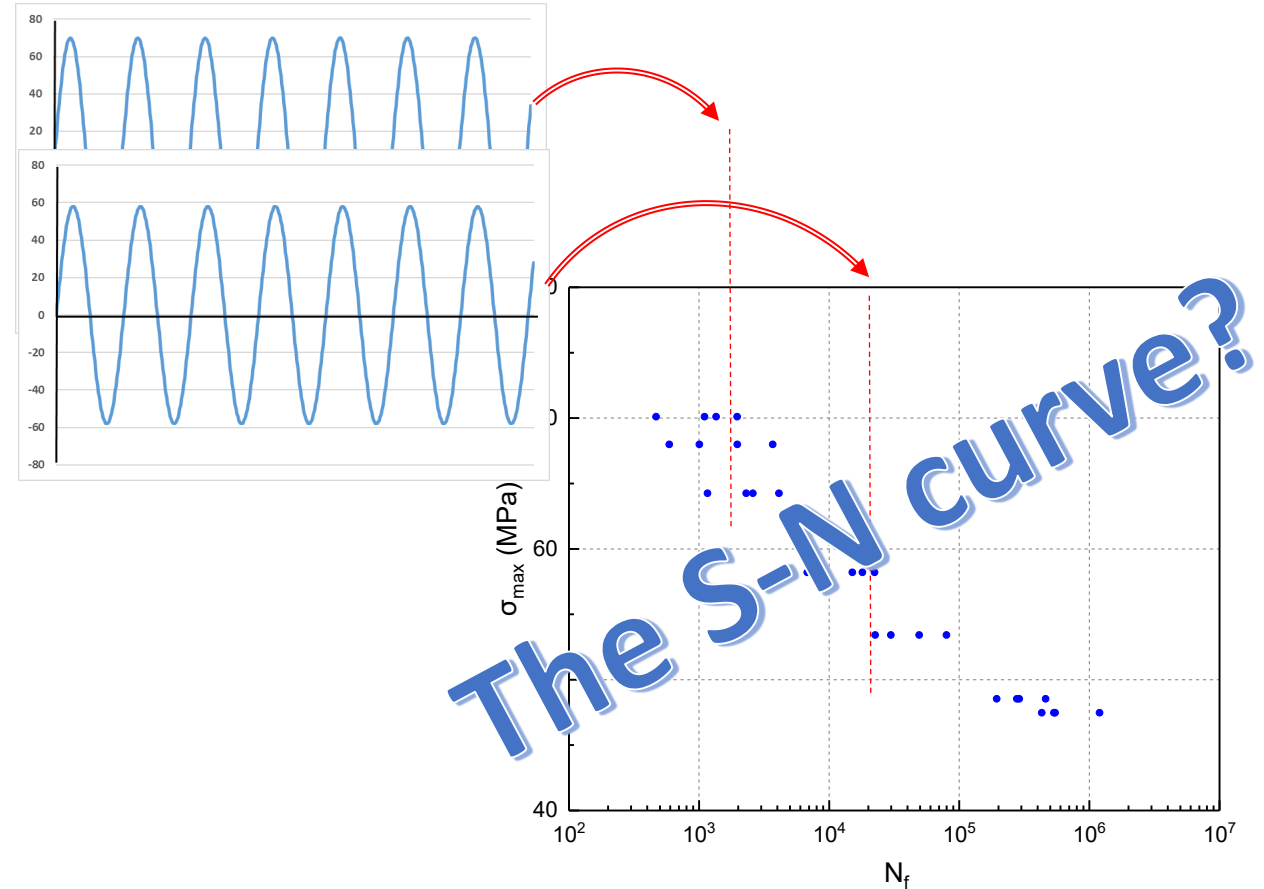
The Wohler experiment



What is the output?



How can I evaluate the loading effect on the lifetime of the axles?



Wöhler supported that the range of stress was more important than the maximum stress (A. Wöhler. "Wöhler's experiments on the strength of metals" Engineering, August 23, 1867, p. 160)

Basquin's curve fitting – The first S-N curve!



And what about the intermediate stress levels?

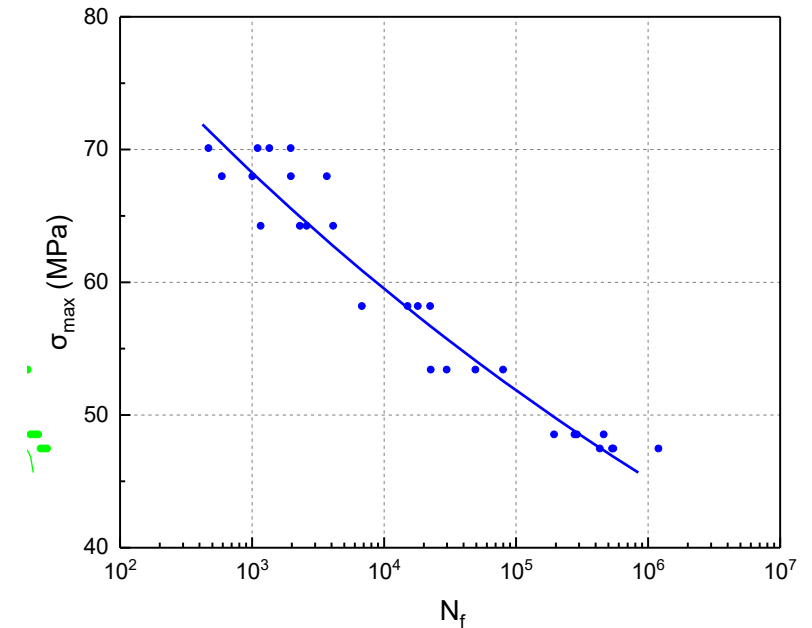
This log-log relationship means that a small change in stress level makes a big change in life (number of cycles)

Basquin, 1910 introduced the log-log linear relationship (after observation of the existing fatigue datasets)

$$\sigma_{max} = \sigma_0 N_f^{-\frac{1}{k}}$$

Olin. H. Basquin "The exponential law of endurance tests"
Proc. ASTM, Vol. 10, Part II, 1910, p 625.

$$\log(\sigma_{max}) = \log(\sigma_0) - \frac{1}{k} \log(Nf)$$



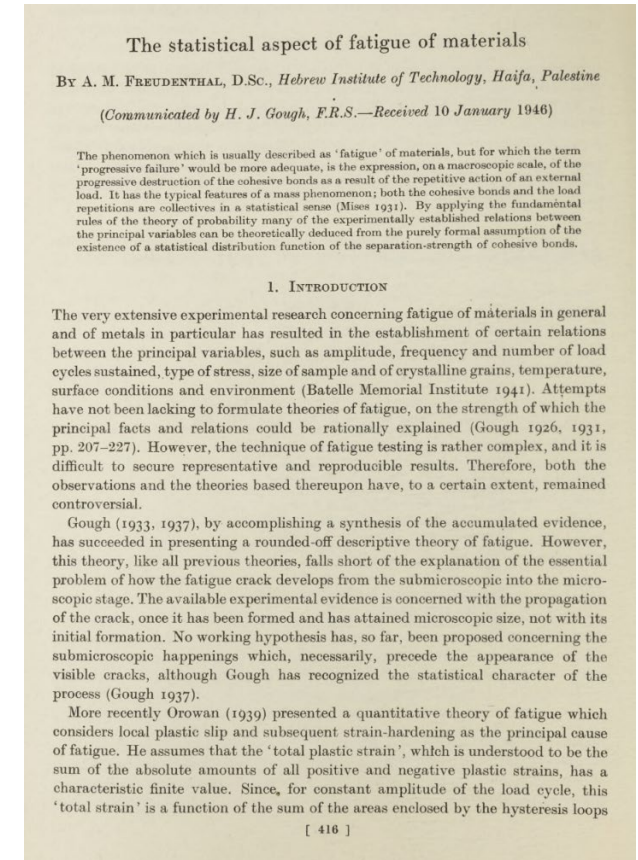
And guess what; We still use the same (Basquin) formulation today!

Curve fitting?

“The relation expresses **the effect of a general mechanism** of fatigue and is more than purely empirical (best fit line of large number of observations) is suggested by the circumstance that it is valid for practically any kind of material, whatever its deformational performance”.

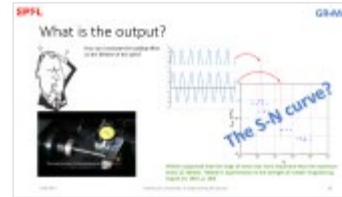
(A. M. Freudenthal “The statistical aspect of fatigue of materials”
Proceedings of the Royal Society of London. Series A. Mathematical and
Physical Sciences, 1946;187(1011):416-429)

Including composites!



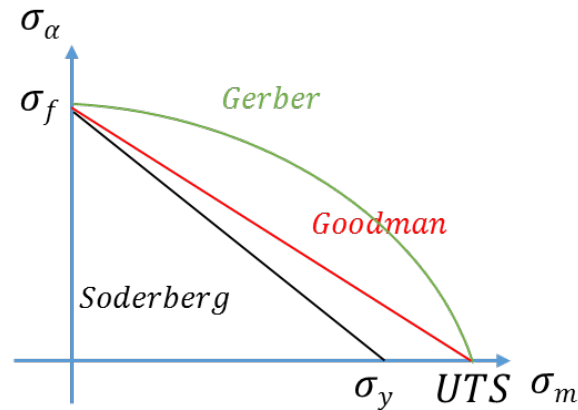
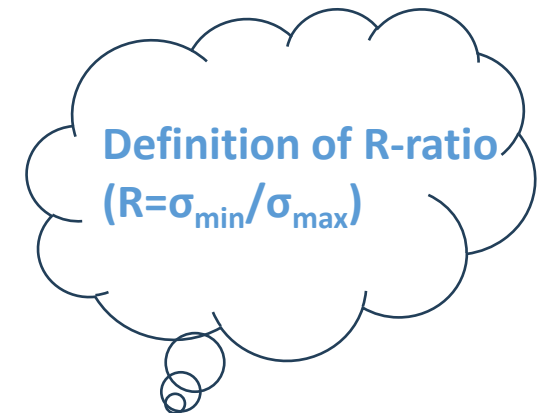
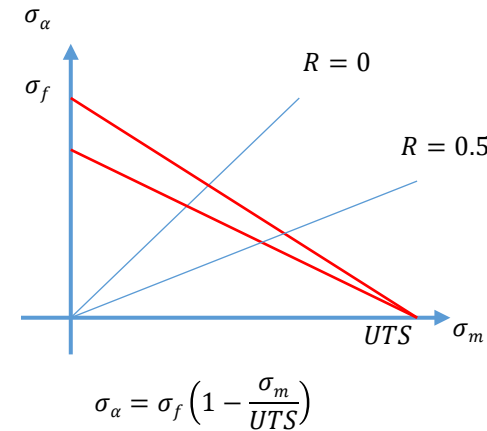
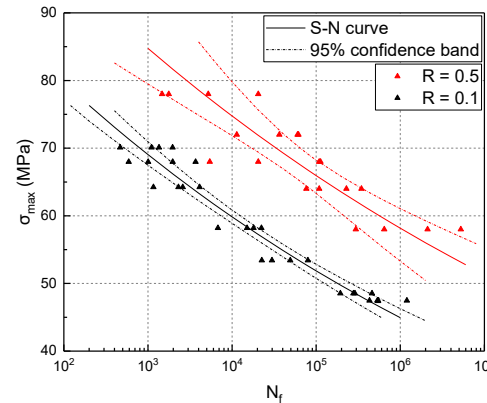
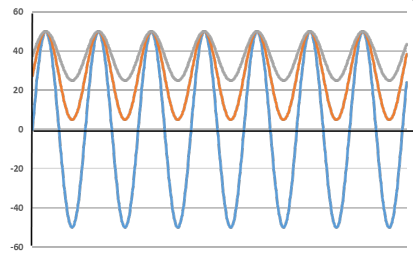
Mean stress effect

- Wöhler supported that the range of stress was more important than the maximum stress



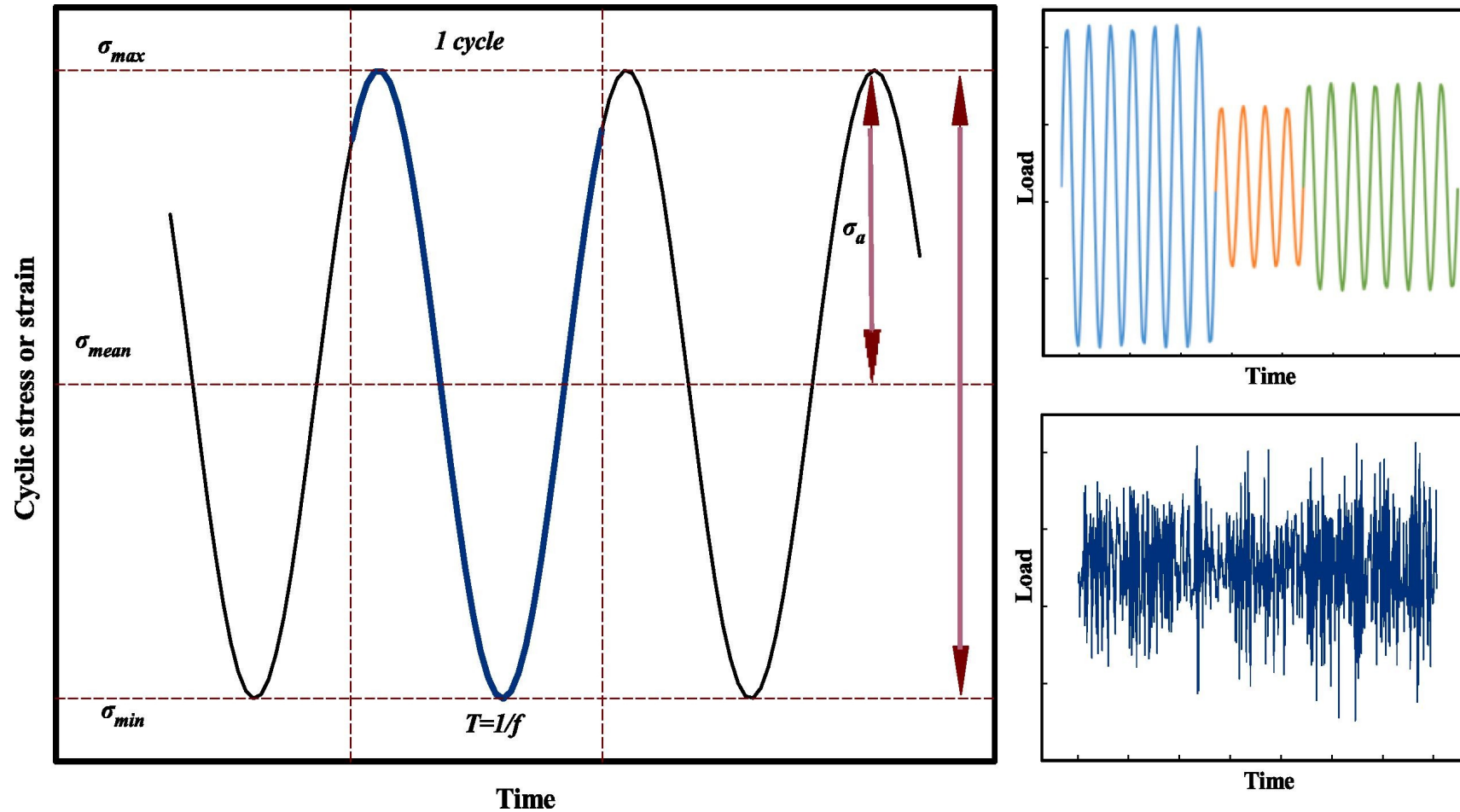
- In 1899 **John Goodman** proposed to quantify the interaction of mean and alternating stress on fatigue life of a material.

Mean stress effect – Goodman diagram



John Goodman (1862-1935) Prof. Civil and Mechanical Eng. Leeds Uni, England. In his publication “Mechanics applied to Engineering” (1st Ed. 1904, 8th Ed. 1914) he wrote: “It is assumed that the varying loads applied to test bars by Wöhler and others produce the same effects as suddenly applied loads”. This statement has been modified for application to actual behavior and gives what is called the “modified Goodman diagram” for mean stress.

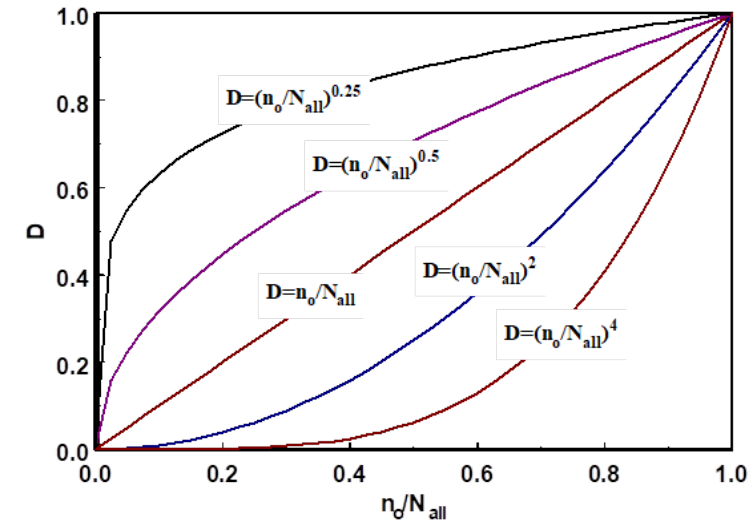
Is that (CA) loading realistic?



[A. P. Vassilopoulos. "The history of fiber-reinforced polymer composite laminate fatigue" Int J Fatigue, 2020;134:105512](#)

Block and variable amplitude loading

- In **1925** Palmgren developed a linear cumulative damage model (although not a lot was known about block and/or variable amplitude loading yet).
 - He has also introduced a four-parameter equation for the S-N curve



- In **1939** Gassner emphasized the importance of variable amplitude loading and promoted the use of an eight-step block loading spectrum for simulation tests.
 - However variable amplitude loading was not very often applied before the **1950s** and the development of the closed-loop electrohydraulic test systems

References

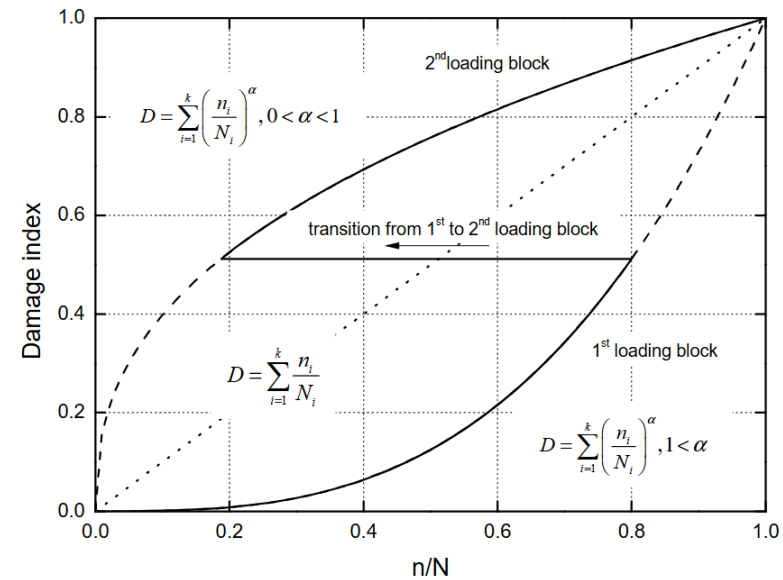
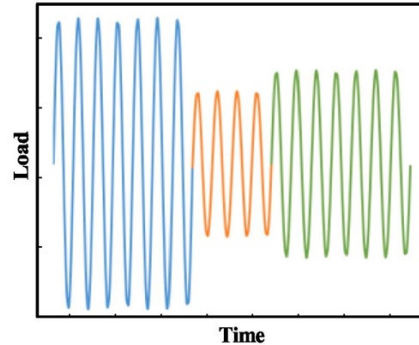
A. Palmgren "Die Lebensdauer von Kugellagern" ZDVI **1924**;68(14):339

E. Gassner "Festigkeitsversuche mit Wiederholter Beanspruchung im Flugzeugbau" Deutsche Luftwacht, Ausg. Luftwissen, **1939**;6:43

Block and variable amplitude loading

- In **1945** Miner “re” introduced the linear cumulative fatigue damage rule

$$\sum \frac{n_i}{N_{fi}} = 1$$

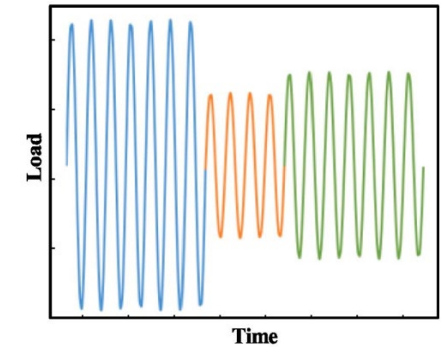
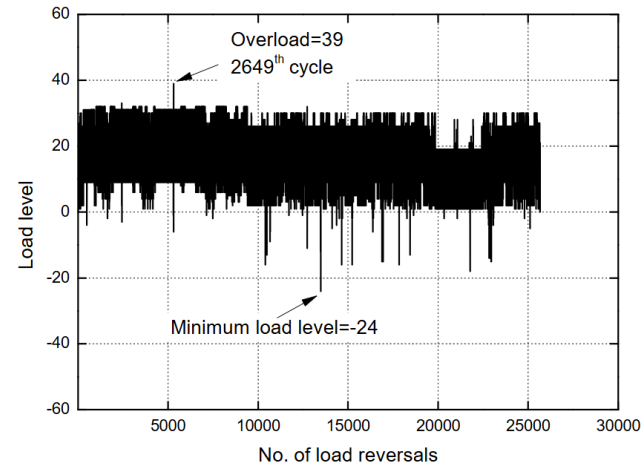
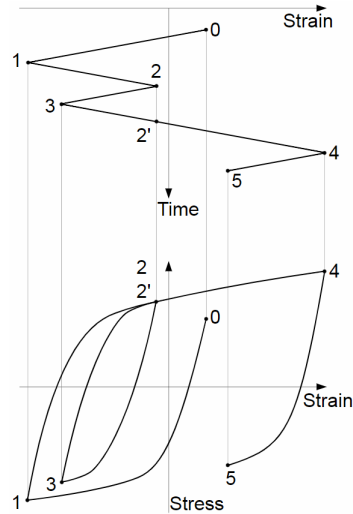


References

M. A. Miner “Cumulative damage in fatigue” Trans. ASME, J. Appl Mech, **1945**;67:A159

Block and variable amplitude loading

- In **1968** Matsuishi and Endo introduced the rainflow counting method



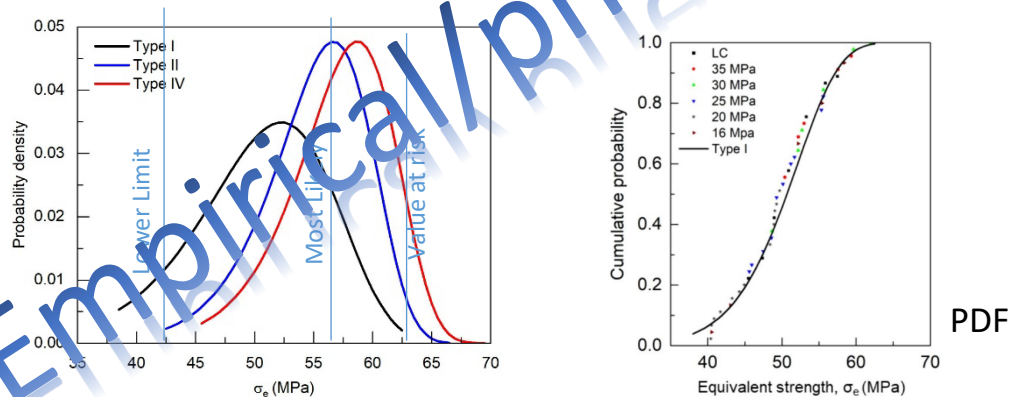
Sequence is lost!

References

M. Matsuishi, T. Endo "Fatigue of metals subjected to varying stress" The Japan Society of Mechanical Engineers, Fukuoka, Japan, 1968

Weibull or Gaussian distributions?

- In **1951** W. Weibull introduced the “Weibull” distribution.
- Weibull carried out thousands of tests on bolts and aluminum to develop a statistical probability distribution for failures.
- He found that fatigue failure probabilities could not be described using classic Gaussian distributions and instead developed his Weibull distribution which included a shape function.



Ernst Hjalmar Waloddi Weibull



WEIBULL, Ernst Hjalmar
WALODDI.
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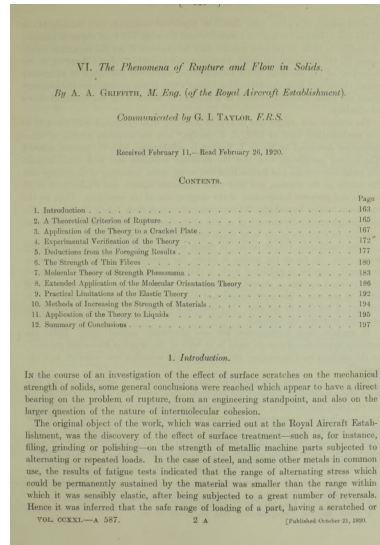
https://en.wikipedia.org/wiki/Waloddi_Weibull

Most investigations based on observations

- Empirical modeling following experience
- Physical modeling/ physics-based understanding of fatigue failure.

Fracture mechanics

- It was Alan Arnold Griffith (1920) who published* the results of his theoretical calculations and experiments on **brittle fracture** using glass.
Griffith developed the basis for fracture mechanics – he thus became the “early father” of fracture mechanics



He introduced the “Griffiths criterion”

$$\sigma\sqrt{\alpha} = const$$

To explain this behavior with thermodynamics theory introduced the “strain energy release rate” known as “ G ” which can physically be understood as: the rate at which energy is absorbed by growth of the crack:

$$G = \frac{\pi\sigma^2\alpha}{E} \quad \text{critical } G, \text{ or } G_c = \frac{\pi\sigma_c^2\alpha}{E}$$

*Griffith, A. A. (1921). "The Phenomena of Rupture and Flow in Solids". Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences. 221 (582–593): 163–198. doi:10.1098/rsta.1921.0006 (open access).

Fracture mechanics

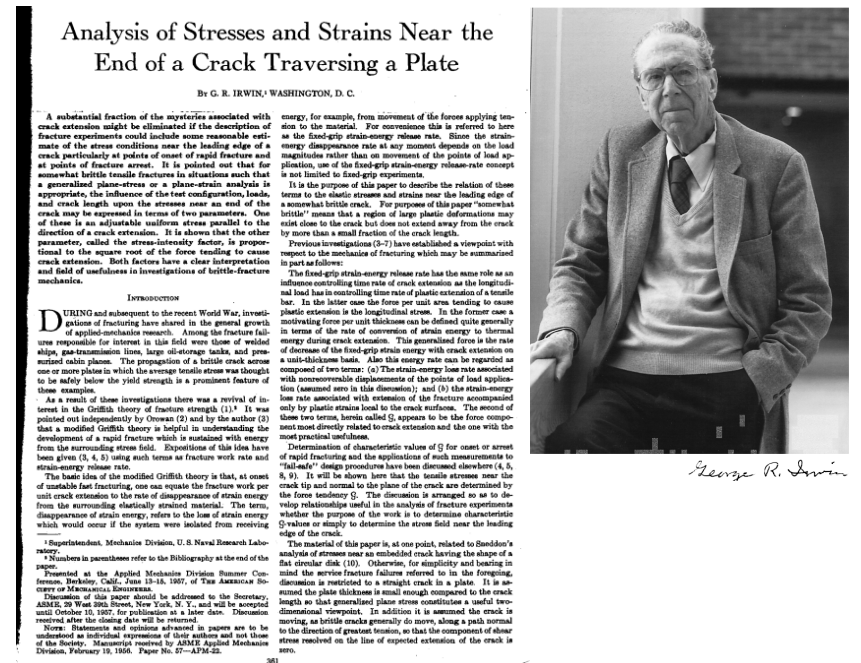
- In **1957** Irwin introduced* the concept of the stress intensity factor (K_I) setting the basis for the linear elastic fracture mechanics.
Irwin invented the term “fracture mechanics” and is assumed as the modern “father of fracture mechanics”

Most engineering materials show some nonlinear elastic and inelastic behavior

He introduced the “Stress intensity factor” to estimate the energy in linear elastic media:

$$\sigma\sqrt{\pi a} = K_I$$

$$K_c = \begin{cases} \sqrt{EG_c} & \text{for plane stress} \\ \sqrt{\frac{EG_c}{1-\nu^2}} & \text{for plane strain} \end{cases}$$

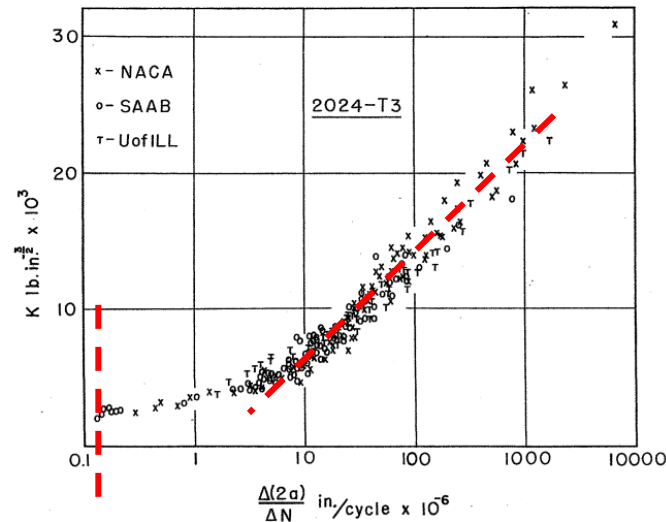


*Irwin G (1957), Analysis of stresses and strains near the end of a crack traversing a plate, Journal of Applied Mechanics 24, 361–364 .

Fracture mechanics

- Paris in **1961** showed that the fatigue crack growth rate (da/dN) is a function of the stress intensity factor range ΔK , (describing what is called today a fatigue crack growth curve - FCG)

$$\frac{da}{dN} = C \cdot \Delta K^m$$



A Rational Analytic Theory of Fatigue

PAUL C. PARIS
Assistant Professor of Civil Engineering

MARIO P. GOMEZ* and WILLIAM E. ANDERSON
Research Engineers, Boeing Airplane Company



P. C. Paris



M. P. Gomez



W. E. Anderson

A great deal of effort has recently centered around examination of the factors influencing the growth of fatigue cracks. Fatigue has been considered a multi-phase problem: e.g., initiation of a crack and its growth are often considered as separate phenomena. In contrast, the objective of this work is to show that the growth of an initial "crack-like" imperfection to a critical size, which causes static failure of a structure, may be described by a single rational theory.

Two loading parameters, the nature of the stress field near the tip of a crack and the variation of this field, are taken to control the rate of crack extension in a given material. This hypothesis is proven by using it to correlate data from three independent investigators. Since it shows a positive correlation of all available data for crack-extension rates from 10^{-7} to 10^{-2} in. per cycle, the hypothesis may be used to formulate a theory of fatigue that permits computing the structural lives of complicated geometries from simple laboratory tests of material properties.

The Stress Distribution Near the Tip of a Crack

The form of the stress distribution in the vicinity of a crack root was given by Sneddon¹ in 1946 and has recently been expanded by Irwin^{2,3} and Williams.⁴ The unique character of this form, as Irwin showed,³ is a controlling factor in attempts to analyze crack extension under static loads. We will show that this same character becomes fundamental in crack extension under cyclic loading upon the addition of new concepts to describe the cyclic nature of the loading.

* Mr. Gomez received his M. S. degree in Metallurgical Engineering in 1958 at the University, after which he worked for Boeing. He is now Senior Scientist at the Missile Systems Division of Lockheed Aircraft Corporation.

Restricting this discussion to cracked bodies in which the geometry and loading of the body are symmetric with respect to the plane of the crack results in very little loss of generality. The nature of cracks

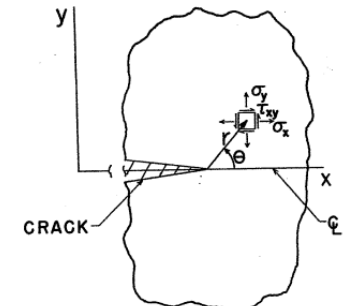


FIG. 1. COORDINATES USED TO DESCRIBE STRESSES NEAR A CRACK TIP (θ ; σ_x ; τ_{xy} ; σ_y)

is to form most often on such planes, i.e., planes perpendicular to maximum-principle tension stresses. Williams⁴ and Irwin³ have given the required forms of stresses for other cases, but these will not be discussed further in this work.

The coordinates of points in a cracked body with

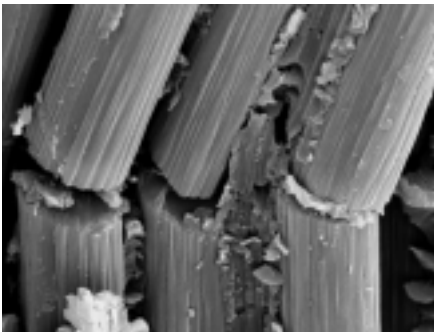
JANUARY, 1961

The Trend in Engineering 13, 9-14 (1961)

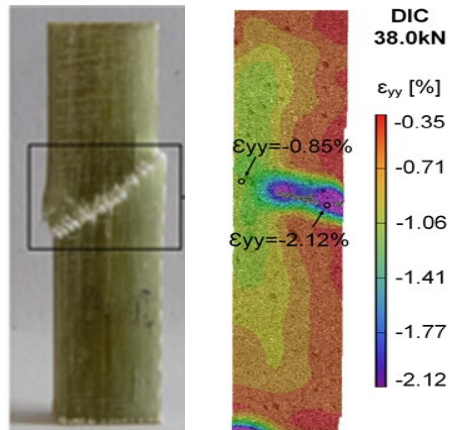
* Paris, P. C.; Gomez, M. P.; Anderson, W. E. (1961). "A rational analytic theory of fatigue". The Trend in Engineering. 13: 9-14

Fracture mechanics – supported by imaging methods

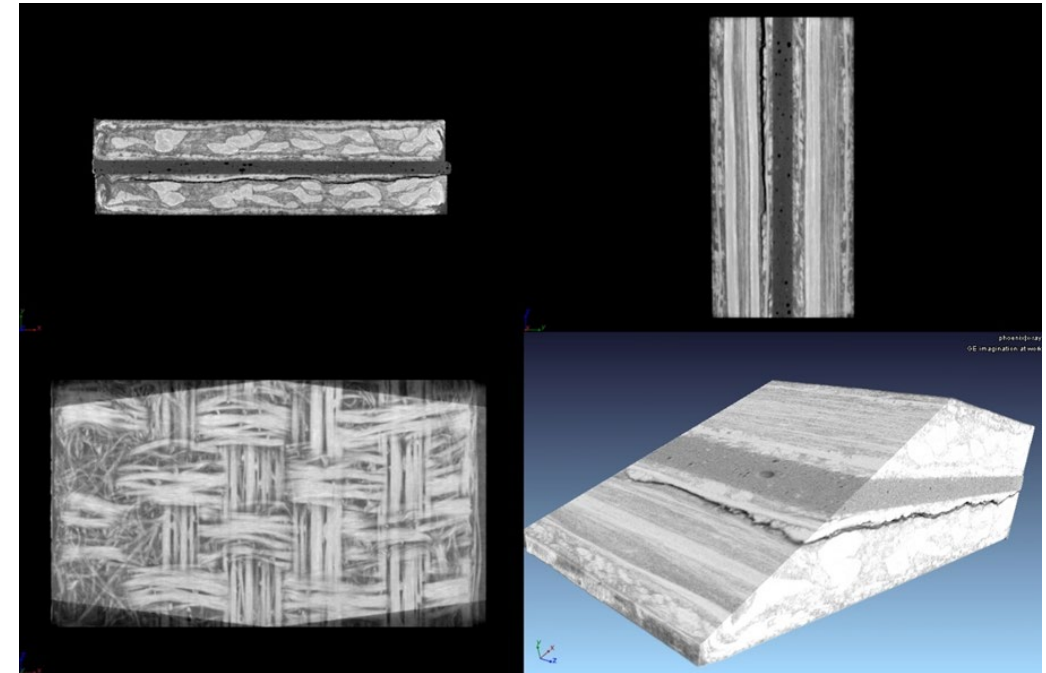
- It was in early **1900s** when Ewin and Humfrey* used the optical microscope to pursue the study of fatigue/fracture mechanisms
- In the **1950s** the scanning electron microscope
- Today DIC/ x-ray tomography



SEM: Fiber kinking due to compression



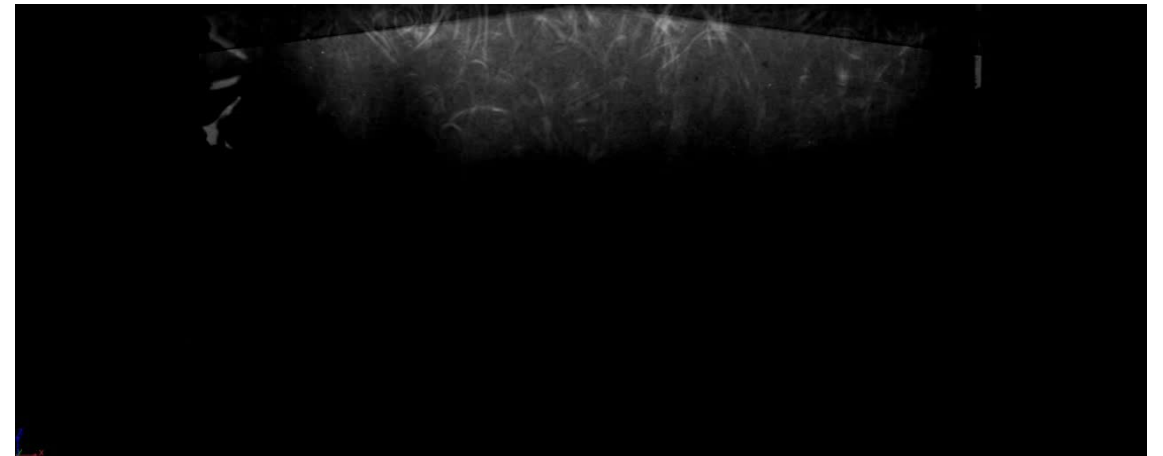
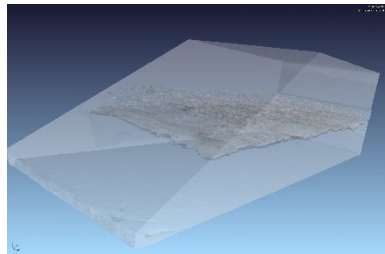
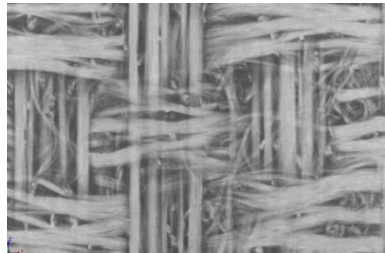
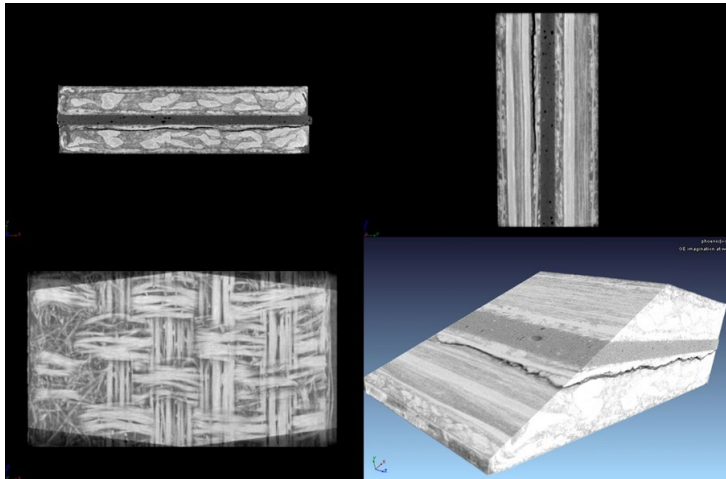
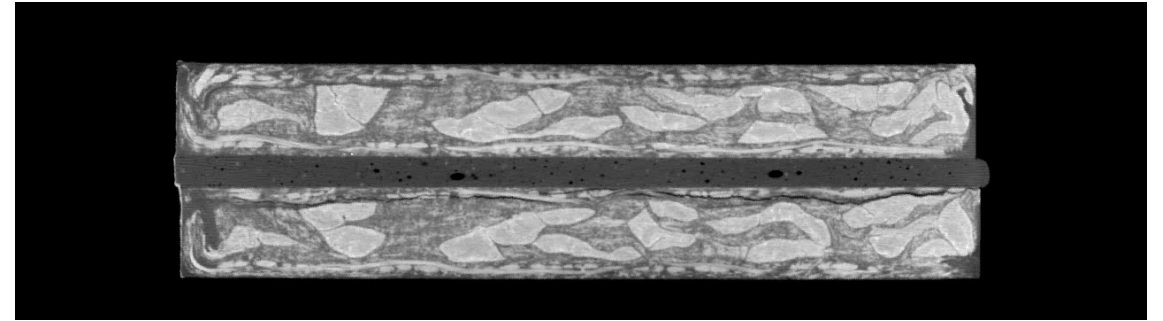
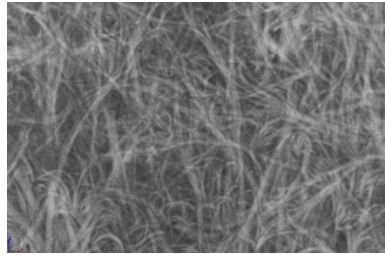
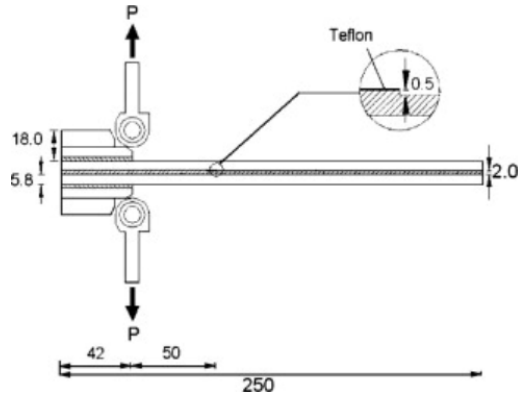
DIC measurement of strain fields



Tomography: Architecture of, and crack in an adhesively bonded joint

*J. A. Ewin, J. C. W. Humfrey. "The fracture of metals under repeated alternations of stress" Philosophical transactions of the royal society A. Mathematical, physical and engineering sciences, 01 Jan 1903

X-ray digital tomography



And composites?

- Yet, nothing was said about fatigue of composites !!!
- Do they actually fail?

Composites are “fatigue insensitive” (?)

Research on carbon fiber-reinforced polymers showed no fatigue failures observed at reasonable numbers of cycles at stress levels below 70% of the mean static strength of the specimens, and therefore, **CFRP materials were designated as fatigue-resistant materials**, (Fatigue insensitive) showing less steep S-N curves than those derived for GFRP laminates.

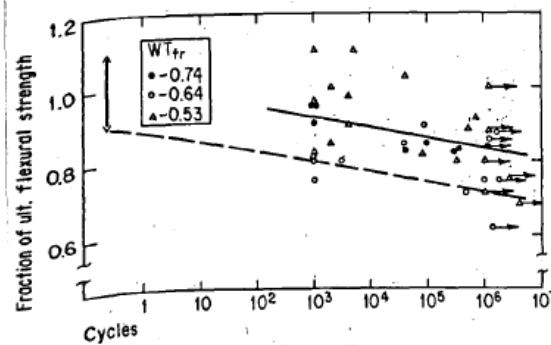


Fig. 9: Normalized flexural curve for polyester carbon fiber RP.

By M.J. Owen* and S. Morris*

Modern Plastics, April 1970

Fatigue resistance of carbon fiber RP

An isophthalic polyester and an epoxy resin were used with untreated high-modulus carbon fibers to prepare unidirectional and cross-ply specimens by the wet layup and prepreg methods. Fatigue properties were evaluated in zero-tension axial and zero-moment flexural modes. **No fatigue failures were observed at stress levels below 70% of the mean static strength of the specimens.** The fatigue behavior of cross-ply carbon fiber laminates is similar to that of unidirectional composites and contrasts with the behavior of glass-reinforced plastics wherein initiation and propagation of debonds produces failures at relatively low stress levels.

A technical feature

Gordon M. Kline, technical editor

form (10,000 filaments) in lengths up to 39 in. The tensile properties supplied by the makers are as follows:

Tensile strength	
Mean ultimate, p.s.i.	271,000
Coefficient of variation, %	25.5
Tensile modulus	
Mean, 10 ⁶ p.s.i.	59.5
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Resins. In most of the work an isophthalic polyester resin (Cellobond A2784, B.P. Chemicals Ltd.) was used in wet layup preparation. One percent of MEKP-RGS (Laporte Industries Ltd.) was used in conjunction with 0.25% of cobalt naphthenate. After gelation, the composites were post-cured for at least 3 hr. at 80° C.

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Importance of fatigue for composite structures

There is a long list of reasons why fatigue is critical for composite structural components or structures*:

- Composites are used for critical structural components and nowadays they participate as a material candidate equal to the traditionally used steel, aluminum or concrete, in emerging structures that must bear significant fatigue loads during operation, such as airplanes, wind turbine rotor blades, leisure boats, foot and vehicular bridges etc. This development changes the common perception concerning the sensitivity of each structure to fatigue. **For example, whereas a concrete road bridge is normally not fatigue-sensitive since the dead loads are significantly higher than the live loads, fatigue becomes an issue for a lightweight composite bridge.**
- Unidirectional composite materials are generally **brittle and behave linearly under load**. Since their failure is sudden, without any prior notice, An understanding of their fatigue behavior and prediction of their fatigue life are of major importance.
- An understanding of composite material fatigue behavior is also valuable for the improvement of product development practices. The hitherto followed product development practice was based on an iterative process whereby a prototype was built and tested against real, or realistic, loading patterns. However, this process is costly and time-consuming. **The ability to simulate the fatigue behavior of the material, structural component and/or structure reduces the cost and allows the development of a wider range of products without the need for increasing the number of physical prototypes.**
- The durability of composite structures is also an important factor. The danger of evaluating durability on the basis of static strength calculations is that the durability impact of cyclic loads is likely to be disregarded. The introduction of fatigue life prediction methodologies into durability simulation procedures allows **the assessment of durability performance early in the product development process and the establishment of clear recommendations for guiding major design choices.**

* A. P. Vassilopoulos, T. Keller. Fatigue of fiber reinforced composites, Springer, 2011

Fatigue of composites

- R-ratio and frequency effect – self-generated heating
- Viscoelastic behavior (matrix and fiber)
- No obvious threshold
- Influence of the constituents and the fiber architecture
- Different tension and compression fatigue behavior → Goodman diagram not applicable

A literature search (www.scopus.com) with keywords “fatigue” and “composites” in the disciplines “Engineering”, “Materials science”, “Energy”, and “Multidisciplinary” in 2010 gave ca. 9500 research articles in the field, with more than 85% of them published after 1980, and around **400** articles per year after **1995**. The same search today gives more than 26000 research articles, an average of ca. **1200** per year since **2015**. This immense increase shows, among others, the high interest of researchers, engineers, and industry in this field*!

Investigations on fatigue of composites started very early, with the introduction of these (structural) materials - in the 1950s.

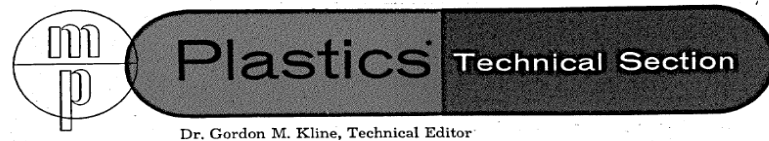
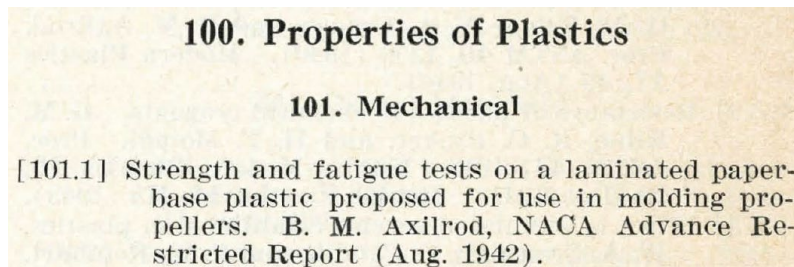
In 1955 Hulbert** wrote “Fatigue and creep are such important aspects of dynamic and static behavior that, among the welter of data accumulated around reinforced plastic materials, **it is astonishing to find little or nothing published about either property**”

* A. P. Vassilopoulos. Fatigue life prediction of composites and composite structures – second edition, 2019, Woodhead Publishing

**G. C. Hulbert. Plastics Structures. J. Royal aeronautical society, 1956;60:114-120

First studies on composites' fatigue

Investigations of the fatigue behavior of composite materials were initiated together [with their introduction as structural elements in several engineering applications, back in the 1950s.](#)



Fatigue strength values are presented for 6 standard and 4 heat-resistant resin laminates reinforced with glass fibers. Fifty-three S-N curves, representing fatigue data between 1 thousand and 10 million cycles, show the effect on fatigue strength of notching, moisture, fabrics, resins, mean stress levels, angles to warp, and temperatures up to 500° F.

45° to the warp at a test frequency of 900 cycles per minute. Various alternating stresses were superimposed on various tensile mean stresses. The complete relationship between alternating stress and mean stress with respect to time was clarified by stress-rup-

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1560



1948

MECHANICAL PROPERTIES OF FIVE LAMINATED PLASTICS

By William N. Findley and Will J. Worley

SUMMARY

Results of mechanical tests of the following laminated plastics are reported: canvas laminate molded at low pressure, grade-C canvas laminate, rayon laminate, paper laminate, and glass-fabric laminate. The following tests were performed on these materials: static tension, compression, and torsion tests; long-time creep tests at different stresses on specimens loaded in tension; fatigue tests of unnotched specimens in bending; fatigue tests of notched specimens in bending; fatigue tests in bending at temperatures of -75°, 77°, and 160° F; and fatigue tests in torsion.

Of the five laminates studied, the glass-fabric laminate had the most desirable mechanical properties for nearly all the properties measured. The properties of the paper laminate were next in order of desirability. It was observed that the creep rate of the glass-fabric laminate increased only slightly (compared with the other laminates) with increase in stress and that temperature made much less difference in the fatigue strength of the glass-fabric laminate than of the other laminates. The mechanical properties of the canvas laminate molded at a pressure of 180 psi were about 30 percent lower than those for the canvas laminate molded at 1800 psi for most of the properties tested. The most pronounced effect of the lower molding pressure was a decrease of 61 percent for the fatigue strength in torsion.

First studies on composites' fatigue

As early as 1955 Hulbert mentioned that “fatigue and creep are such important aspects of dynamic and static behavior that, among the welter of data accumulated around reinforced plastics materials, it is astonishing to find **little or nothing** published about either property”.

However, for Hulbert, the main problem lay in the “welter of data” signifying the inconsistency of the existing data (for several reasons, including **specimen preparation** and **new problems arising during fatigue testing of composites**) that result in less confidence in fiber-reinforced materials for primary structures.

Almost 20 years later, Harris wrote, “The abundance of experimental data does not contribute to our understanding about the fatigue of composites, **and there is not yet (in 1977) any general agreement about what kind of design data are most useful for reinforced polymers**”

Fatigue and accumulation of damage in reinforced plastics

B. HARRIS

COMPOSITES . OCTOBER 1977

Plastics Structures

by

G. C. HULBERT, B.Sc.,

(Polyester Technical Service, Scott Bader & Co. Ltd.)

1. Introduction

More than two years ago Pollard^(*) in a paper given to the Society made out the case for the use of fibre-reinforced plastics in the structural field. He and others have shown that materials based on strong, inert fibres, such as glass or asbestos, have mechanical properties which, when stabilised by suitable resins, permit them to function with a high degree of efficiency within certain well defined parameters of loading.

Four distinct groups of resin are used in combination with such fibres, phenolic, polyester, epoxide and silicone. With the present exception of silicone resins, each can be used at low or even contact pressures during moulding and thus do not impose serious processing limitation upon the size, quantity or complexity of the structures which can be contemplated.

In addition to their mechanical and manufacturing possibilities, individually these materials possess certain valuable secondary properties. It was by making use of such properties as radar transparency that their use was first established. Such use continues to the present time and in fact still constitutes the reason for adopting the material in the majority of existing applications.

The growth of structures making use of the mechanical properties for important load bearing applications has been slow, despite the courage of certain companies who have built important prototypes, such as complete wings, which have undergone searching tests with reassuring results.

The serious use of any material is bound to be proportionate to the confidence which can be placed in it. It is the intention of this paper therefore to examine the experience and knowledge of the material, to isolate, if possible, some factors which hinder this necessary growth of confidence and, where possible, to offer encouragement to those, who, like the author, believe in the ultimate valuable service of which the material is capable.

2. Factors

Within the scope of a single paper it will not be possible to examine all the factors, the study of which could help to further the use of the material. The author has therefore chosen the following four for consideration. They have been chosen firstly because of their immediate importance to the position and secondly, because it is felt that a state of knowledge has already been reached where something immediate and constructive can be done about them. They are:—

1. Consistency of test results and its effect upon the design safety factor.
2. Permanence of behaviour when exposed to weather and other normal external agencies.
3. Fatigue and creep behaviour.
4. Temperature resistance.

3. Consistency of Test Results

Plastics materials, particularly those containing reinforcement of glass or asbestos fibres, have unfortunately gained a reputation for a high degree of scatter in test results. So long as this persists designers will suffer the handicap of a high safety factor in their designs. At present this is approximately double that operating for metal structures and largely nullifies the strength/weight advantage the material has over its competitors. It must be ascertained therefore whether this scatter is inherent in the material or whether it is due to the treatment of it either in manufacture, processing or in testing. Fortunately the experience of at least one large company indicates that scatter inherent in the material is neither more nor less than in any other and that, once correct and full control of treatment has been established, its mechanical behaviour is fully consistent. Correct and full control of treatment would therefore seem to be the key and it is necessary to ascertain what this involves.

It must first be realised that in producing a moulding of reinforced plastics material we have largely telescoped into one moulding operation many factors which, in metal it would have been possible to control and test at various clearly defined stages such as foundry, rolling mill and finally fabrication of the raw material into the component. Further, that it is impossible in the reinforced material to find a true equivalent to the raw material stage for a metal sheet. Reinforced plastics is not a single homogeneous material but, even in a simple test piece, is a structural alliance of two dissimilar components. It is therefore obvious that all factors in the make-up, handling and processing together of the glass, asbestos or resin can affect the final behaviour of the structure. Only now is a state of knowledge being reached when it is possible to recognise the factors and to select those which can be regarded as key factors, the control of which will ensure consistent behaviour of the final product.

At this point a comparison between the state of knowledge about the asbestos-reinforced phenolic materials and the many combinations of glass fibre and resin is of interest.

With the asbestos phenolic resin materials a limited number of forms are available, two or, at the most, three

^(*)A Section Lecture given to the Royal Aeronautical Society on 3rd May 1955.

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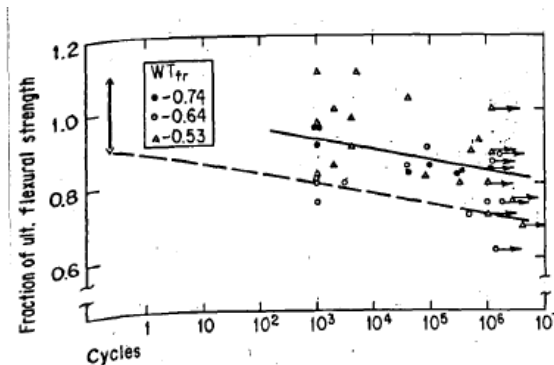


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Modern Plastics, April 1970

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A technical feature

Gordon M. Kline, technical editor

form (10,000 filaments) in lengths up to 39 in. The tensile properties supplied by the makers are as follows:

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Resins. In most of the work an isophthalic polyester resin (Cellobond A2784, B.P. Chemicals Ltd.) was used in wet layup preparation. One percent of MEKP-RGS (Laporte Industries Ltd.) was used in conjunction with 0.25% of cobalt naphthenate. After gelation, the composites were post-cured for at least 3 hr. at 80° C.

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Composites used (not only) in aerospace

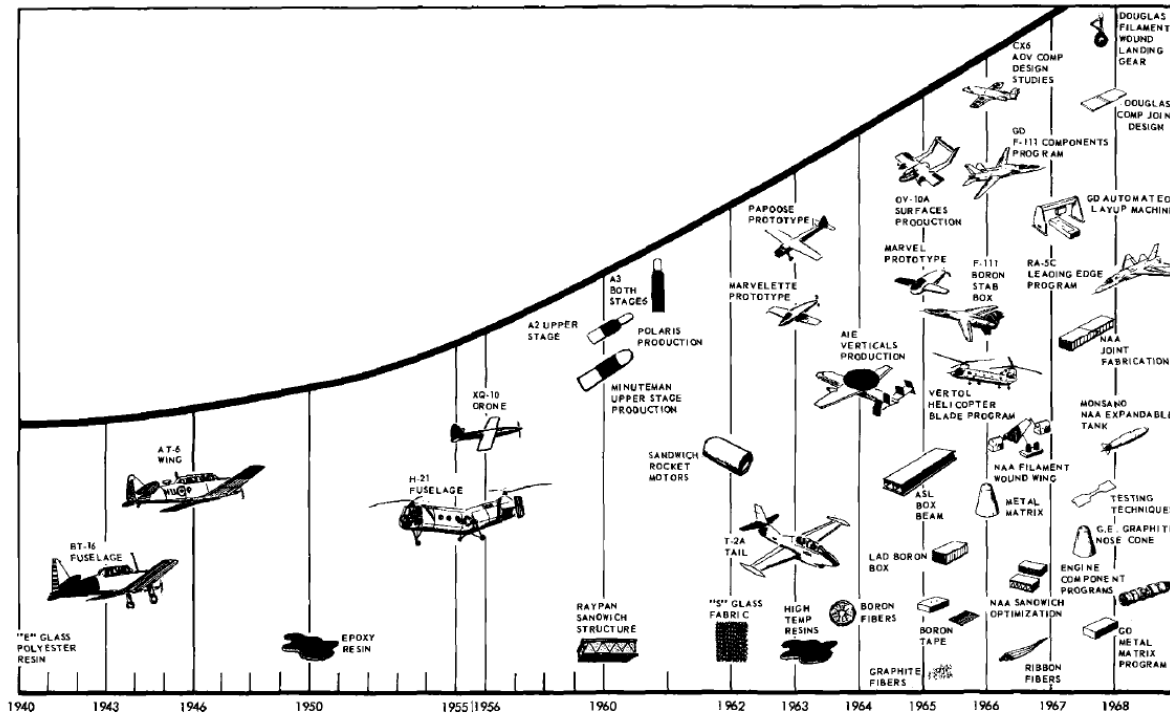
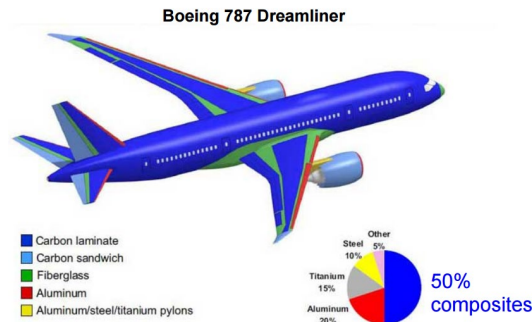


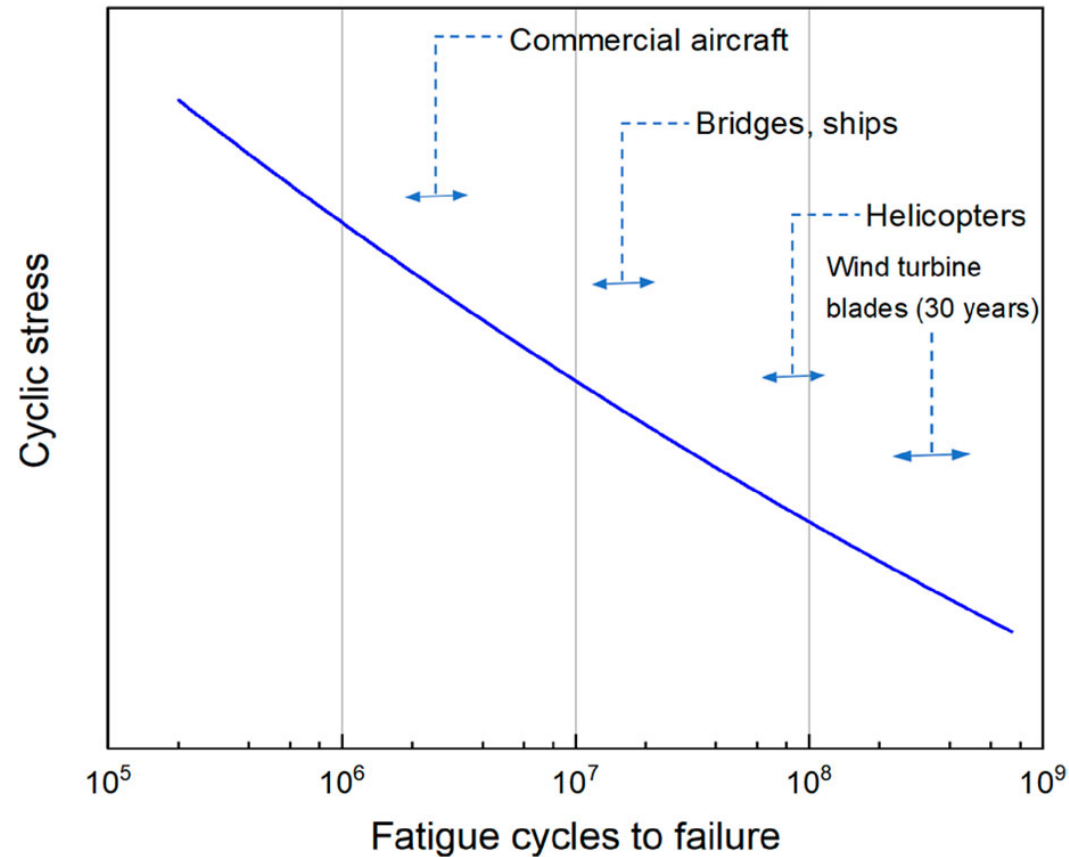
FIG. 1—Chronology of composites usage.



<https://discovercomposites.com/what-are-composites/history-of-composites/index.html>

Structures that are facing intensive fatigue loadings during their lifetime

Fatigue in (composite) structures



107 meters, LM Wind Energy's blades for the Haliade-X, 12MW

INTERNATIONAL MATERIALS REVIEWS
2021, VOL. 66, NO. 5, 313–338
<https://doi.org/10.1080/09506608.2020.1845110>

iom
The Institute of Materials,
Minerals and Mining

ASM
INTERNATIONAL

Taylor & Francis
Taylor & Francis Group

Check for updates

FULL CRITICAL REVIEW

Review of fatigue of bulk structural adhesives and thick adhesive joints

Peiyuan Zuo^a and Anastasios P. Vassilopoulos^{io^b}

What we know about composites' fatigue

- More troublesome to test composites in fatigue...
- Self heating
 - low frequencies
- Fiber reorientation
 - For MD and angle ply laminates
- Strain rate
 - Important for composites (especially more susceptible to viscoelastic effects)

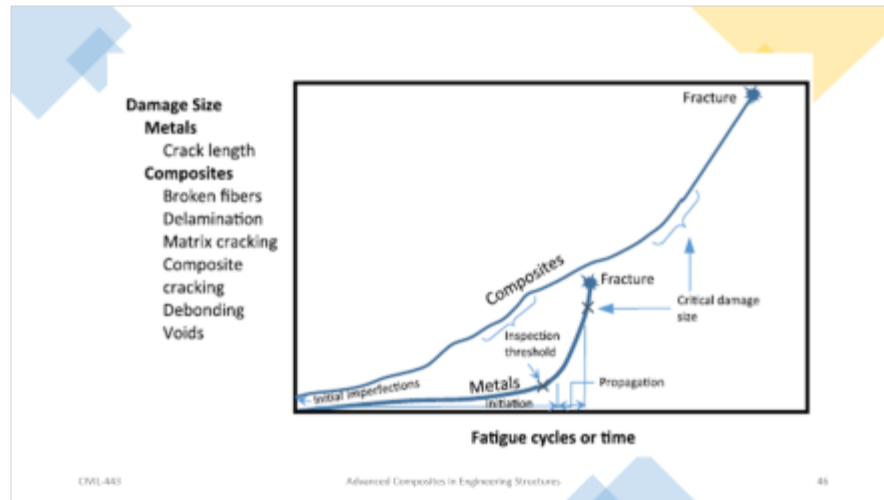
What we know about composites' fatigue

- Manufacturing defects
 - Voids in adhesives
 - Misalignment of fibers
- Material (pre)conditioning
 - Expose to humidity, temperature, solvents, etc
- Specimen geometry -
 - (Many) Tab failures
- Standards e.g. ASTM for Type I, Type II, Type IV – different results!

Damage modes are known!

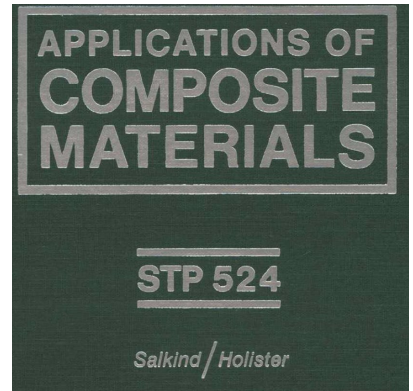
By the end of 1970s' scientists already knew almost everything about failure modes, damage mechanisms, and material behavior of composites (almost as much as we know today!). It was common knowledge that composites exhibit several damage modes including delamination, matrix crazing, fiber failure, void growth, matrix cracking, and composite cracking.

A structure may exhibit any or all of these damage modes and it is difficult to predict, a priori, which mode will dominate and cause failure. Therefore, although all damage that occurs in a composite material can be identified, it is extremely difficult to accurately quantify its effect on fatigue life.



M. J. Salkind^{*}

Chapter III – VTOL Aircraft



REFERENCE: Salkind, M. J., "VTOL Aircraft," *Applications of Composite Materials*, ASTM STP 524, American Society for Testing and Materials, 1973, pp. 76-107.

ABSTRACT: Fibrous composites offer significant potential for structural improvement in VTOL (vertical take off and landing) aircraft. In addition to the potential for light weight fuselage structures, composites offer the unique capability of providing dynamic tuning of the fuselage. Composites provide substantial potential for helicopter rotor blades because of improved fatigue capability, good damage tolerance, and ability to be molded in complex aerodynamic configurations.

This chapter summarizes the major design considerations in VTOL aircraft and reviews composite hardware which has been developed to date.

KEY WORDS: composite materials, fibers, aircraft, helicopters, rotary wings, fatigue (materials)

Damage Size

Metals

Crack length

Composites

Broken fibers

Delamination

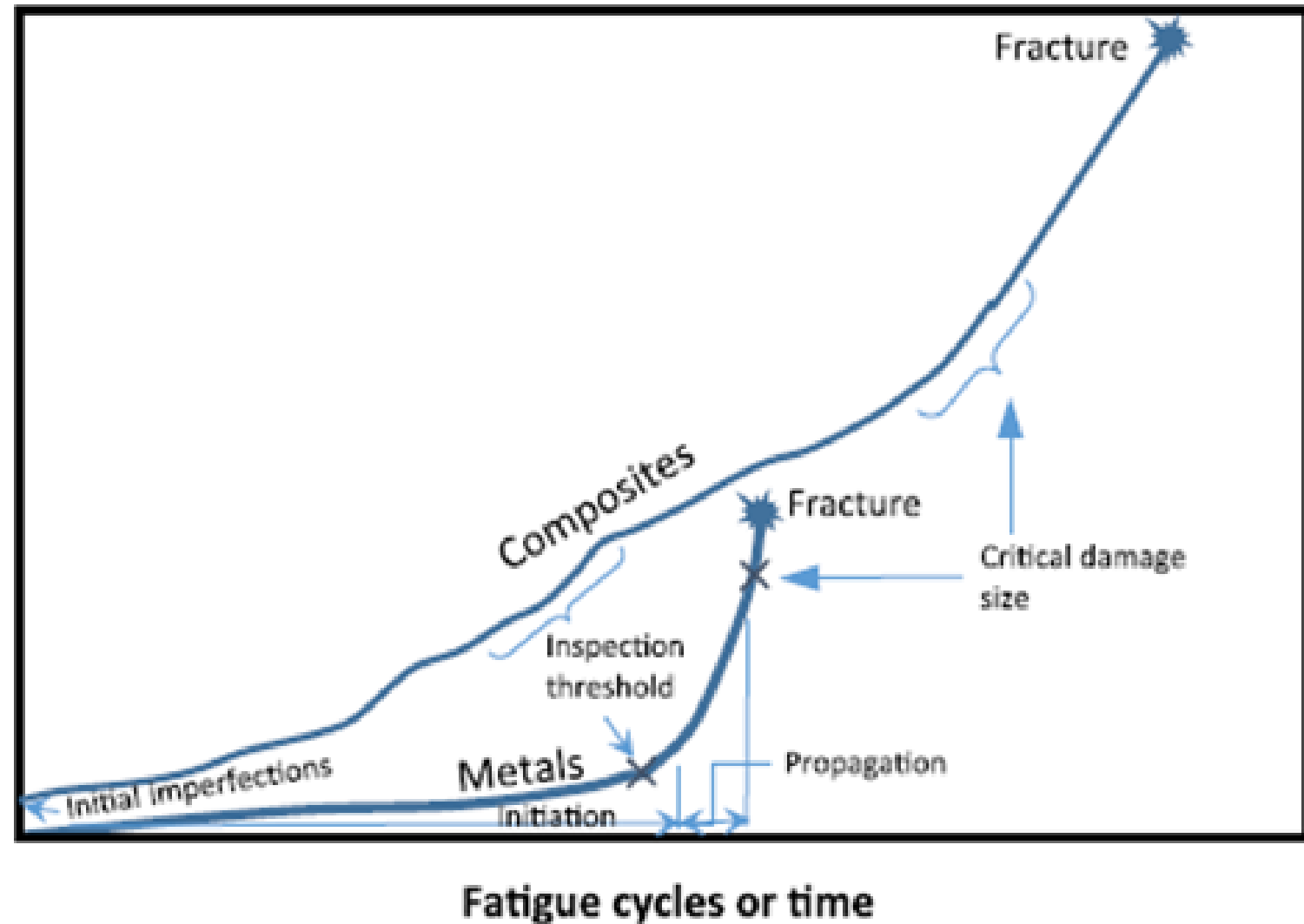
Matrix cracking

Composite

cracking

Debonding

Voids

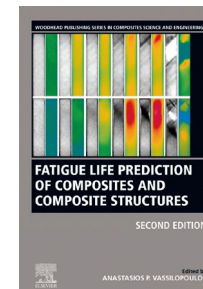
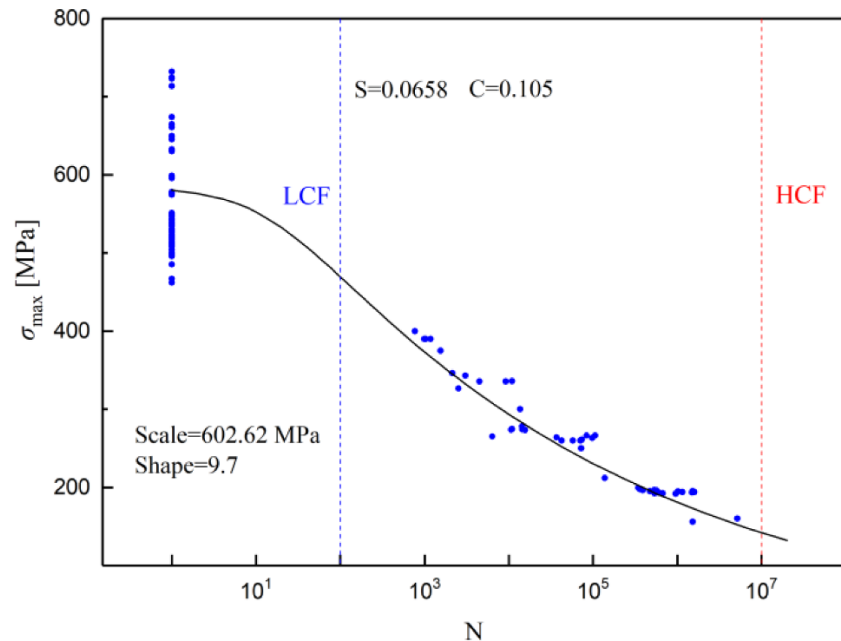


What we know about composites' fatigue

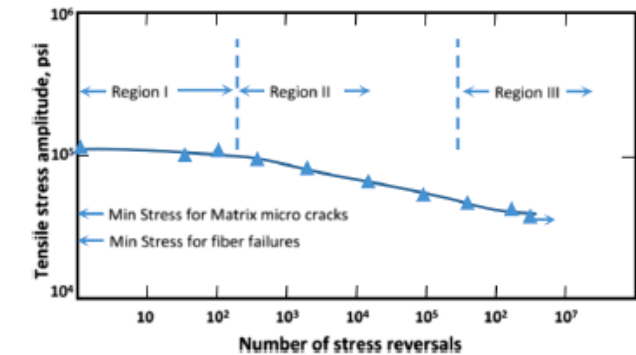
More troublesome to test composites in fatigue...

Composites **ARE NOT** fatigue insensitive!

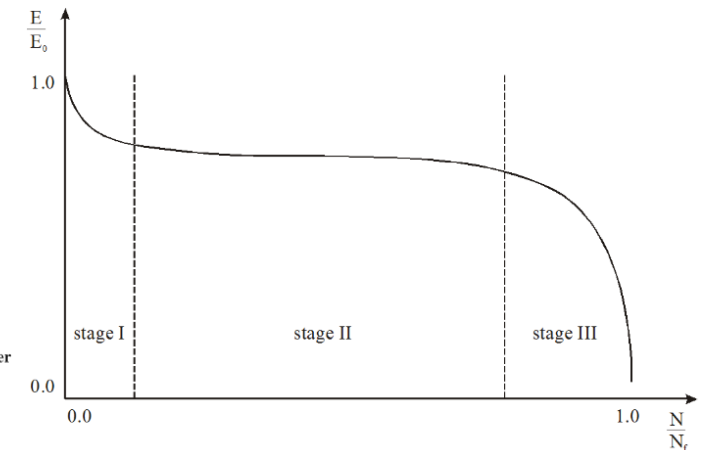
They **DO** fail in fatigue – show no fatigue threshold...



13 Prediction of fatigue crack initiation in UD laminates under different stress ratios
R.D.B. Sevenois and W. Van Paepegem



S-N curve showing three different slopes assigned to three different damage mechanisms



What we know about composites' fatigue

A specimen of a certain rank in the static strength probability distribution has the same rank in the fatigue life distribution!

Introduction of the "SLERA" by Hahn and Kim in 1975.

Proof Testing of Composite Materials

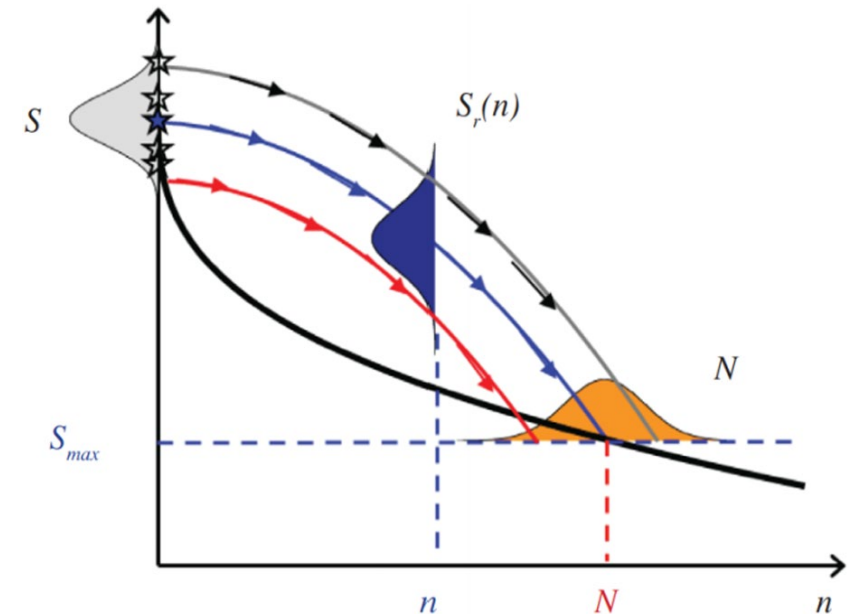
H. T. HAHN AND R. Y. KIM

University of Dayton Research Institute
Dayton, Ohio 45469

(Received July 7, 1975)

ABSTRACT

The paper presents a concept of proof testing for composite materials. A unique relationship between the static strength and time to rupture is demonstrated for a unidirectional glass/epoxy composite subjected to static fatigue. The maximum significance level at which a Weibull distribution is applicable to represent scatter is 35% for the static strength and 61% for the fatigue life. Limitations and variations of the strength degradation model for the life prediction are discussed.



Schematic representation: Stress life equal rank assumption

Life Prediction Methodologies for Materials and Structures

M. Kaminski, F. Laurin, J.-F. Maire
(ONERA)
C. Rakotoarisoa
(Snecma, Safran Group)
E. Hémon
(Safran Composites, Safran Group)

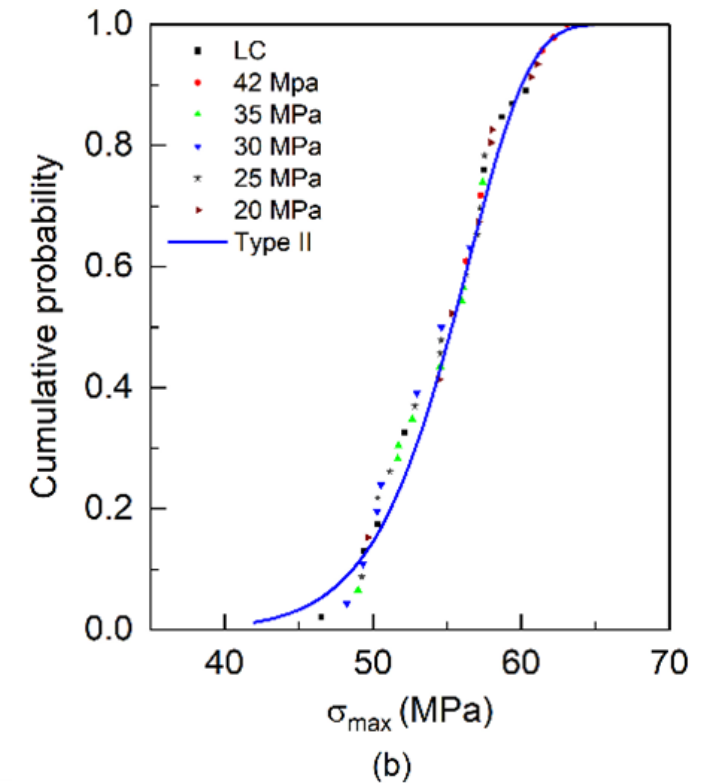
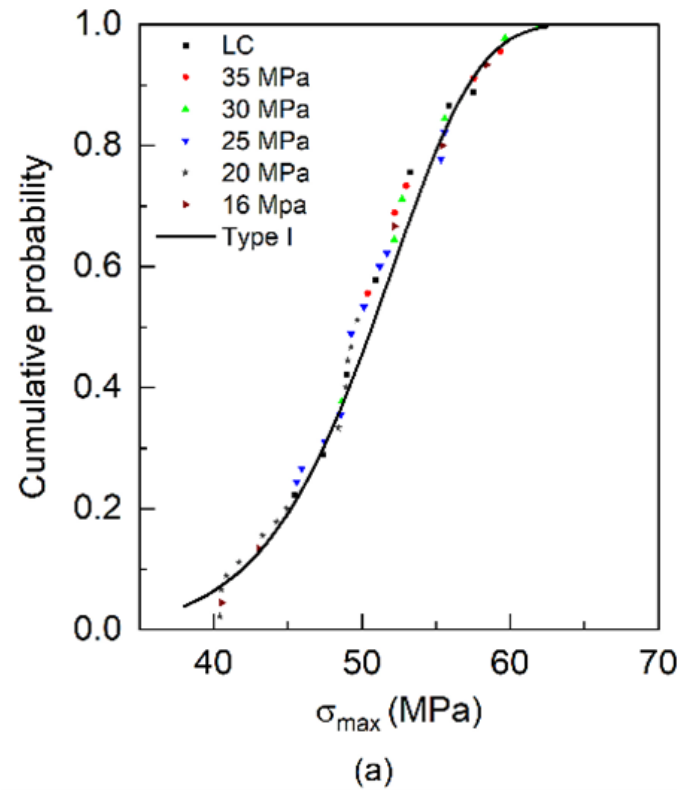
E-mail: Myriam.Kaminski@onera.fr

DOI: 10.12762/2015.AL09-06

**Fatigue Damage Modeling
of Composite Structures:
the ONERA Viewpoint**

What we know about composites' fatigue

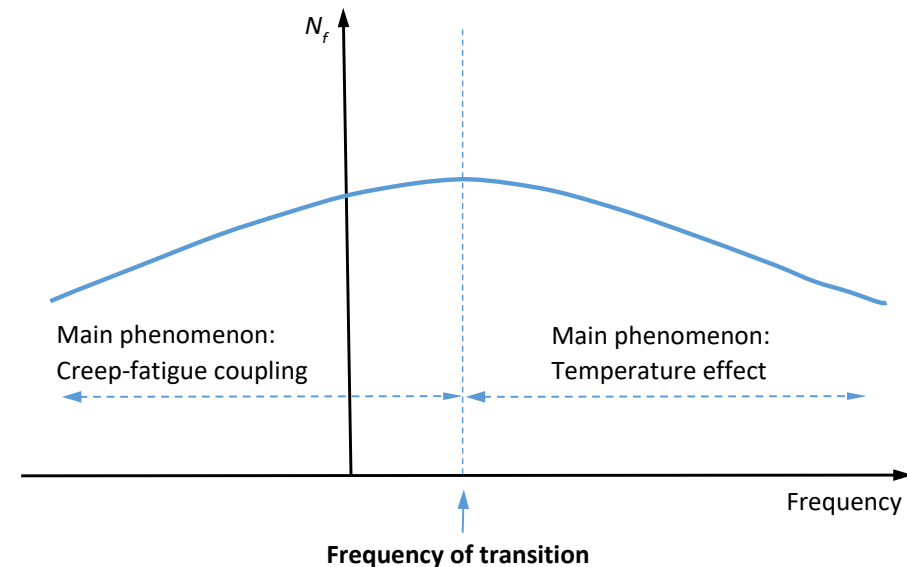
Composites follow a Weibull strength probability distribution



What we know about composites' fatigue

Low frequencies – avoid self-heating

Frequencies lower than 10Hz, sometimes as low as 1Hz are used to avoid self-heating



What we know about composites' fatigue



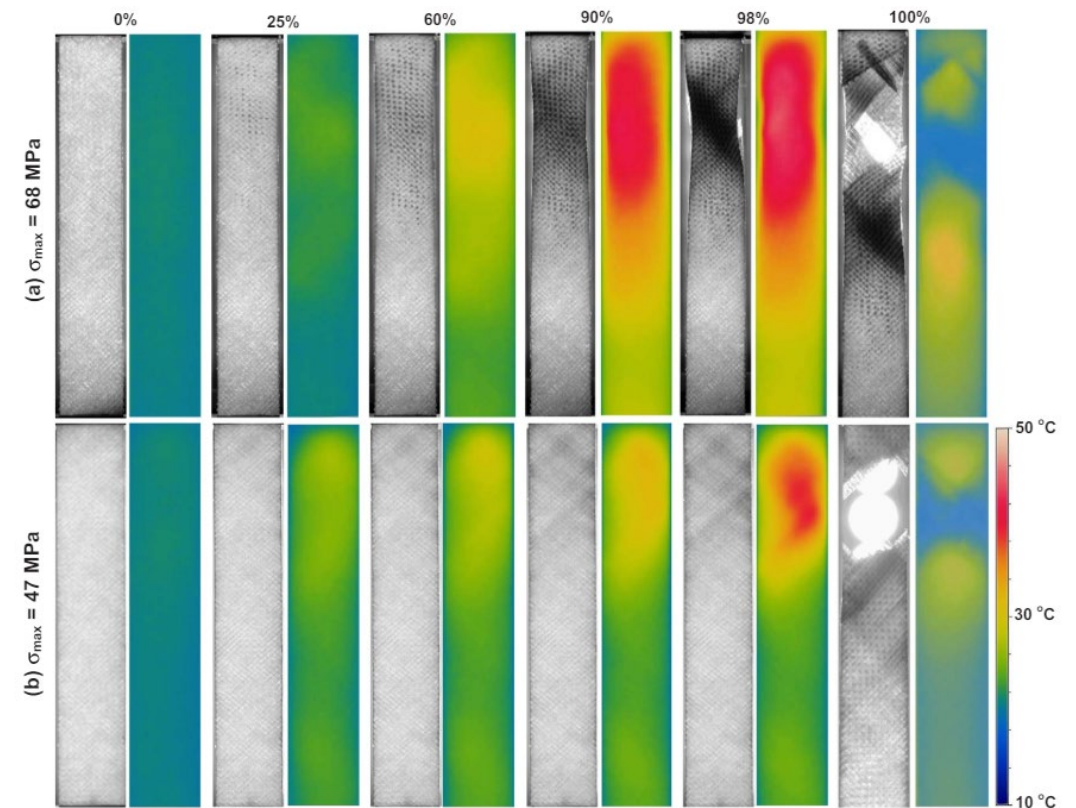
Fatigue damage in angle-ply GFRP laminates under tension-tension fatigue

A. Vahid Movahedi-Rad, Thomas Keller, Anastasios P. Vassilopoulos*

Composite Construction Laboratory (CCLab), Ecole Polytechnique Fédérale de Lausanne (EPFL), Station 16, Bâtiment BP, CH-1015 Lausanne, Switzerland



Light transmittance (damage indicator) and self-generated temperature at different percentages of fatigue life at two stress levels



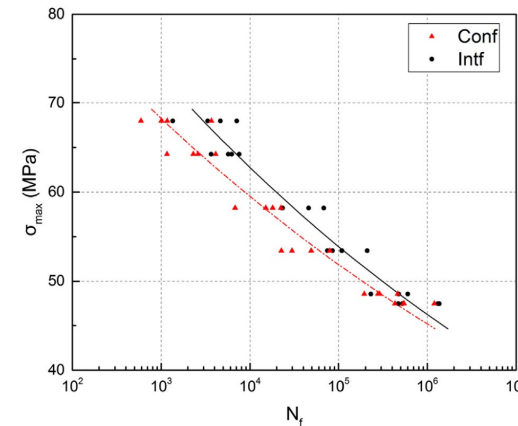
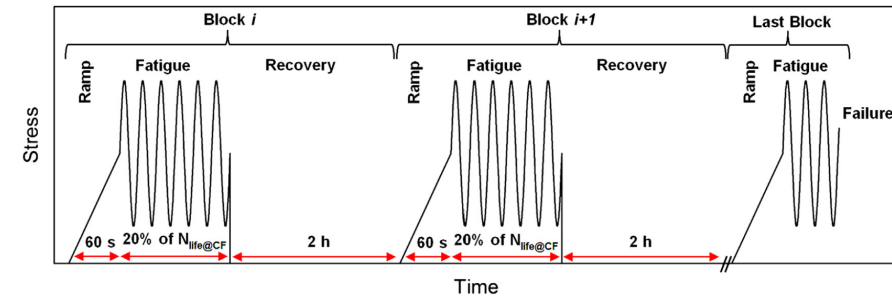
What we know about composites' fatigue

Viscoelastic material behavior (even for thermosets)



Interrupted tension-tension fatigue behavior of angle-ply GFRP composite laminates

A. Vahid Movahedi-Rad, Thomas Keller, Anastasios P. Vassilopoulos*



Exp. results

What we know about composites' fatigue

Viscoelastic material behavior (even for thermosets)

International Journal of Fatigue 123 (2019) 144–156

Contents lists available at ScienceDirect

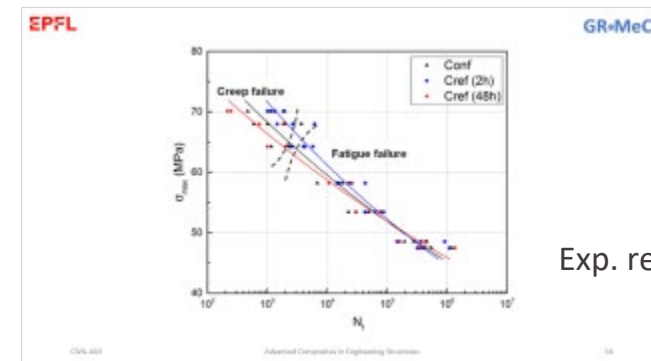
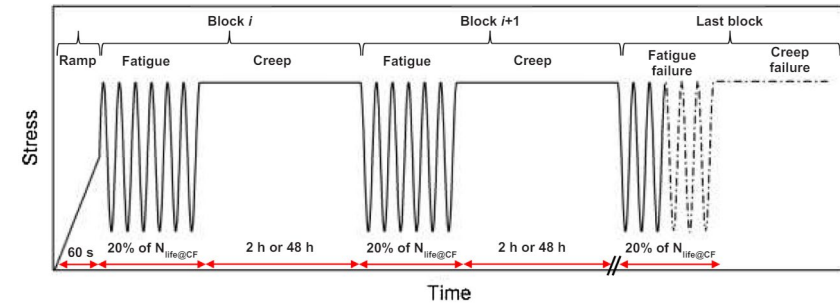
International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

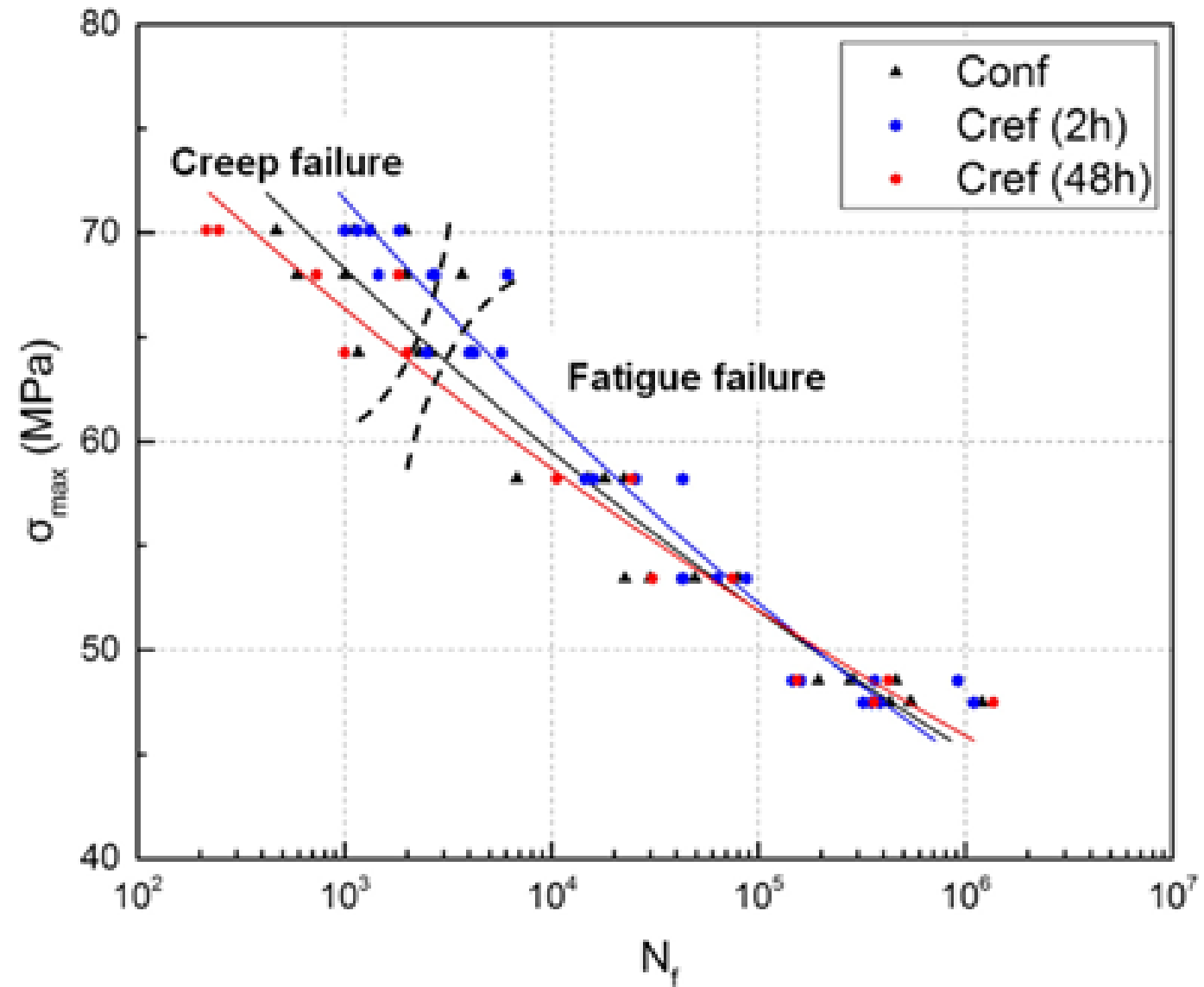


Creep effects on tension-tension fatigue behavior of angle-ply GFRP composite laminates

A. Vahid Movahedi-Rad, Thomas Keller, Anastasios P. Vassilopoulos*



Exp. results



What we know about composites' fatigue

Goodman... is not good any more 😊

- Constant life diagrams are used instead reflecting the combined effect of mean stress and material anisotropy on the fatigue life
- Predictive tools...

Main parameters for defining a CLD:

The mean cyclic stress, σ_m ,

The cyclic stress amplitude, σ_a ,

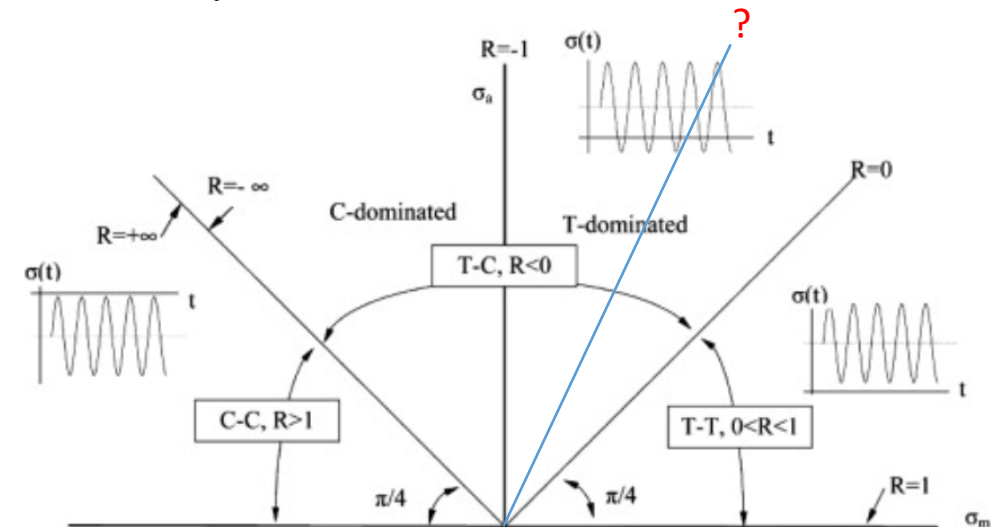
The R -ratio $R = \sigma_{min}/\sigma_{max}$

Three sectors for three different loading types...



Influence of the constant life diagram formulation on the fatigue life prediction of composite materials

Anastasios P. Vassilopoulos*, Behzad D. Manshadi, Thomas Keller



What we know about composites' fatigue

Goodman... is not good any more ☺

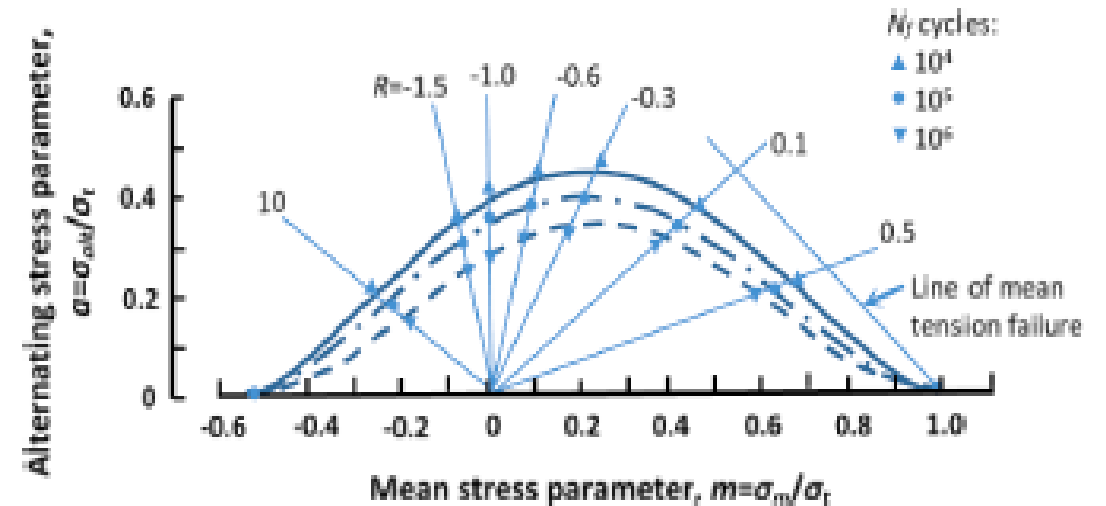
Constant life diagrams are used instead
– different behavior in tension and compression!

JOURNAL OF MATERIALS SCIENCE 23 (1988) 3732–3743

Fatigue behaviour of hybrid composites:

Part 1 Carbon/Kevlar hybrids

G. FERNANDO, R. F. DICKSON*, T. ADAM, H. REITER, B. HARRIS
Schools of Materials Science and Mechanical Engineering, University of Bath,
Bath BA2 7AY, UK



Constant life diagram for $[(\pm 45/0_2)_2]_S$ laminate

What we know about composites' fatigue

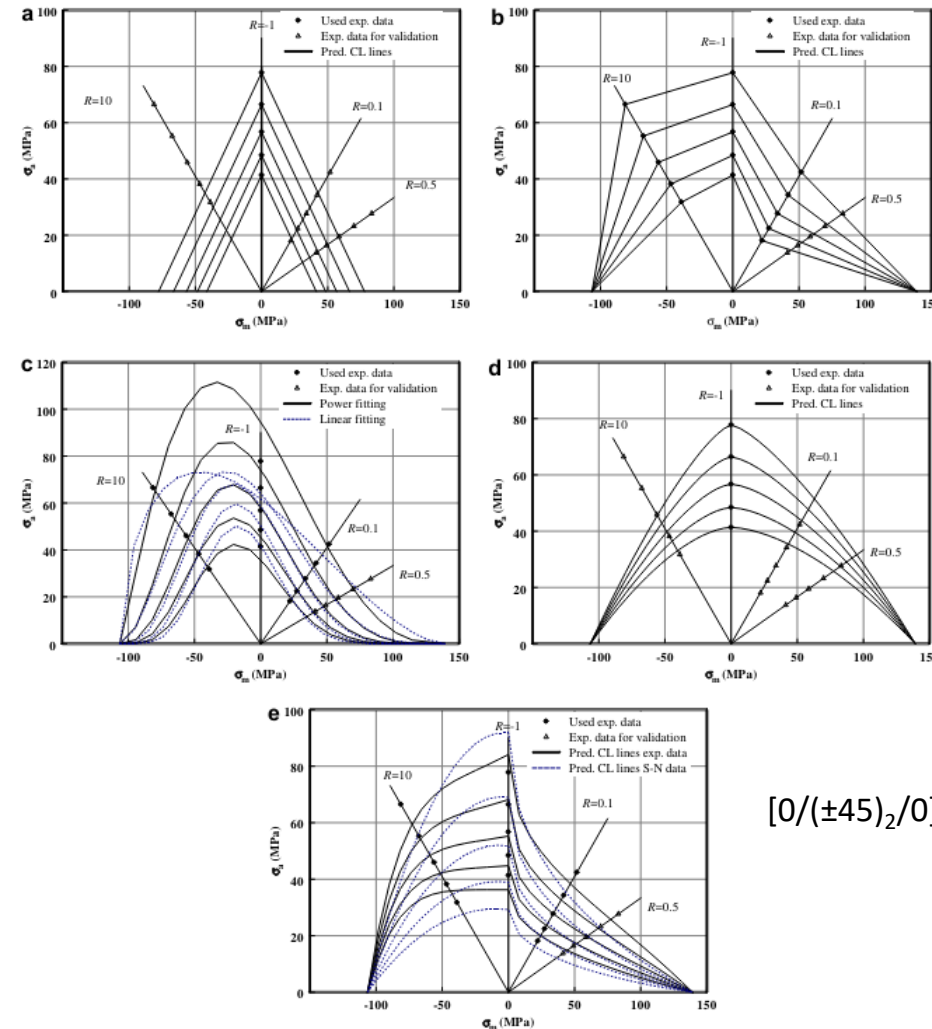
Goodman... is not good any more ☺

CLDs depend on the material system



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Anastasios P. Vassilopoulos*, Behzad D. Manshadi, Thomas Keller



More fatigue

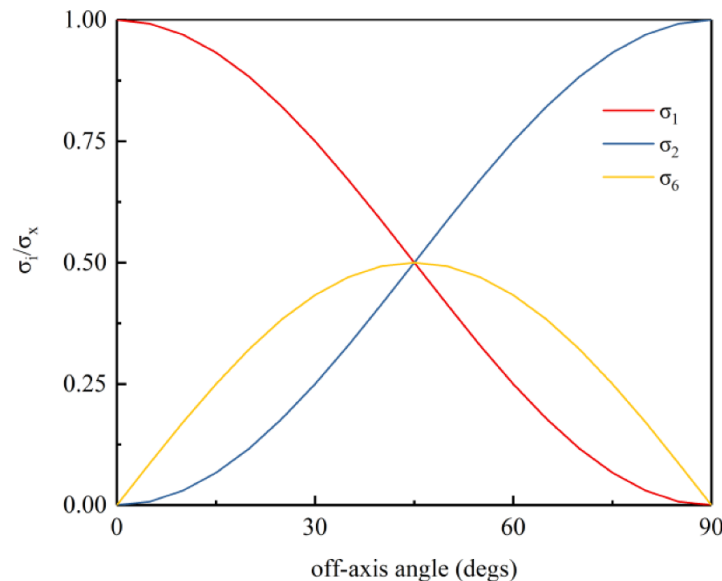
More creep

$[0/(\pm 45)_2/0]_T$

Fig. 2. Constant life diagrams for $N = 10^3 - 10^7$, material #1 (linear-a, piecewise linear-b, Harris-c, Kawai-d, Boerstra-e).

What we know about composites' fatigue

Multiaxial stress states – even under uniaxial loading!



Multiaxial stress state at principal coordinate system for off-axis layer.

Neglecting some of the stress components of plane-stress states is misleading and overestimates substantially the fatigue life of composite structural components



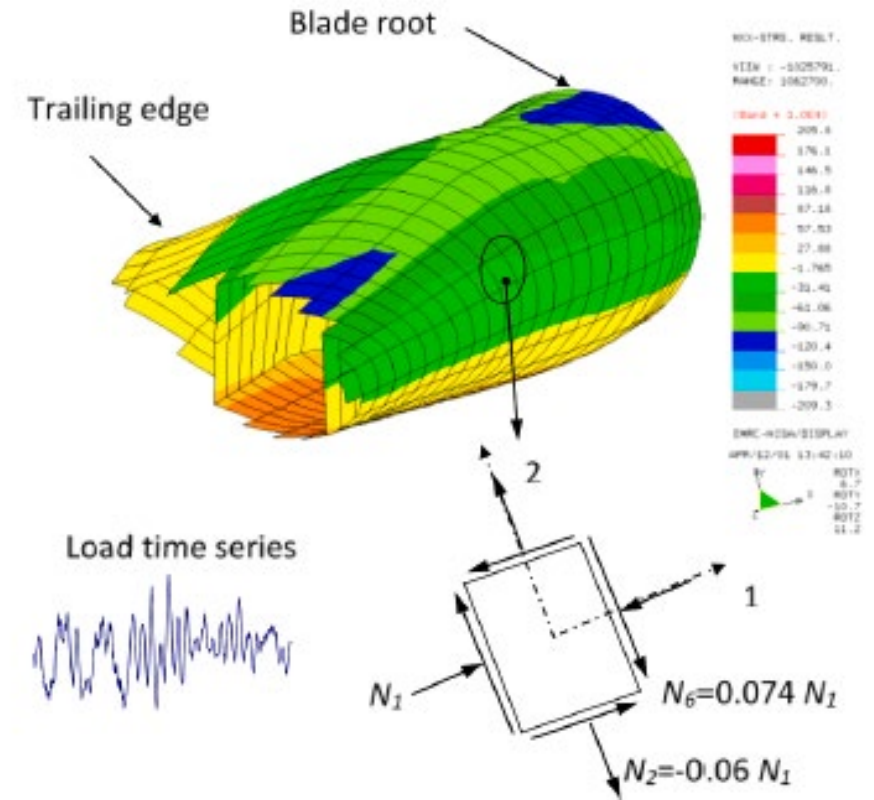
International Journal of Fatigue 24 (2002) 825–830



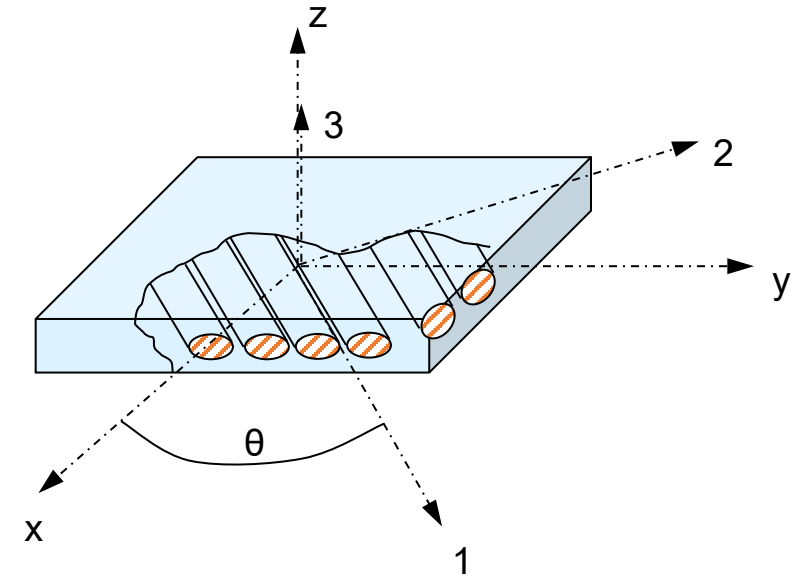
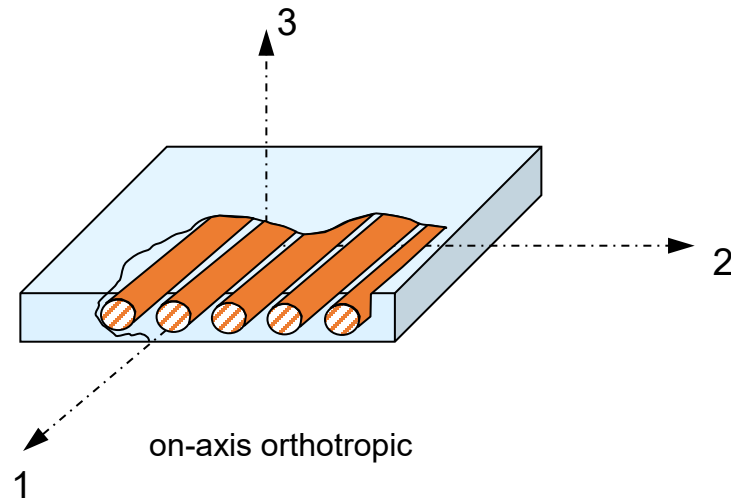
Complex stress state effect on fatigue life of GRP laminates.
Part II, Theoretical formulation

Theodore P. Philippidis*, Anastasios P. Vassilopoulos

Department of Mechanical Engineering and Aeronautics, University of Patras, PO Box 1401, 26500 Patras, Greece



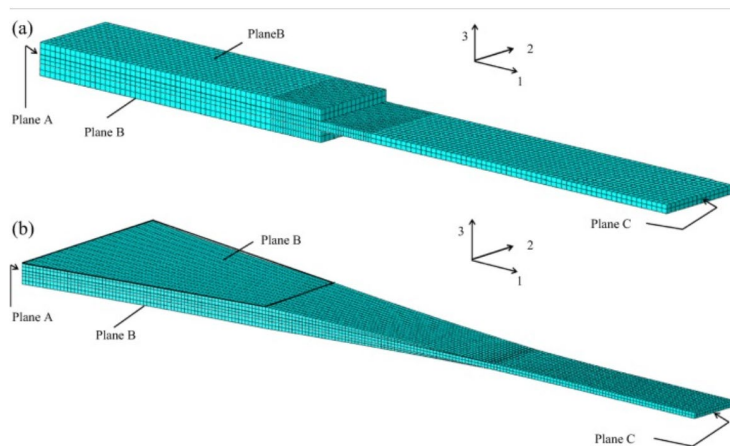
What we know about composites' fatigue



$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix}$$

What we know about composites' fatigue

Specimen type and size play a role!

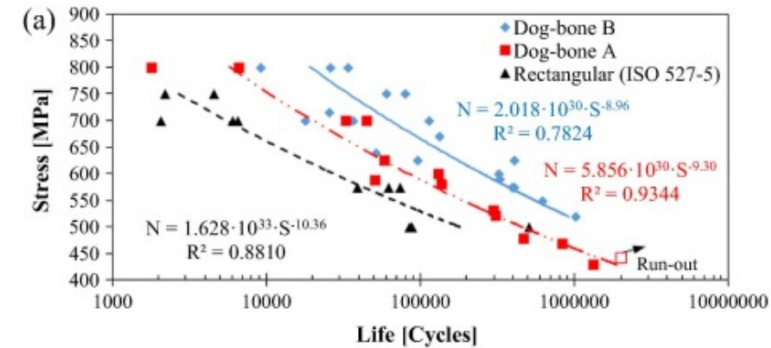


International Journal of Fatigue 85 (2016) 114–129

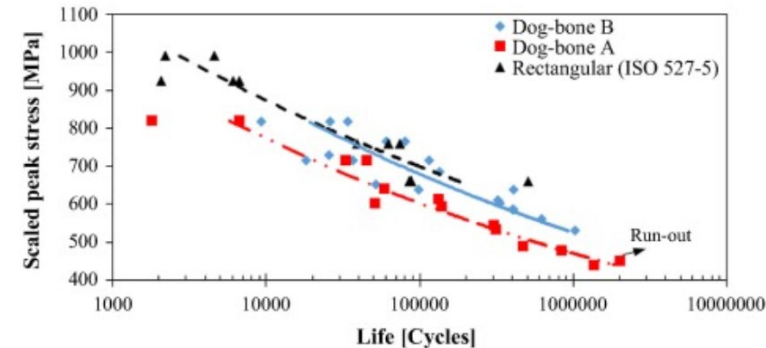


Influence of specimen type and reinforcement on measured tension–tension fatigue life of unidirectional GFRP laminates

Samuli Korkiakoski ^{a,*}, Povl Brøndsted ^b, Essi Sarlin ^c, Olli Saarela ^a



Exp. results

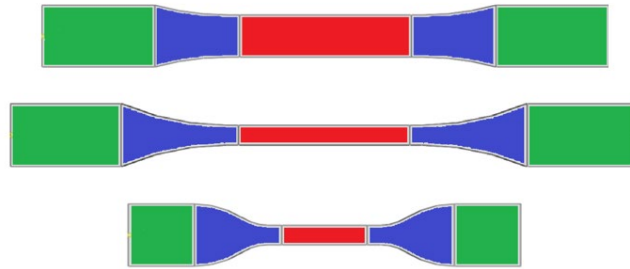


Scaled results by stress concentration factors

What we know about composites' fatigue

Specimen type and size play a role!

ASTM D7791-22, refers to ASTM D638-22



International Journal of Fatigue 190 (2025) 108600

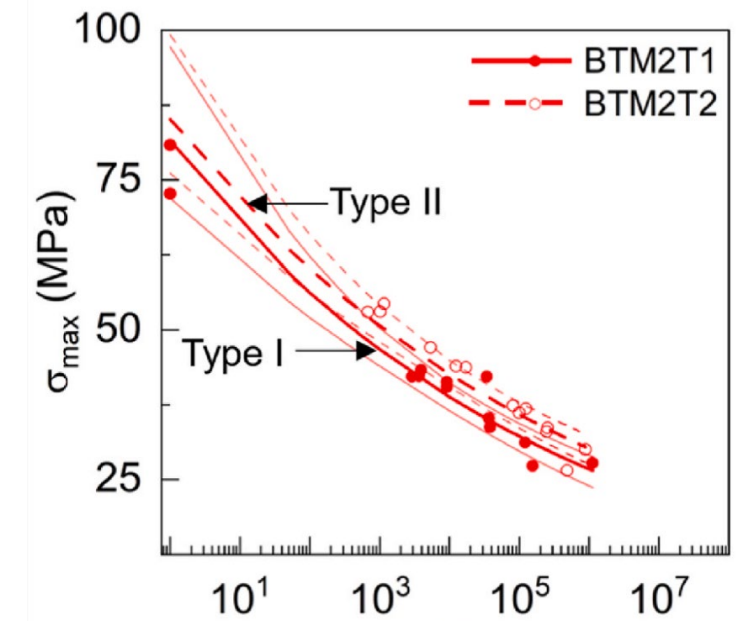
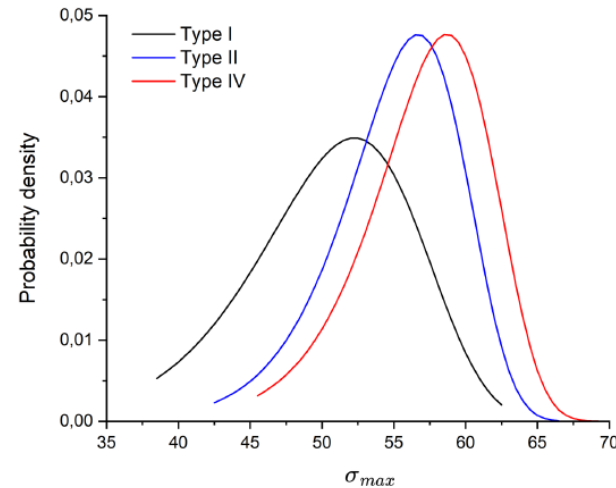


Standard specimen geometries do not always lead to consistent fatigue results for epoxy adhesives

Filippo Mannino^{a,b}, Dharun V. Srinivasan^a, Daniele Fanteria^b, Anastasios P. Vassilopoulos^{a,*}

^a Mechanics of Composites Group (GR-MeC), Ecole Polytechnique Fédérale de Lausanne (EPFL), Station 16, Batiment BP, Lausanne CH-1015, Switzerland

^b University of Pisa, Civil and Industrial Engineering Department, L.go Lucio Lazzarini, Pisa 56122, Italy



Polymer Testing 121 (2023) 107975



Fatigue performance of wind turbine rotor blade epoxy adhesives

Dharun Vadugappatty Srinivasan, Anastasios P. Vassilopoulos^{*}

Composite Construction Laboratory (CCLab) Ecole Polytechnique Fédérale de Lausanne (EPFL) Station 16, CH-1015, Lausanne, Switzerland

What we know about composites' fatigue

New materials, new joints, new geometries...

Also know that methods used for metals (**and composites for aerospace**) are not any more appropriate for composites used in other domains!



What we know about composites' fatigue

New materials, new joints, new geometries...

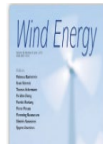


Thick adhesive joints

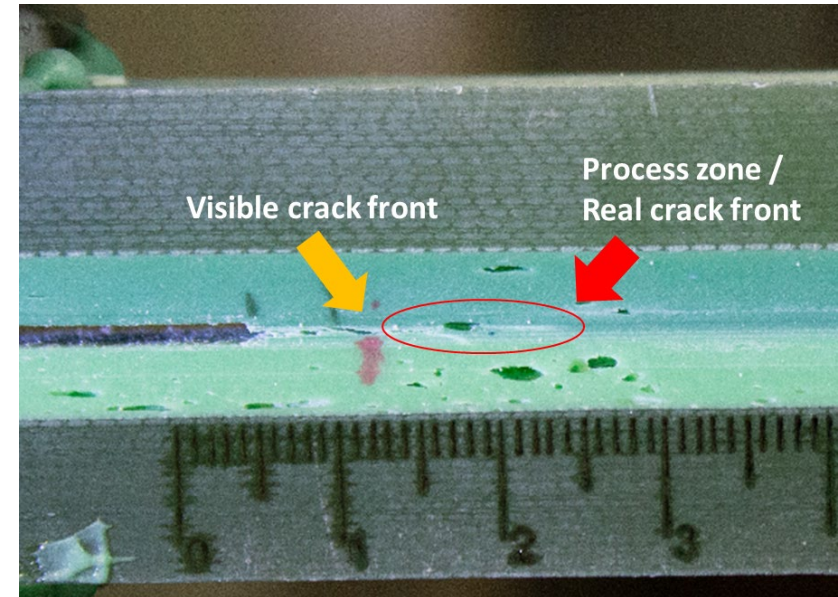
Fracture analysis of adhesive joints in wind turbine blades

M. A. Eder, R. D. Bitsche

First published: 01 April 2014 | <https://doi.org/10.1002/we.1744> | Citations: 37



Volume 18, Issue 6
June 2015
Pages 1007-1022



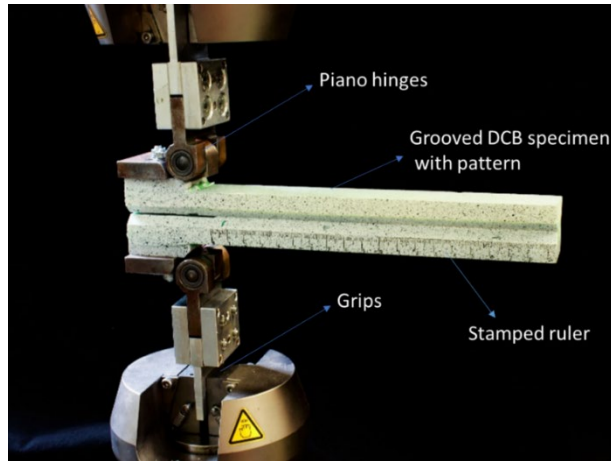
The behavior of the bulk adhesive dominates the joint performance



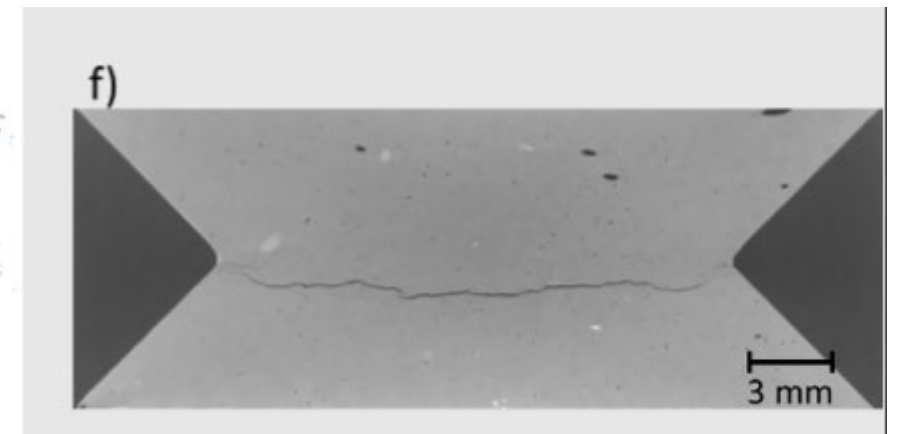
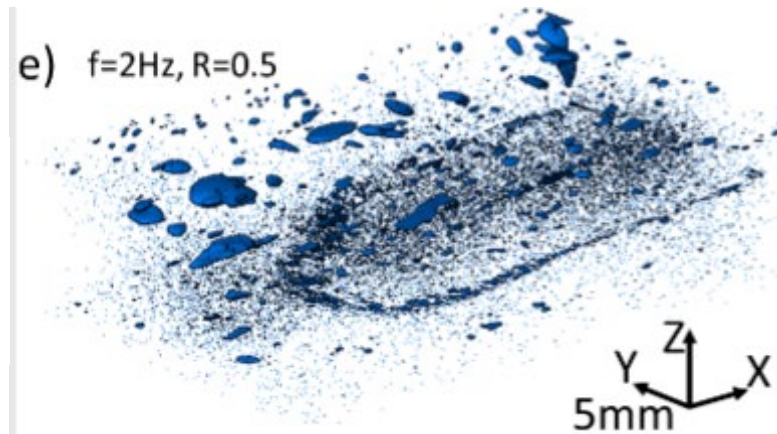
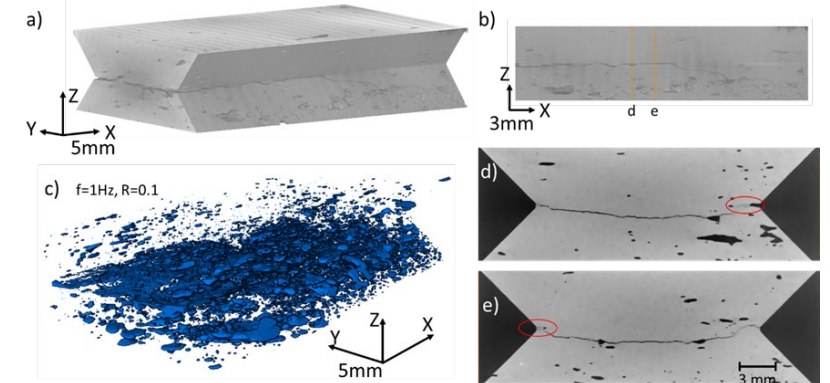
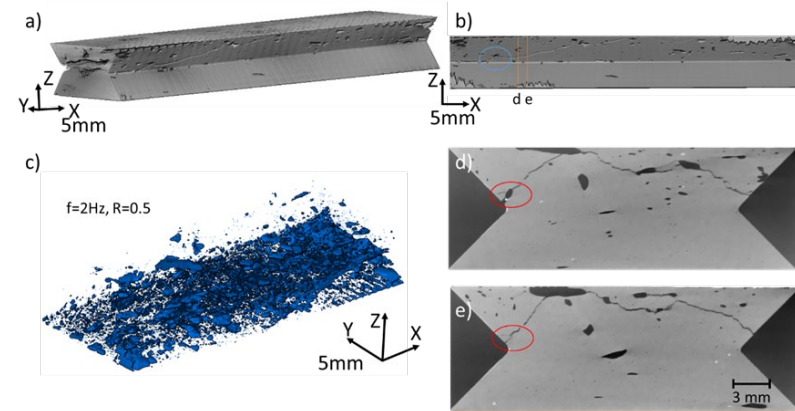
Mode I fracture of thick adhesively bonded GFRP composite joints for wind turbine rotor blades

Jialiang Fan^a, Anastasios P. Vassilopoulos^b, Veronique Michaud^{a,*}

What we know about composites' fatigue



Thick adhesive joints



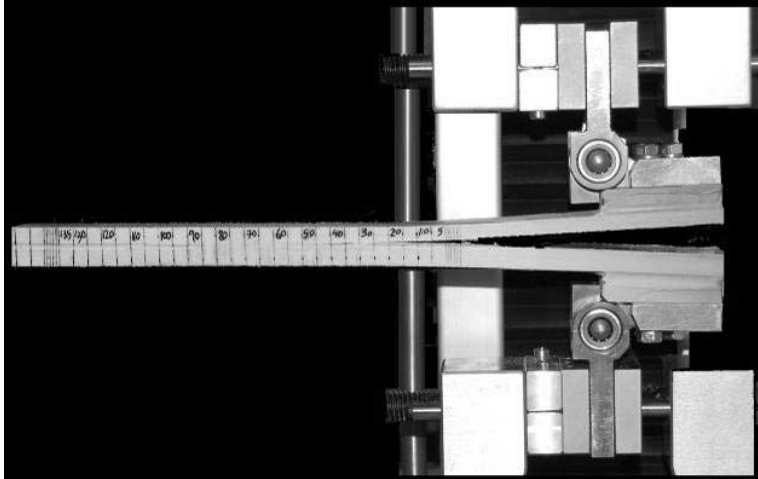
Void content and displacement ratio effects on fatigue crack growth in thick adhesively bonded composite joints under constant amplitude loading

Jialiang Fan^a, Kei-ichi Ikeda^b, Anastasios P. Vassilopoulos^c, Veronique Michaud^{a,*}

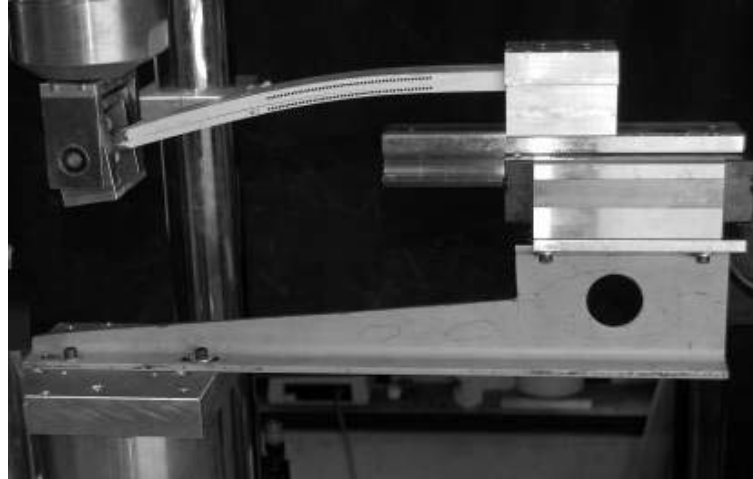
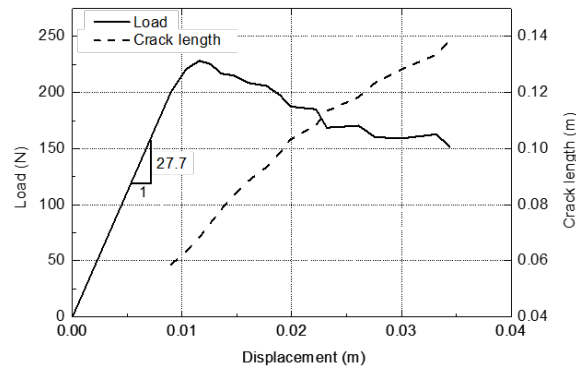
What we know about composites' fatigue

Fracture mechanics – G for K

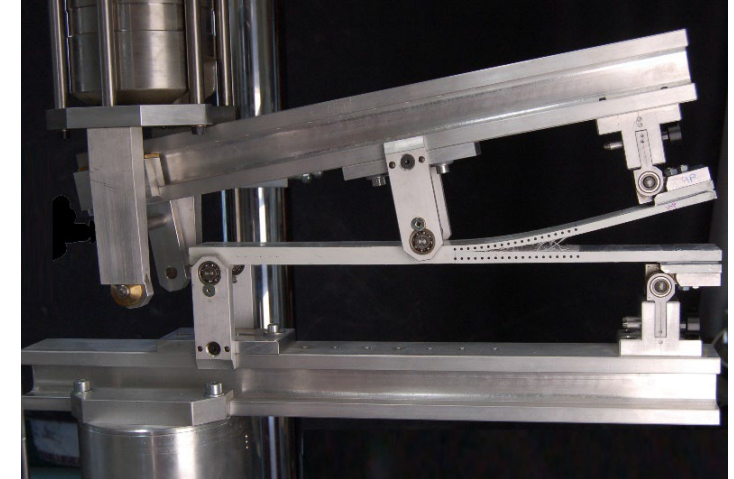
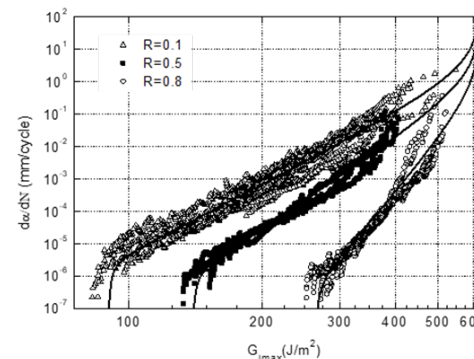
or maybe ... ΔG ? or $\sqrt{G}$? or...



Mode I



Mode II

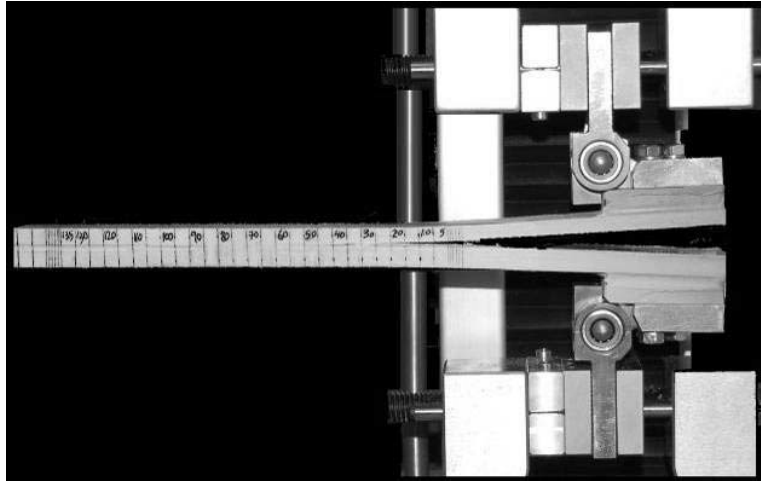


Mixed-Mode I/II

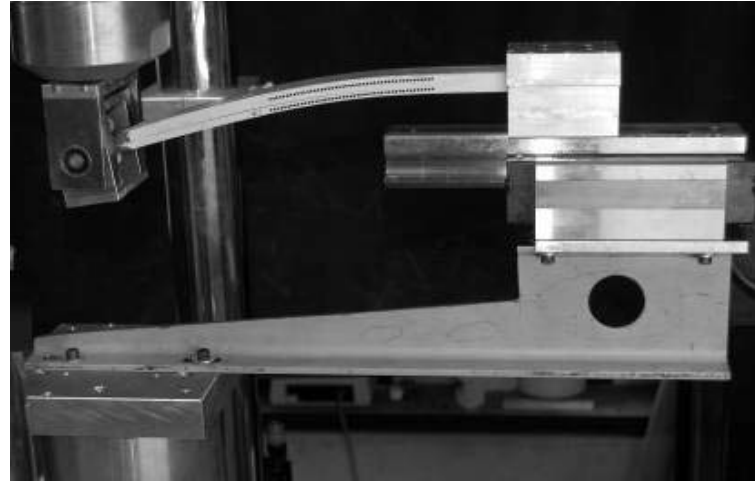
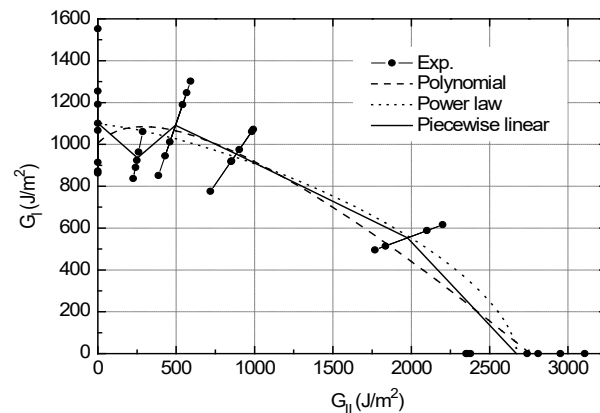
$$\frac{da}{dN} = D(G_{\max})^m \frac{\left(1 - \left(\frac{G_{th}}{G_{\max}}\right)^{Q_1}\right)}{\left(1 - \left(\frac{G_{\max}}{G_c}\right)^{Q_2}\right)}$$

*M. Shahverdi, A. P. Vassilopoulos, T. Keller "A total fatigue life model for the prediction of the R-ratio effects on fatigue crack growth of adhesively-bonded pultruded GFRP DCB joints" Composites A, 2012;43(10):1783-1790

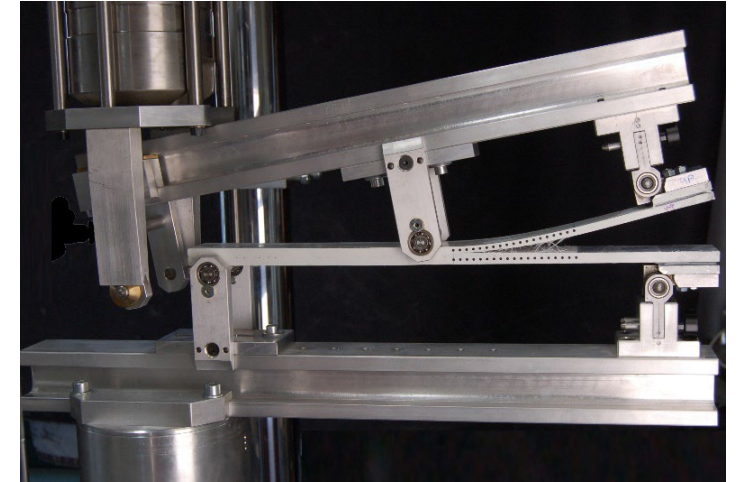
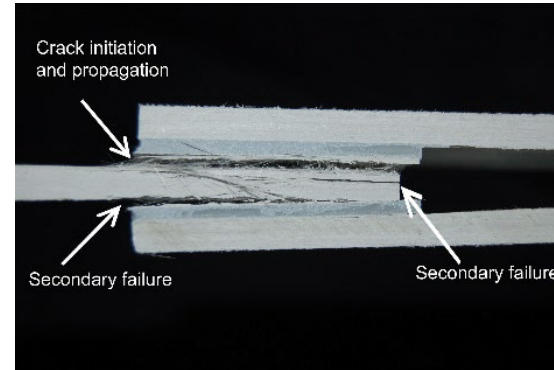
What we know about composites' fatigue



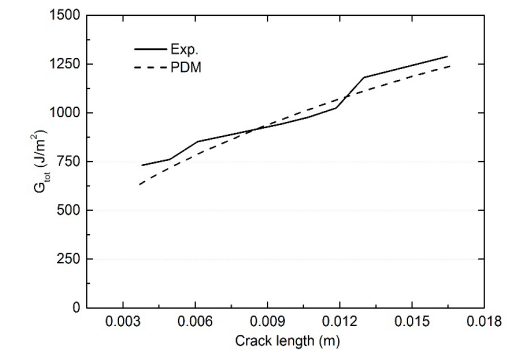
Mode I



Mode II



Mixed-Mode I/II



*A. Cameselle-Molares, R. Sarfaraz, M. Shahverdi, A. P. Vassilopoulos, T. Keller "Fracture mechanics-based progressive damage modelling of adhesively bonded fibre-reinforced polymer joints" FFEMS, 2017;40(12):2183-2193

What is expected?

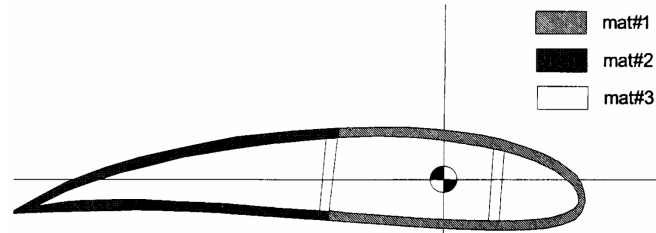
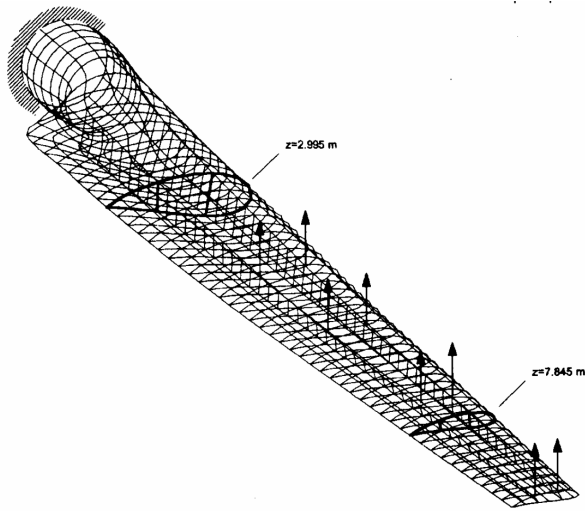
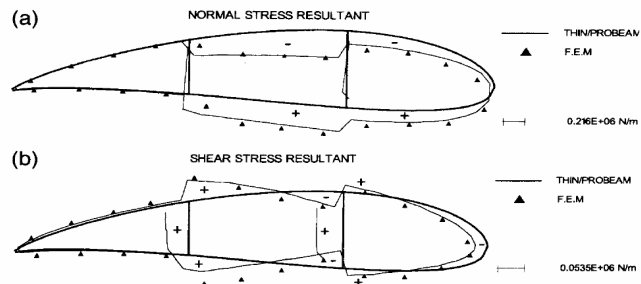
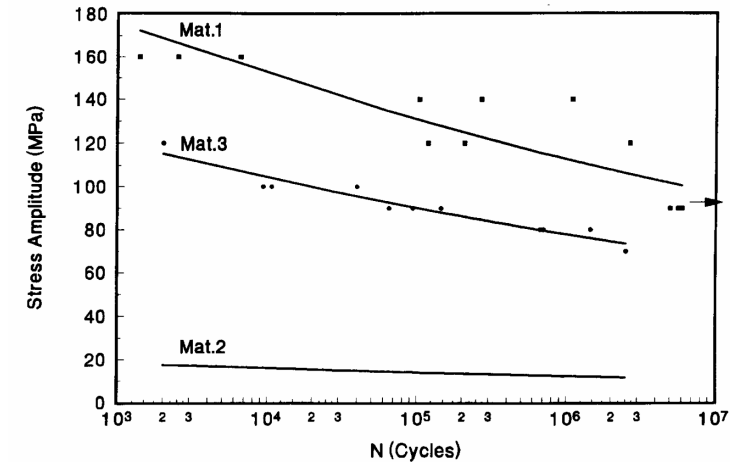
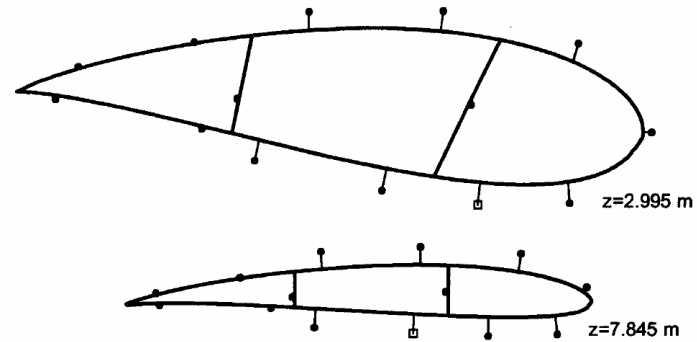


Figure 7. Location of different laminates in typical airfoil section.



CCfatigue



Fatigue lifetime prediction!
New and existing structures

Conclusions #1

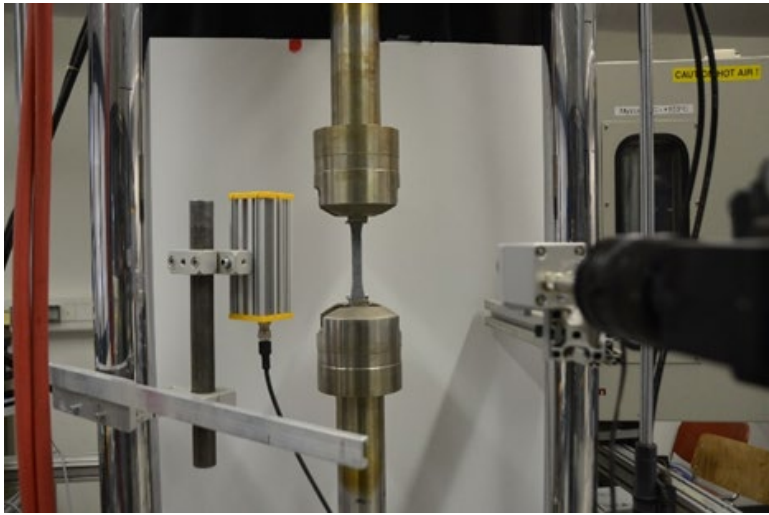
- Fatigue failures are the most common in components, and structures in all engineering fields
- Fatigue – a **BIG** problem in design of engineering structures
- Fatigue failure - The combined (synergistic) effect of the load, environment, geometry, material architecture, time, defects... (seems complicated ☹)
- Fatigue can be addressed by different approaches
- Prediction capacity limited due to the lack of exp. data for validation

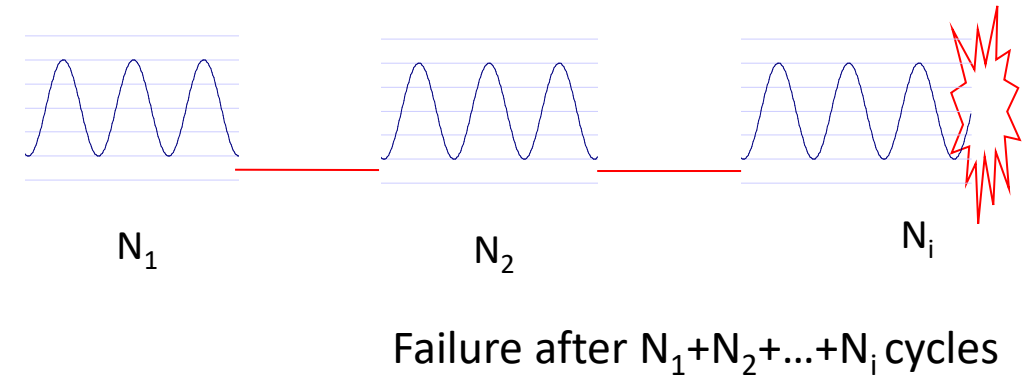
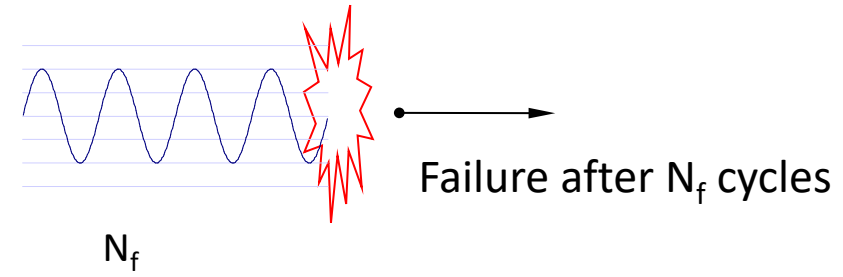
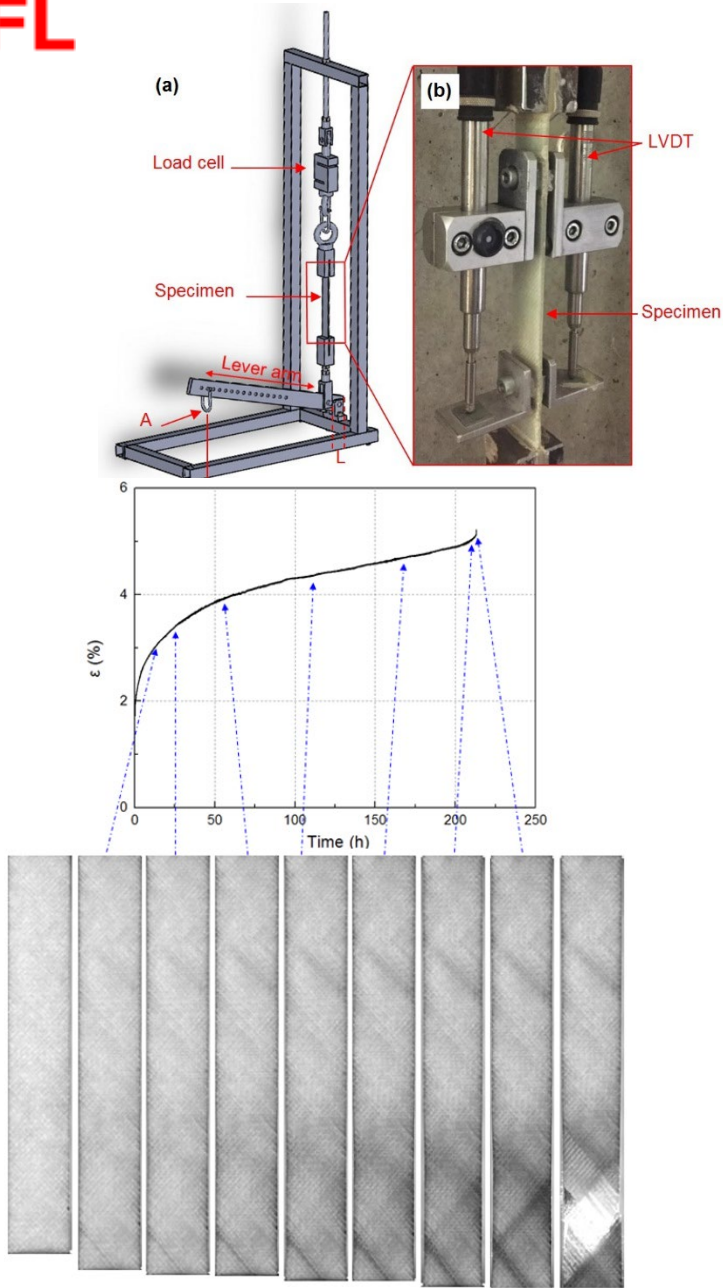


Prof. Anastasios P. Vassilopoulos

Fatigue of Composites
Lecture VIII – Part B:
Experimental investigations and
modeling approaches







Experimental characterization of composite laminates

Objectives of experimental investigation:

Identify material performance (qualitative)

Identify failure modes

Collect **data** for life estimation/durability/performance (quantitative)

Use **data** for modeling / create databases!

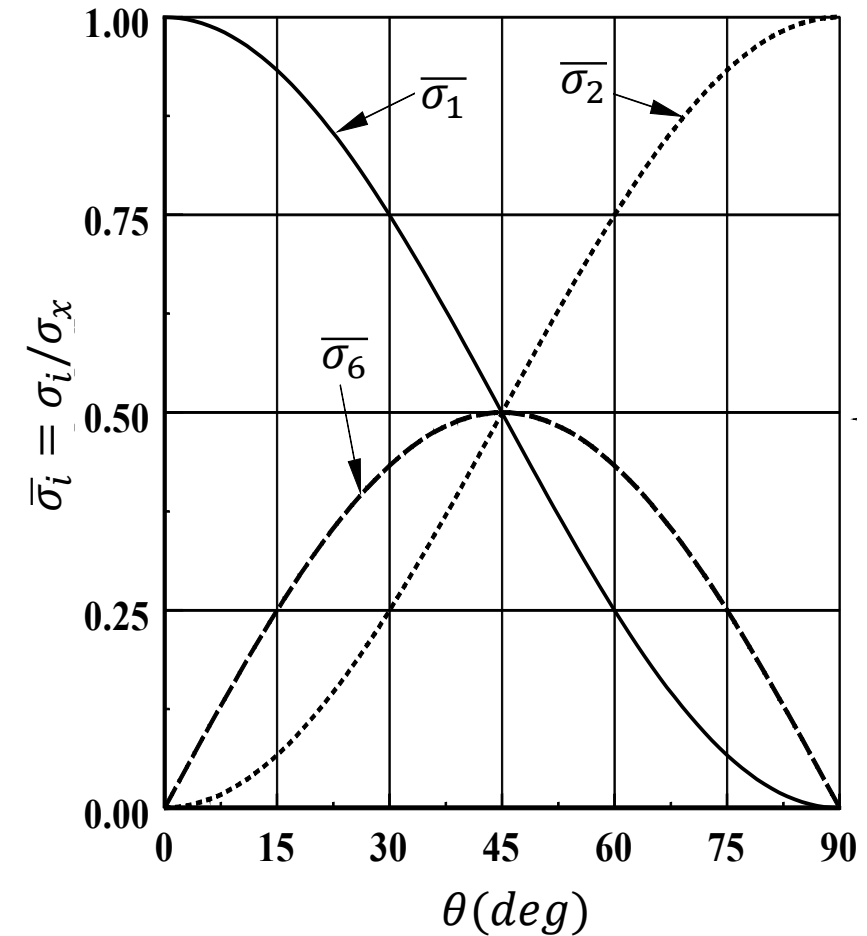
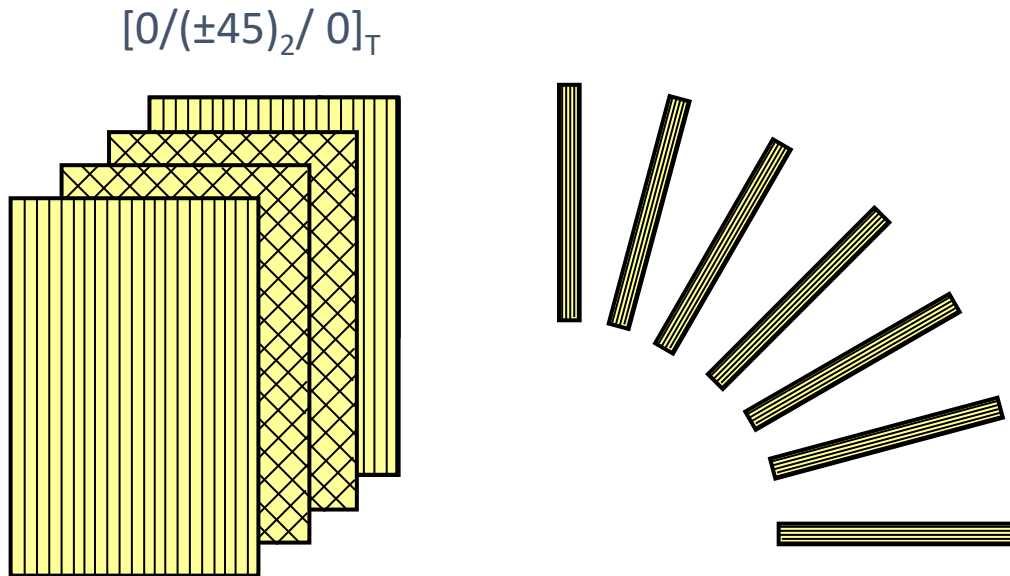
Experimental data are essential – either in **quasi static** or in **fatigue** data are needed to validate any theoretical estimation!

Fatigue is more complicated (and more costly ☹️) – The material properties vary with loading (not linearly and history plays significant role!)

What are the (usual) expected results?

- **Depending on the needs!**
- Collect lifetime data (S-N curve) life estimation
 - Measure load
- Stiffness degradation performance
 - Measure load and strain
- Strength degradation
 - Interrupt the test and perform static strength test
- Crack density

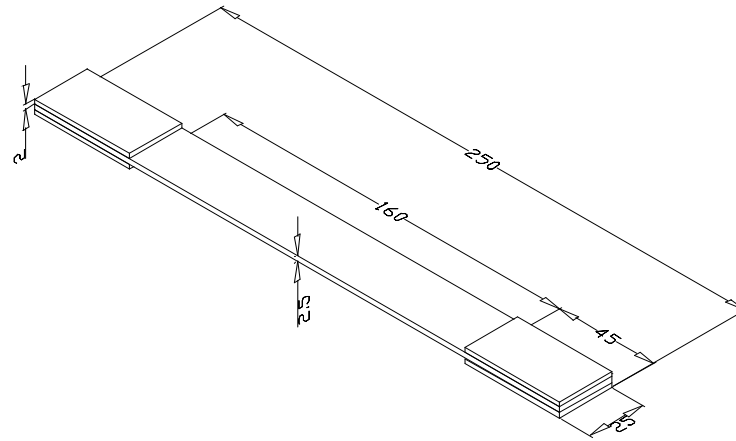
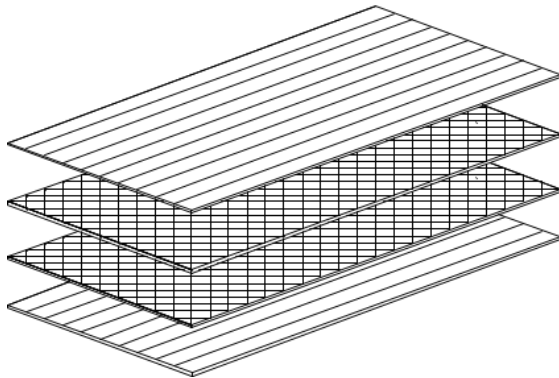
In plane stress state



M#1: Multidirectional laminate results

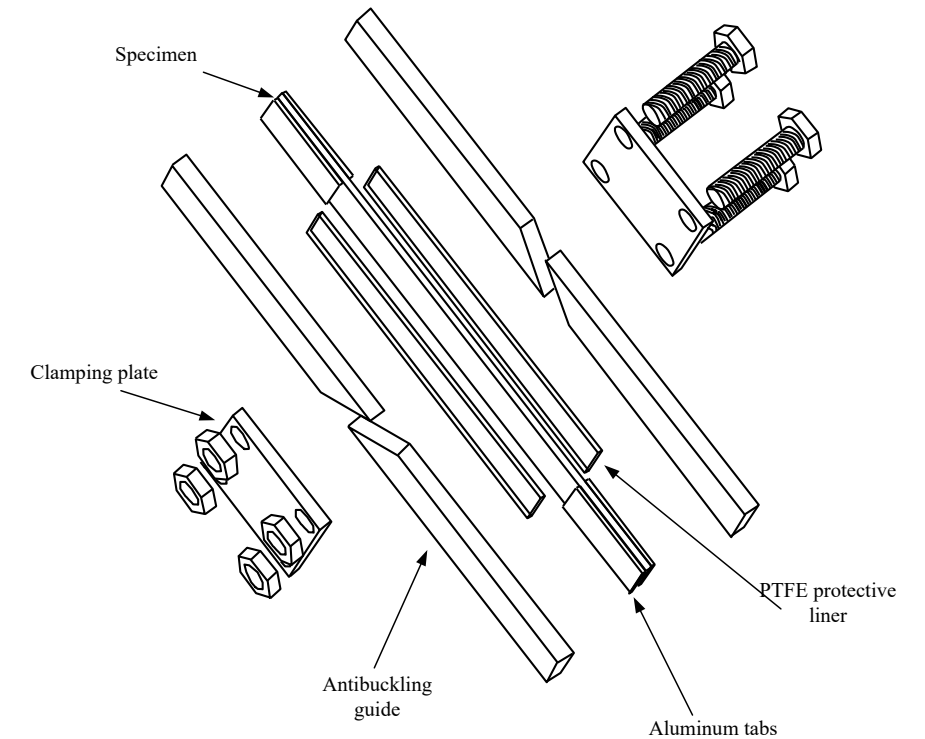
Specimen geometry and stacking sequence

$[0/(\pm 45)_2/0]_T$



Why this geometry?

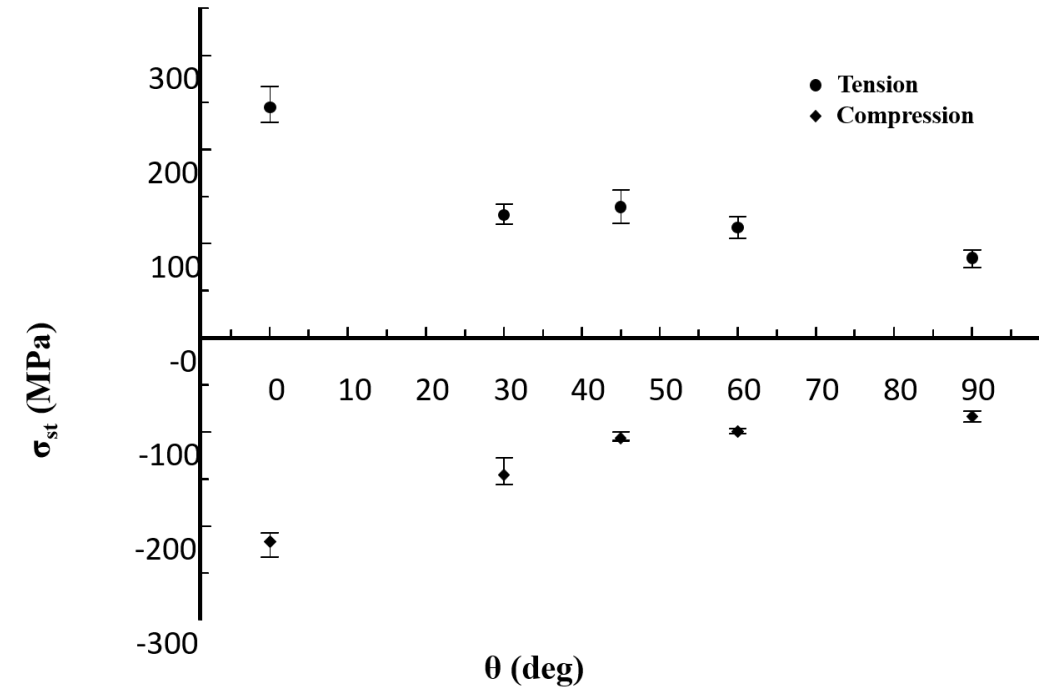
Anti buckling device



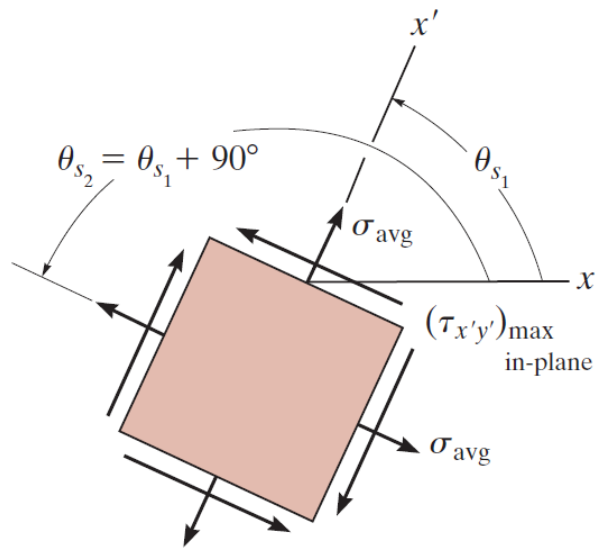
M#1: Multidirectional laminate results

QS results

	UTS (MPa)		UCS (MPa)	
	mean value	s.d.	mean value	s.d.
	Displacement control, 1mm/min			
0°	244.84 (4)	18.08	216.68 (4)	14.67
30°	130.52 (2)	14.23	145.52 (2)	13.94
45°	139.12 (2)	25.61	106.40 (2)	2.69
60°	117.26 (2)	16.64	99.52 (2)	3.59
90°	84.94 (3)	2.06	83.64 (3)	5.37
Shear	61.38=(139.12+106.40)/2.2			
	Load control, 40 kN/sec			
0°	417.49 (5)	74.86		



...mechanics...



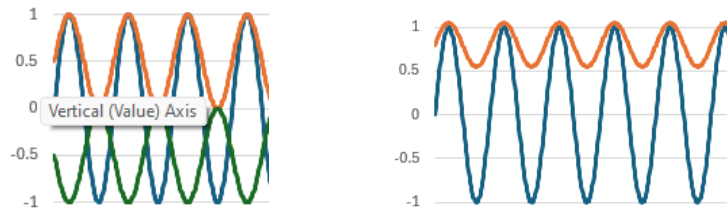
Maximum in-plane shear stresses

$$\tau_{max,in-plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

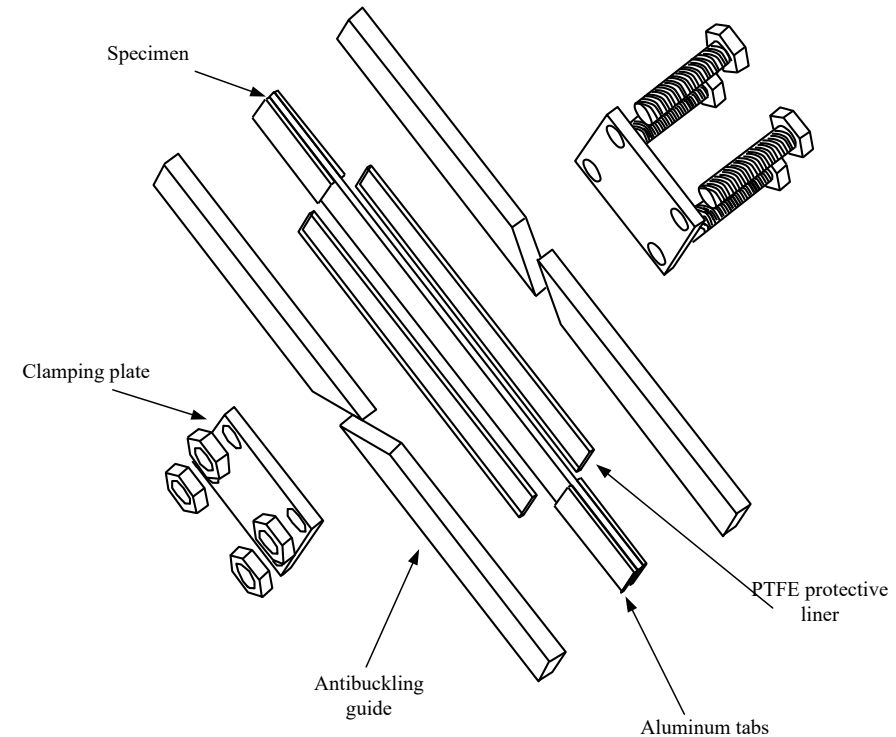
M#1: Multidirectional laminate results

CA Fatigue loading conditions!

1. Frequency 10Hz
2. R-ratio ($R = \sigma_{\min} / \sigma_{\max}$) = 0.1, 10, -1, 0.5
3. Laboratory conditions ($\pm 22^\circ\text{C}$, 50% RH)

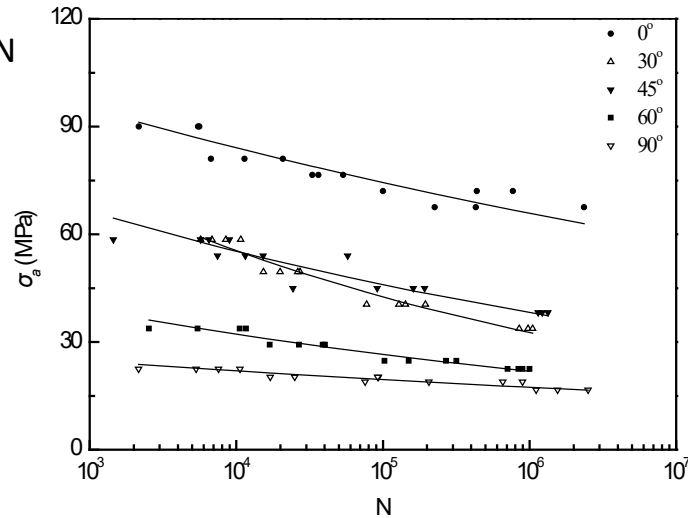


Anti buckling device

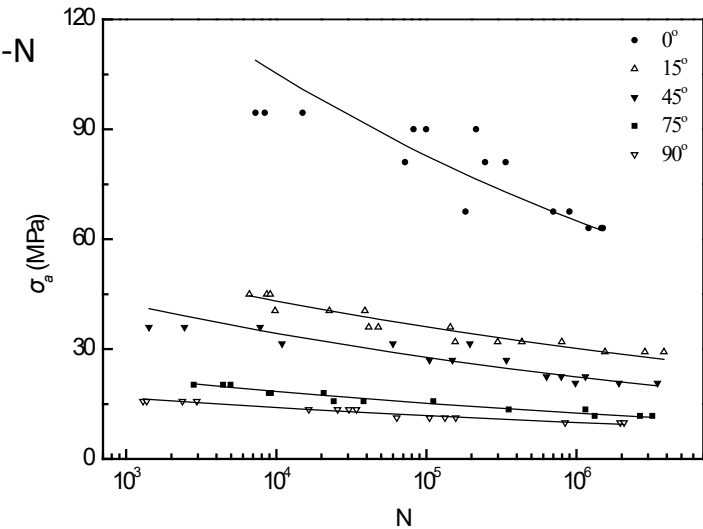


Stress life data (life estimation)

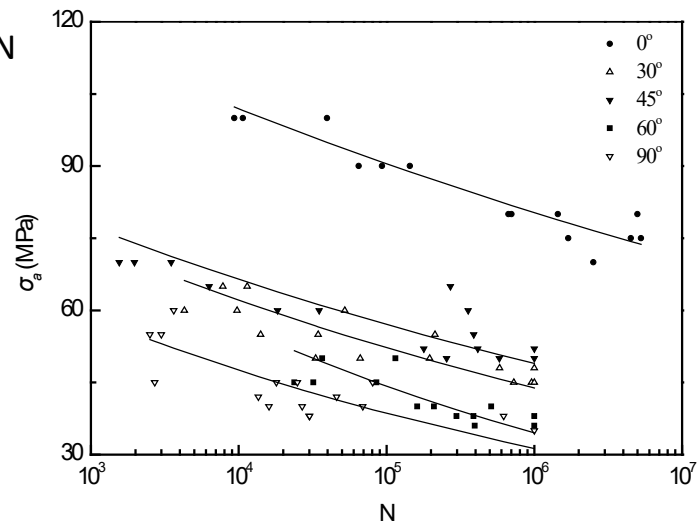
Fatigue data and S-N curves for R=10



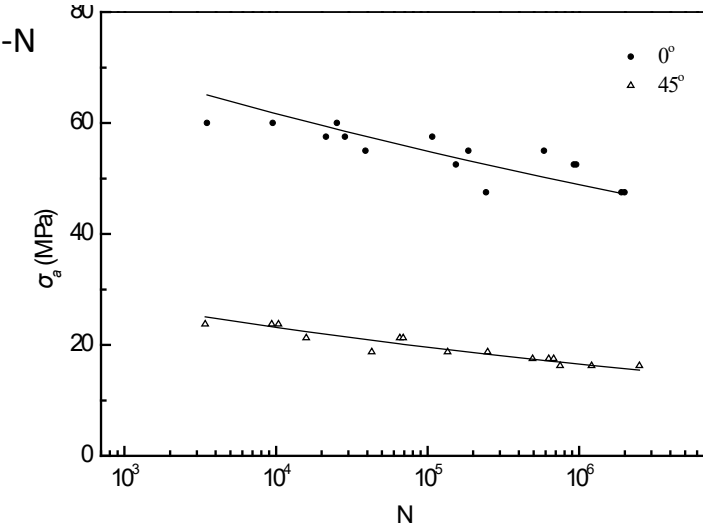
Fatigue data and S-N curves for R=0.1



Fatigue data and S-N curves for R=-1



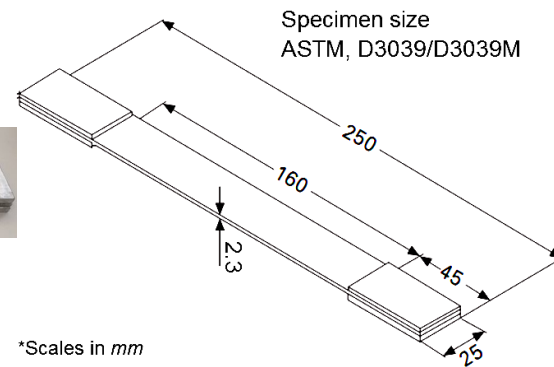
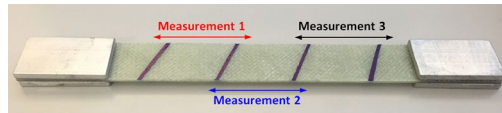
Fatigue data and S-N curves for R=0.5



Specimens and manufacturing

Fabrication:

- ❖ Used material: Glass/epoxy (GFRP)
- ❖ Specimen layup $(\pm 45)_{2s}$
- ❖ Vacuum assisted hand-lay up technique

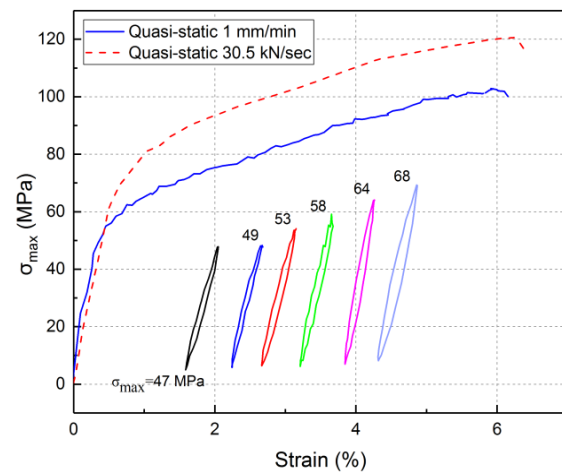
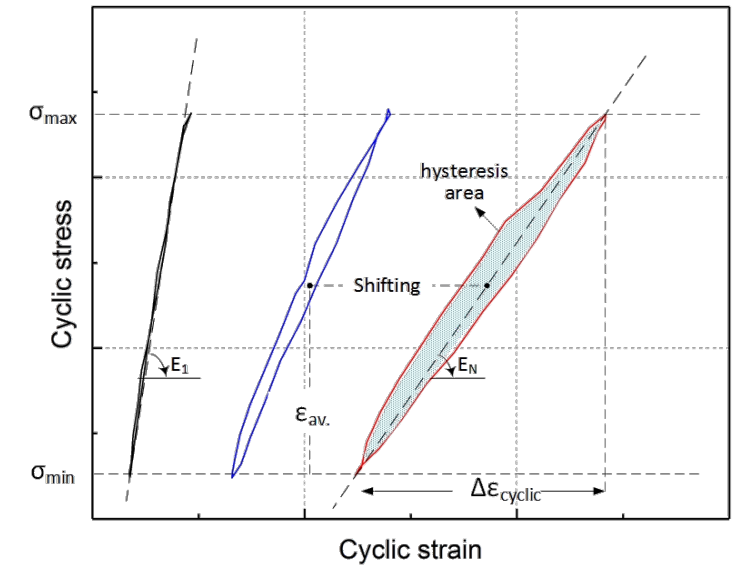
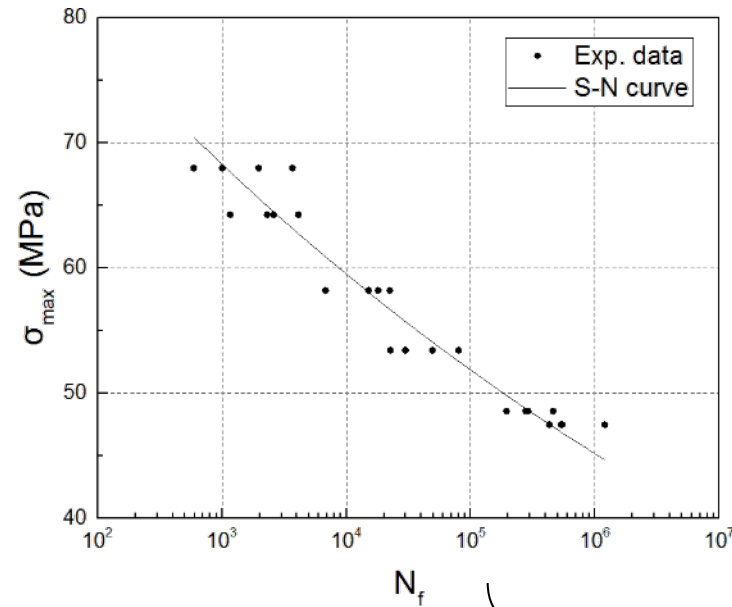
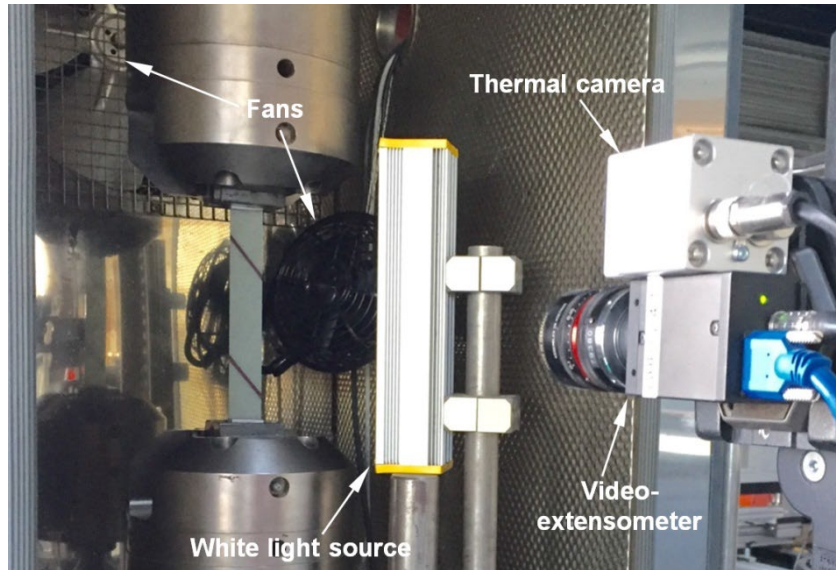


Burn off test:

- ❖ 62% Fiber content
- ❖ 38% Matrix content



Experimental results: S-N curves



Fatigue behavior

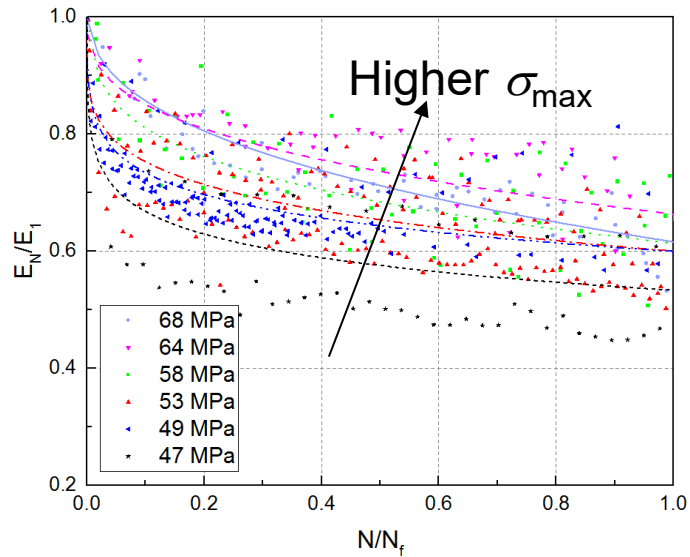
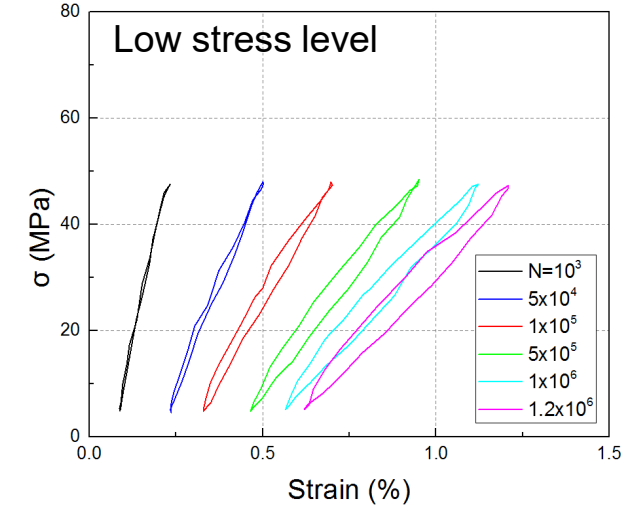
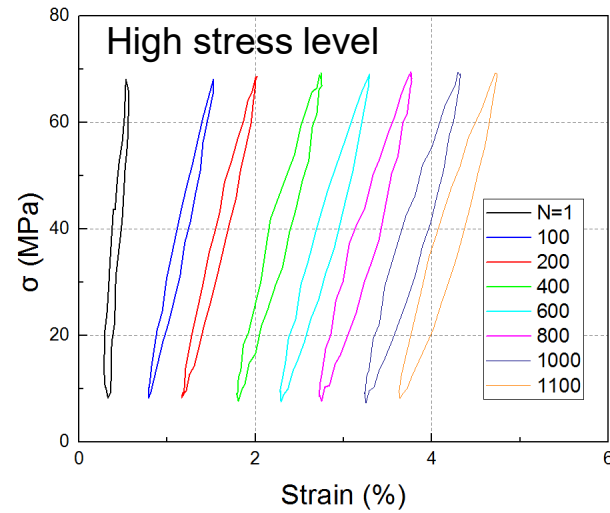
Fatigue life

Fatigue stiffness

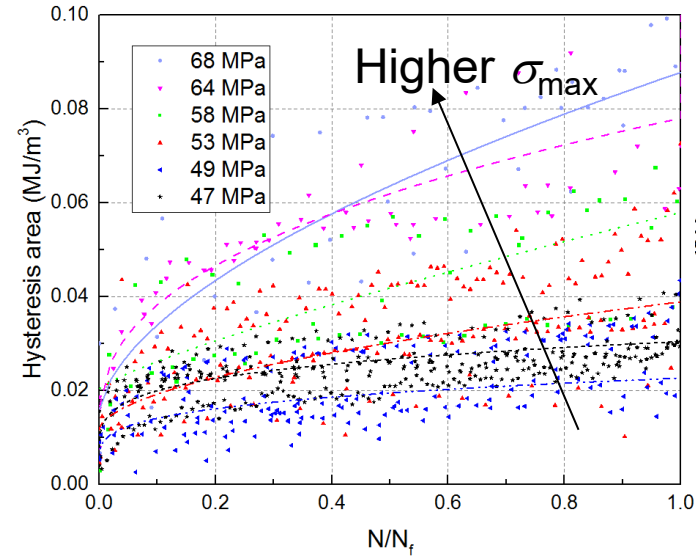
Hysteresis loop area

Cyclic creep: $(\epsilon_{\max} + \epsilon_{\min})/2$

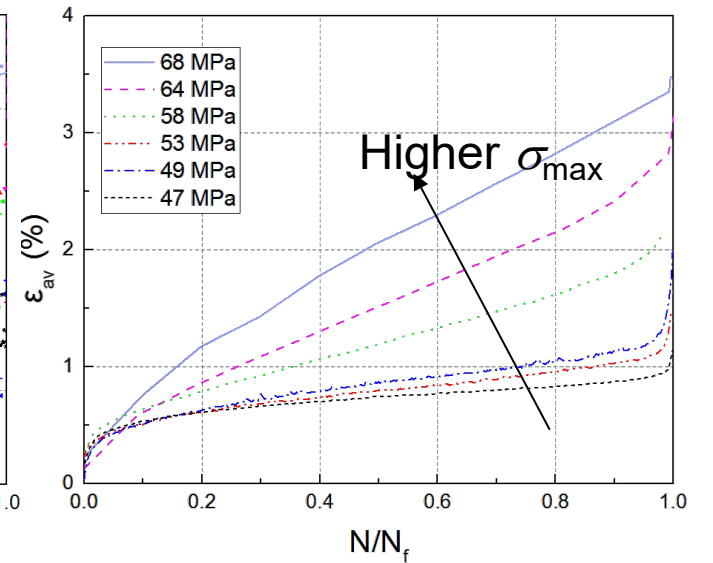
CA Fatigue performance



✓ Damage formation and accumulation

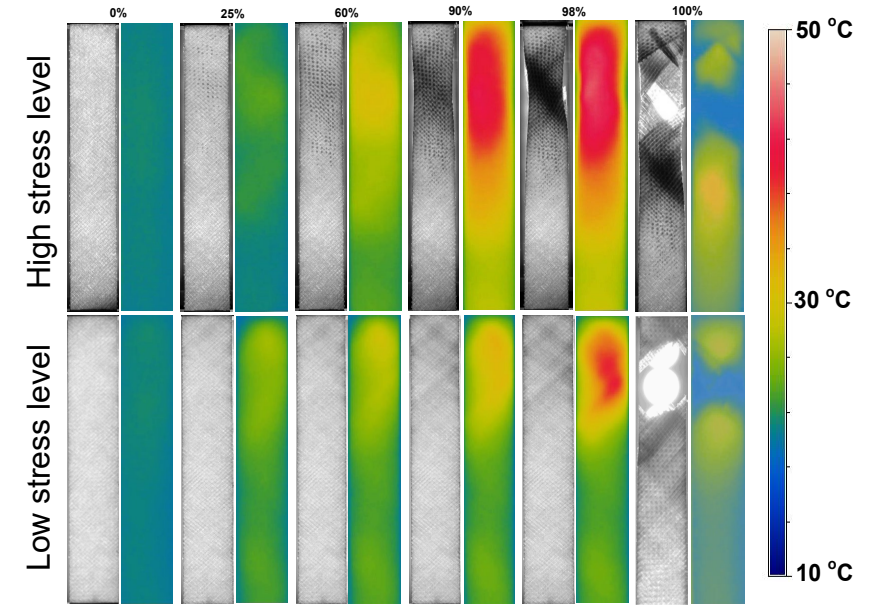
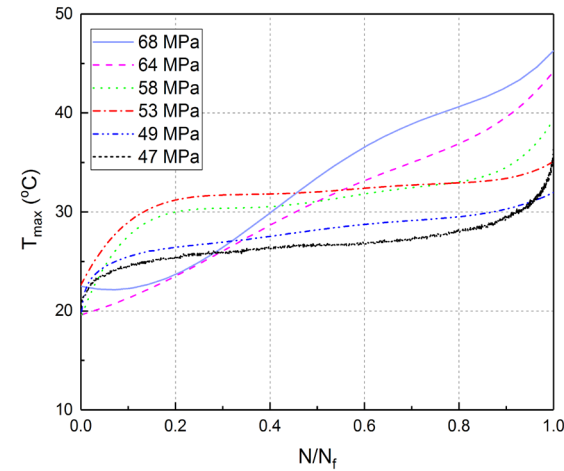
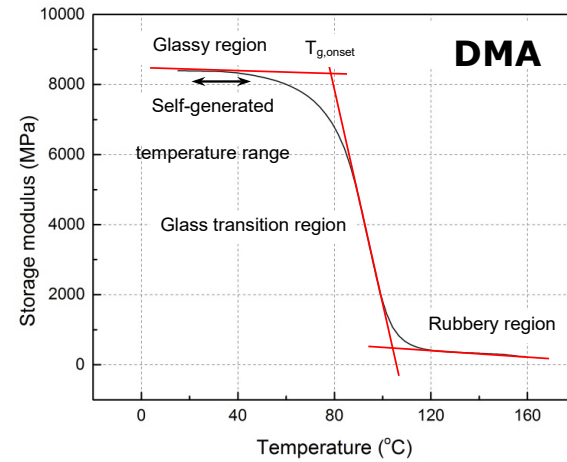
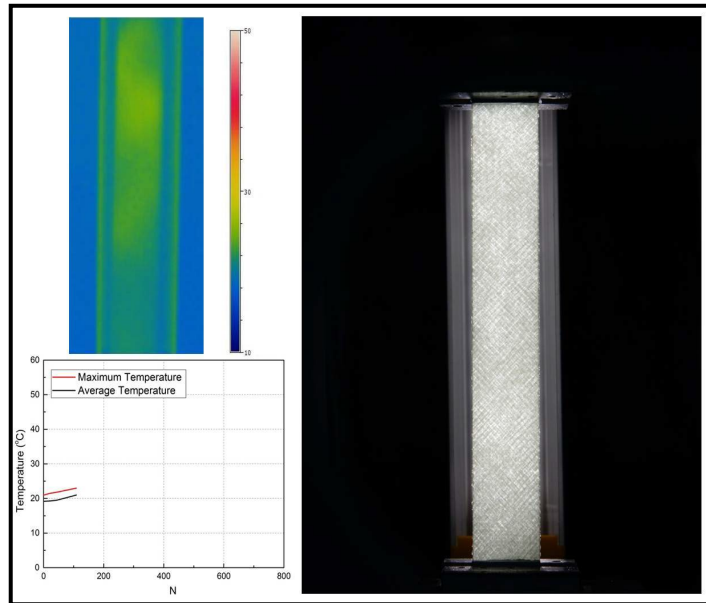


✓ Internal friction



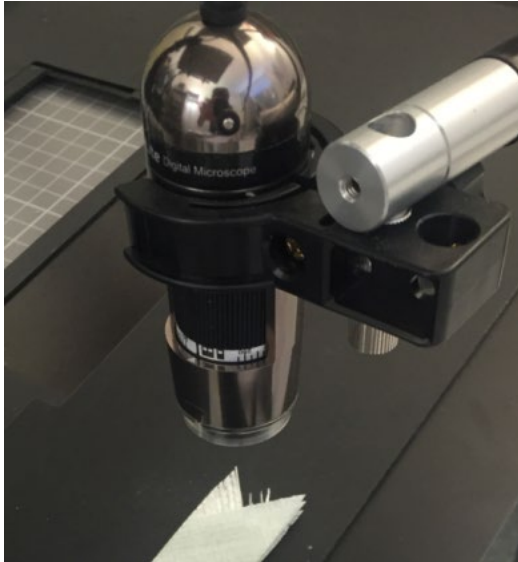
✓ Viscoelastic deformation
✓ Damage accumulation

Thermal behavior/observation ($R=0.1$)

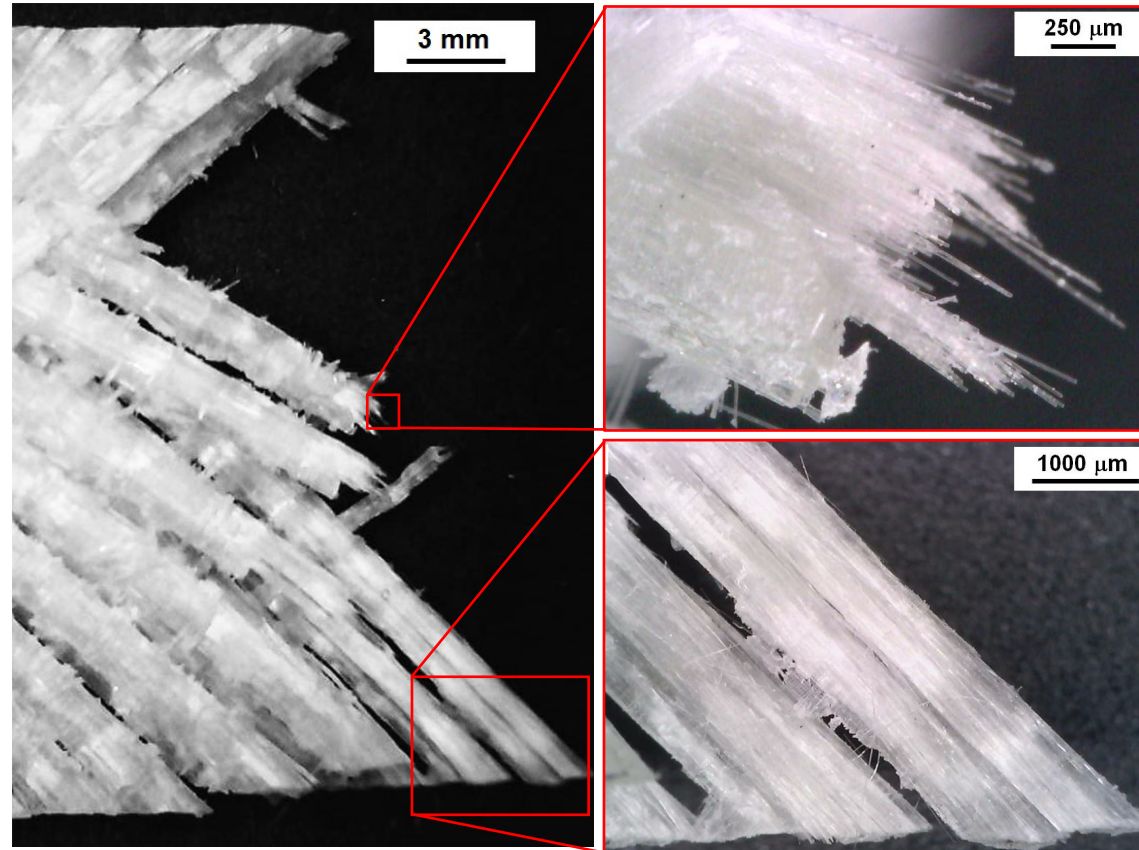


Damage development with stress level

Failure mechanisms



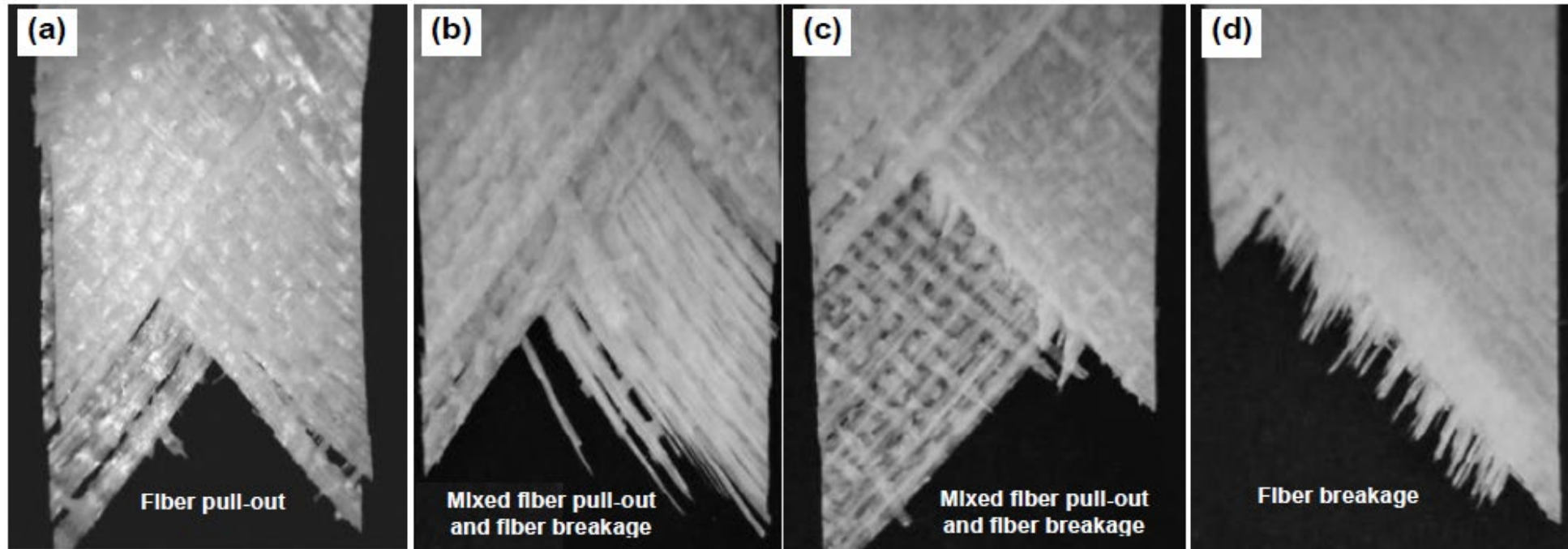
Digital handheld
microscope



1. Fiber breakage

2. Fiber pull-out

Failure mode

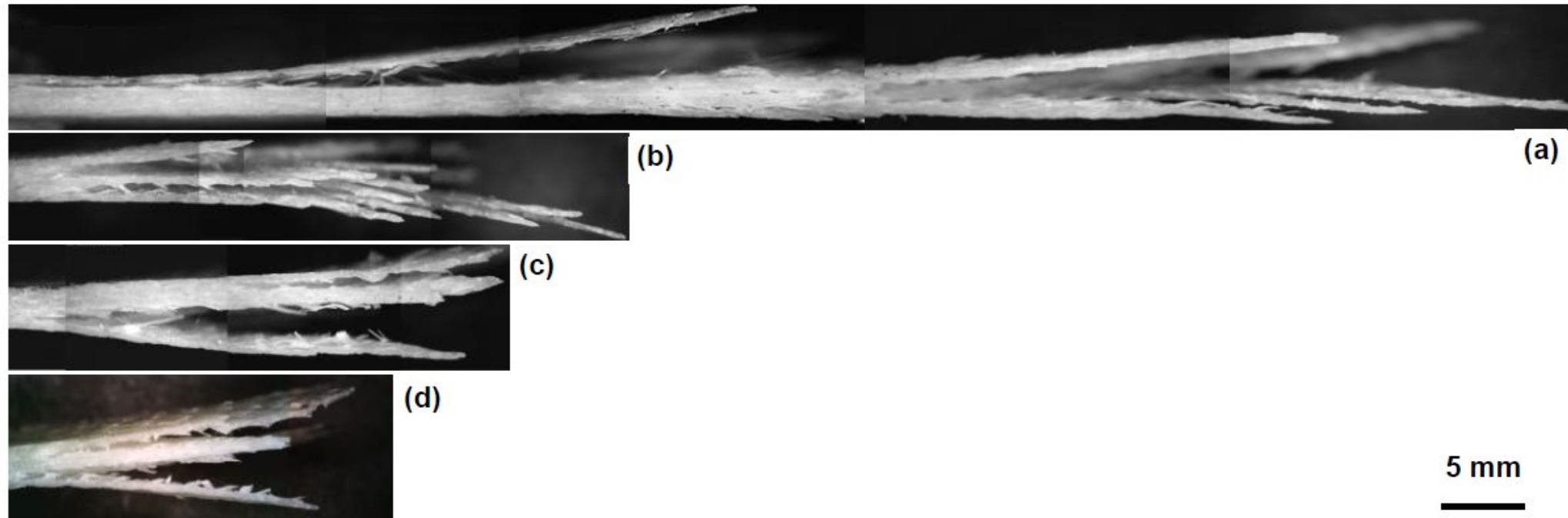


Fatigue fracture surface, (a) $\sigma_{\max} = 68$ MPa (Conf-0.1-68-c), (b) $\sigma_{\max} = 58$ MPa (Conf-0.1-58-c), (c) $\sigma_{\max} = 53$ MPa (Conf-0.1-53-c), and (d) $\sigma_{\max} = 49$ MPa (Conf-0.1-49-a).

All fracture surfaces exhibited a diagonal damage pattern

High stress = fiber pull-out, **Intermediate and low stress** = significant necking in the failure zone, mixed fiber pull-out and fiber breakage (partially fiber breakage and partially fiber pull-out), **low stress** = extensive fiber breakage

Failure mode



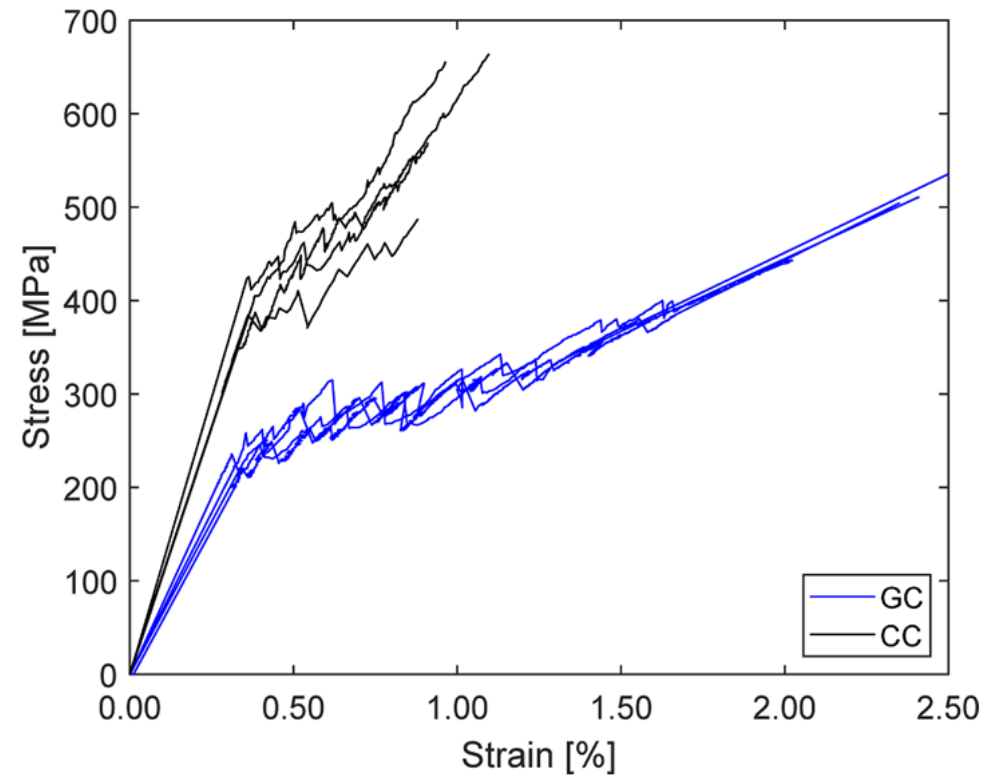
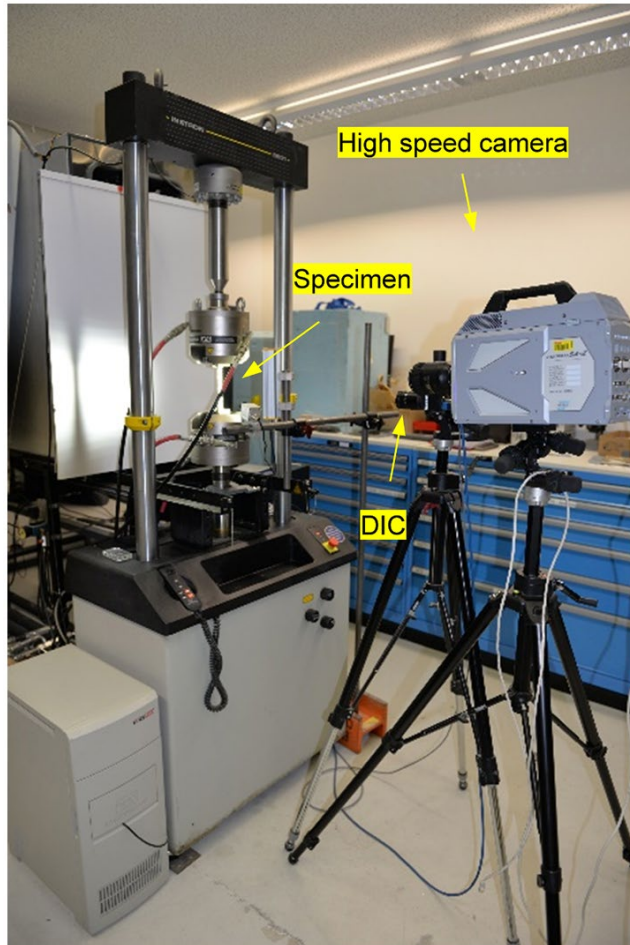
Delamination occurred in the resin-rich regions between the plies as a result of damage accumulation (cracks) in the matrix. The delamination size is longer for specimens examined at higher stress levels, i.e. exhibiting fiber pullout.

Demonstrate fatigue results with exp programs

- **Material #3: Hybrid G/CFRP composite laminate**
- Material system - 3 UD fabrics:
 - ultra high modulus carbon (S&P C-Sheet 640)
 - high strength carbon (S&P C- Sheet 240)
 - E-glass (S&P G-sheet E 90/10).
- A solvent-free, transparent 2-component epoxy resin (S&P Resin Epoxy 55 HP)
- Quasi-static and fatigue experiments, CA fatigue for hybrid glass/carbon
- **Objective: Investigate hybrid effect/fatigue performance**

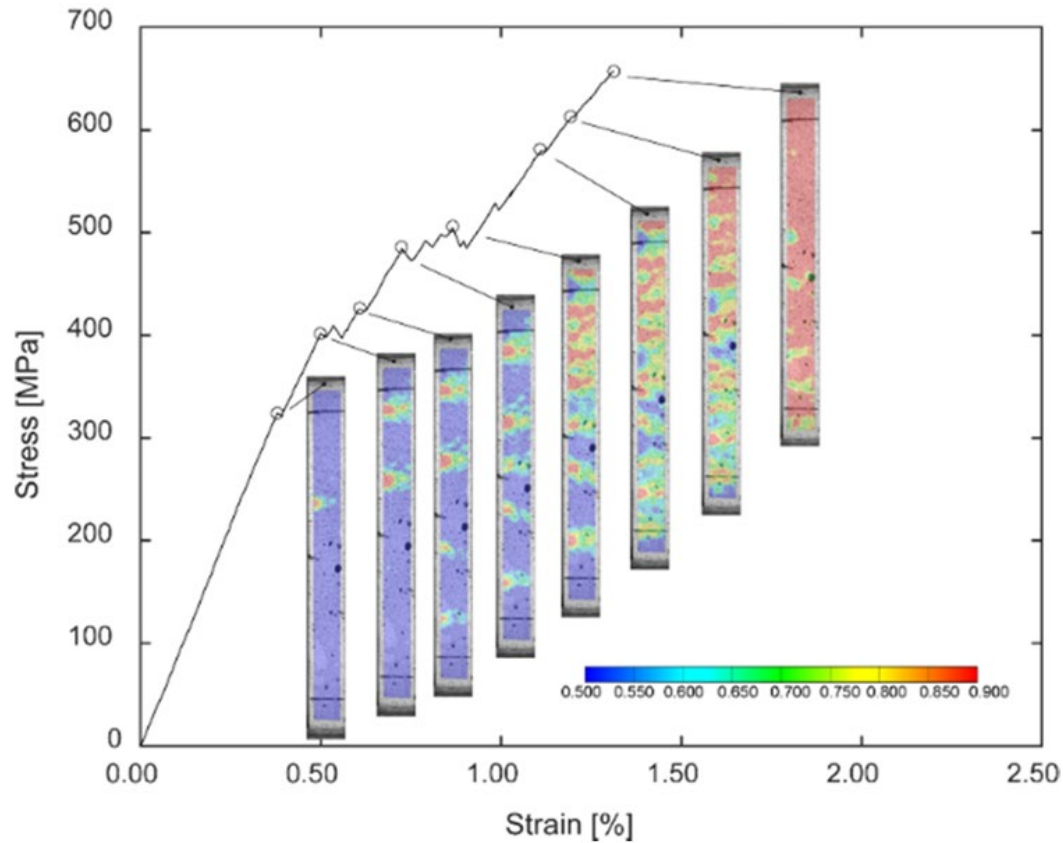
F. Ribeiro, J Sena Cruz, A. P. Vassilopoulos. "Tension-tension fatigue behavior of hybrid glass/carbon and carbon/carbon composites" Int J Fatigue, 2021;146:106143

Experimental results



Pseudo-ductile tensile responses of GC and CC combinations.

Failure process/failure modes



(a)

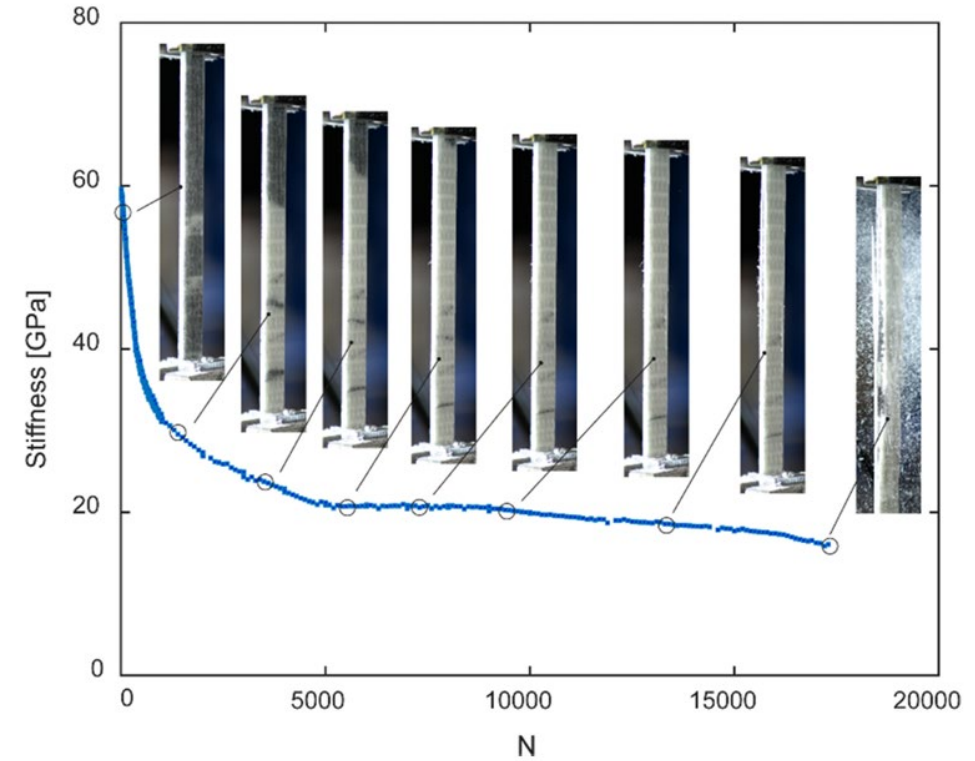
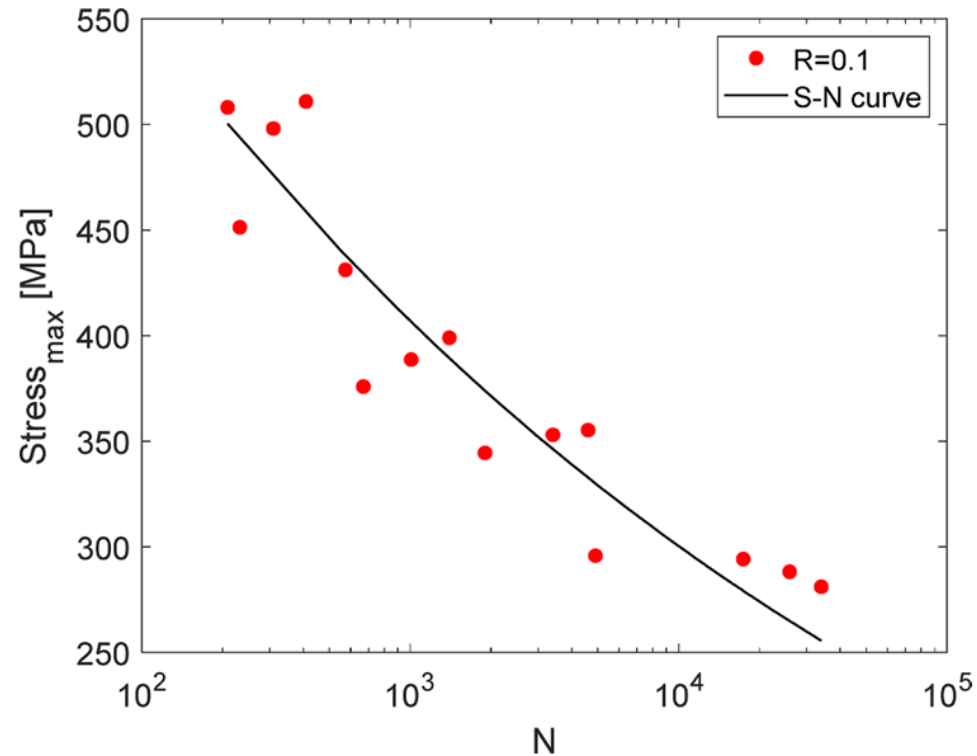


(b)

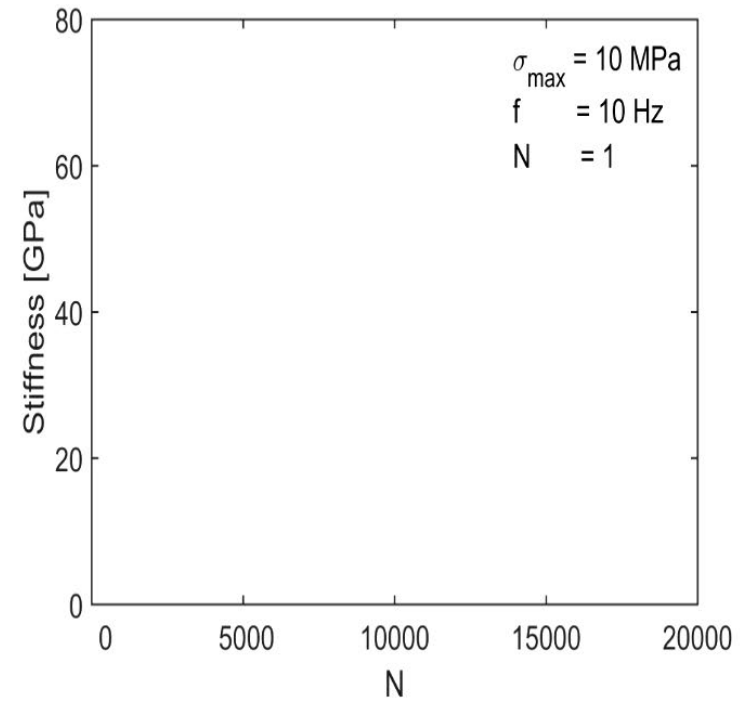
Failure modes: (a) GC and (b) CC combinations.

Strain field of CC combination (CC-QS-03 specimen) during loading

Fatigue results (GC laminate)



Damage and stiffness degradation during the fatigue test at lowest maximum stress level.



Modeling and prediction in fatigue

- Experimental investigations
 - Describe and **understand(?)** the fatigue behavior
- Models were established to simulate the exhibited behavior
 - Actually, to **Fit** the material behavior and derive mathematical representations (e.g. S-N curves, stiffness degradation models)
- Models for prediction
 - Models to “estimate” the material behavior under “unseen” **loads*** (CLDs, fatigue failure criteria etc.)

*Not necessarily possessing any **behavior explanation ability** – called as well as empirical/phenomenological models

Objective of modeling approaches

- Replace the need for excessive experimental campaigns
- Develop virtual testing environments
- Validated analytical/numerical models can be used for:
 - The **simulation** of the material's **response** under selected loading patterns, (S-N, E-N, CLDs).
 - The **lifetime estimation** under unseen loading patterns, including VA (failure criteria – damage rules).
 - **PDM**, identifying damage modes, and predicting eventual material failure.
 - **Material selection**/optimization, especially through bottom-up micro-mechanical multiscale approaches
 - able to assist in selecting the appropriate mix of materials, as well as joining techniques.

Descriptive modeling

- Methods of investigation (rely mainly on experimental data)
 - S-N curves and CLD*
 - Stiffness degradation theories
 - Residual strength theories

*CLDs also used for predictions

S-N curve types and CLD modeling

- Empirical methods relying on the quality of the available data
- S-N curve – fitting the fatigue life data (cyclic stress or strain (ϵ -N in that case) vs. cycles to failure
- CLD – constant life diagram – Shows the mean stress effects (summary of S-N data for different loading patterns)

S-N curves

- S-N curves simulate the fatigue behavior – mean fatigue life @ a given stress level
- They are simple representations of the material response – have no physical meaning
- The best S-N curve is that equation that **FITS** better the fatigue data in all fatigue regions*

* A. M. Freudenthal “The statistical aspect of fatigue of materials” 1946 - “The relation expresses the effect of a general mechanism of fatigue and is more than purely empirical (best fit line of large number of observations) is suggested by the circumstance that it is valid for practically any kind of material, whatever its deformational performance”.

The S-N curve

- One of the most **explicit and straightforward** ways to represent experimental fatigue data is the S–N diagram.
- It is **preferred** to other approaches for the modeling of the fatigue life of FRP composite materials, e.g. those based on stiffness degradation, or crack propagation measurements during lifetime, since it requires **input data** (applied load and corresponding cycles to failure) that can be **collected using very simple recording devices**
- Usually, fatigue data for preliminary design purposes are gathered in the region of fatigue cycles ranging between 10^3 and 10^7 .
- Depending on the application, **high- or low-cycle fatigue regimes can be of interest**. Additional data are needed in such cases to **avoid the danger of poor modeling due to extrapolation into unknown spaces**.

R. Sarfaraz, A. P. Vassilopoulos, T. Keller “A hybrid S–N formulation for fatigue life modeling of composite materials and structures” Comp A, 2012;43:445-453

The S-N curve

- Although for the high-cycle fatigue (**HCF**) regime long-term and time-consuming fatigue data must be acquired.
- The situation seems **easier (?)** for low-cycle fatigue (**LCF**) where, static strength data can apparently be used in combination with the fatigue data.
- When the **static strength data** are considered in the analysis, other problems can arise especially when they have been acquired under **strain rates** much lower than those used in fatigue loading.
- The S-N curve flattens at the LCF region – this has been attributed to the sensitivity of FRP composites to high strain ranges that occurred under high stress level.

R. Sarfaraz, A. P. Vassilopoulos, T. Keller "A hybrid S-N formulation for fatigue life modeling of composite materials and structures" Comp A, 2012;43:445-453

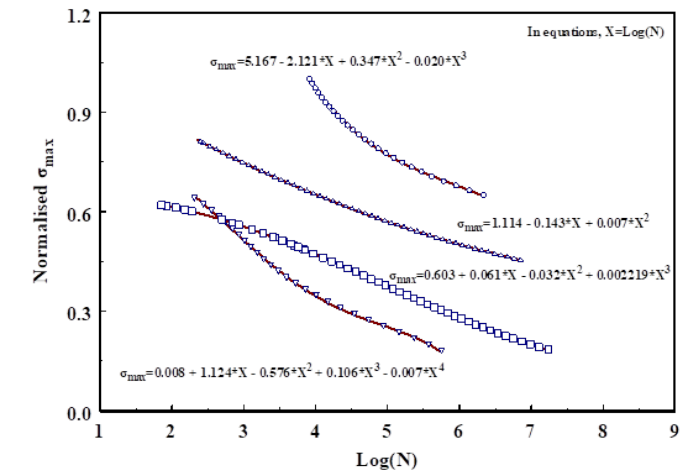
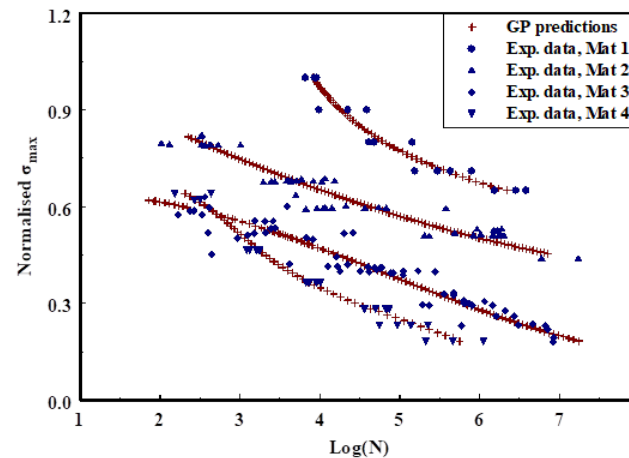
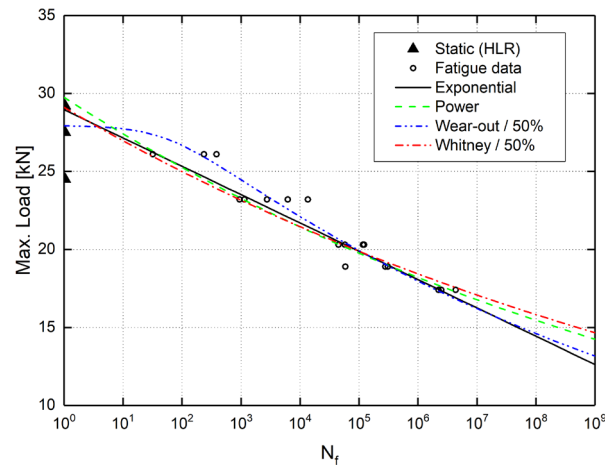
S-N curve types

- Which type of S-N curve is the best?
- That one that fits better the fatigue data!

R. Sarfaraz, A. P. Vassilopoulos, T. Keller "A hybrid S-N formulation for fatigue life modeling of composite materials and structures" Comp A, 2012;43:445-453

S-N curve types – methods for derivation

- Linear regression (Power law, logarithmic, ASTM etc)
- Statistical based S-N curves (Whitney, Sendekyj, etc)
- Polynomial fitting (Harris, Vassilopoulos)
- “ML” methods for fatigue analysis of composites (Vassilopoulos)**



*R. Sarfaraz, A. P. Vassilopoulos, T. Keller “Modeling the constant amplitude fatigue behavior of adhesively bonded pultruded GFRP joints” J. Adhes Sci Technol, 2013;27(8):855-878

** A. P. Vassilopoulos, E. Georgopoulos “Novel Computational methods for fatigue life modeling of composite materials” in Fatigue and life prediction of composites and composite structures, 2010

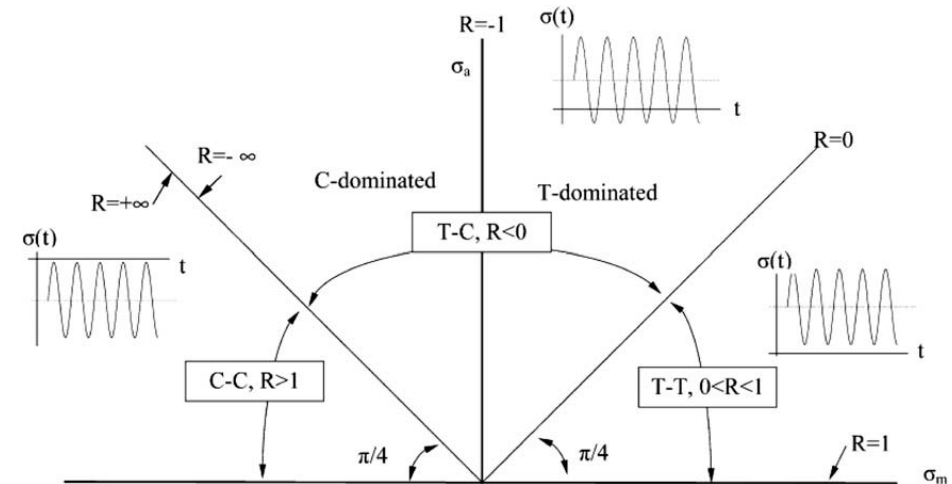
Constant life diagrams

- Constant life diagrams **reflect the combined effect of mean stress and material anisotropy** on the fatigue life of the examined composite material. Furthermore, they offer a predictive tool for the estimation of the fatigue life of the material under loading patterns for which no experimental data exist
- The **main parameters** that define a CLD are the mean cyclic stress, σ_m , the cyclic stress amplitude, σ_a , and the R -ratio defined as the ratio between the minimum and maximum cyclic stress, $R = \sigma_{min}/\sigma_{max}$

- Three sectors...
- Radial lines emanating from the origin = S-N curves

$$\sigma_a = \left(\frac{1 - R}{1 + R} \right) \sigma_m$$

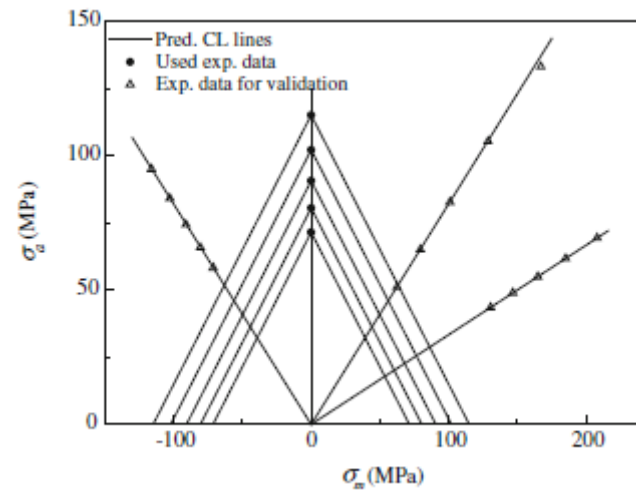
- CLDs are formed by connecting data in various S-N curves



Constant life diagram types

- Linear CLD
- The concept of the linear CLD model is based on a single S–N curve that must be experimentally derived. All other S–N curves can be determined from the given one by simple calculations. This simplified formulation assumes **identical failure mechanisms** in tension and in compression when the load amplitude is the same.

$$\frac{\sigma_a}{\sigma_o} + \frac{\sigma_m}{\sigma_o} = N^{-1/k}$$



Constant life diagram types

- Piece wise linear (PWL) CLD
- Derived by linear interpolation between known values in the $(\sigma_m - \sigma_a)$ -plane
- Requires a limited number of experimentally determined S–N curves, and UTS, UCS
- Usually curves for $R=0.1$ for T-T loading, $R=-1$ for T-C loading and $R=10$, for C-C loading patterns

If R in TT region

$$\sigma'_a = \frac{UTS}{\frac{UTS}{\sigma_{a,1TT}} + r' - r_{1TT}}$$

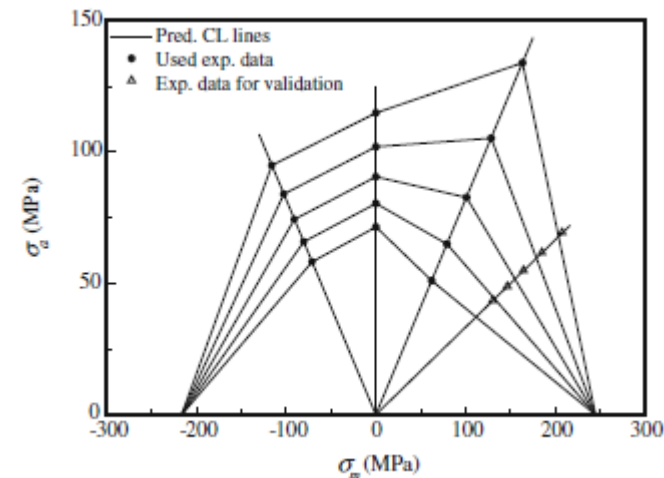
If R in CT region

$$\sigma'_a = \frac{\sigma_{a,i}(r_i - r_{i+1})}{(r_i - r')\frac{\sigma_{a,i}}{\sigma_{a,i+1}} + (r' - r_{i+1})}$$

If R in CC region

$$\sigma'_a = \frac{UCS}{\frac{UCS}{\sigma_{a,1CC}} - r' + r_{1CC}}$$

$$r_i = (1 + R_i)/(1 - R_i)$$



Harris CLD

- A semi-empirical equation based on fatigue test data

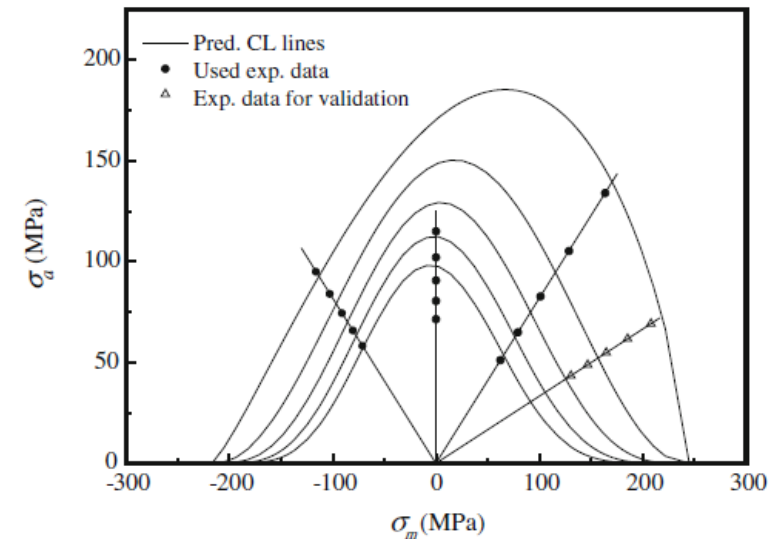
$$a = f(1 - m)^u(c + m)^v$$

- where a is the normalized stress amplitude component, σ_a/UTS ,
- m the normalized mean stress component, σ_m/UTS ,
- c the normalized compression strength, UCS/UTS
- In this equation, f , u and v are three adjustable parameters functions of fatigue life.

$$f = A_1 \log(N) + B_1$$

Requests 3 S-N curves: $u = A_2 \log(N) + B_2$

$$v = A_3 \log(N) + B_3$$



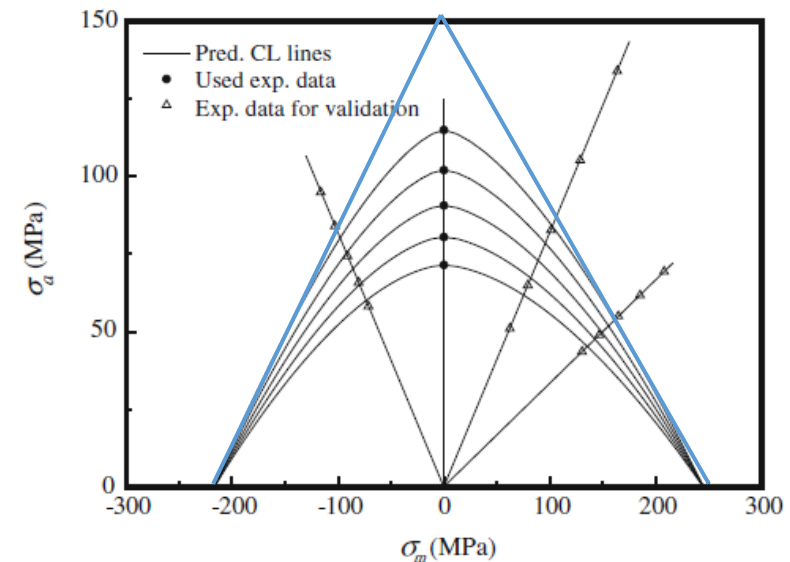
Kawai CLD

- Asymmetric CLD based on the assumptions:

1. The stress amplitude, σ_a , for a given constant value of fatigue life N is greatest at the critical stress ratio,
2. The shape of the CFL curves changes progressively from a straight line to a parabola with increasing fatigue life
3. The diagram is bounded by the static failure envelope

$$\frac{\sigma_a^\lambda - \sigma_a}{\sigma_a^\lambda} = \begin{cases} \left(\frac{\sigma_m - \sigma_m^\lambda}{UTS - \sigma_m^\lambda} \right)^{(2-\psi_\lambda)}, & UTS \geq \sigma_m \geq \sigma_m^\lambda \\ \left(\frac{\sigma_m - \sigma_m^\lambda}{UCS - \sigma_m^\lambda} \right)^{(2-\psi_\lambda)}, & UCS \leq \sigma_m \leq \sigma_m^\lambda \end{cases}$$

Critical stress ratio: $\psi_\lambda = \frac{\sigma_{\max}^\lambda}{\sigma_B}$

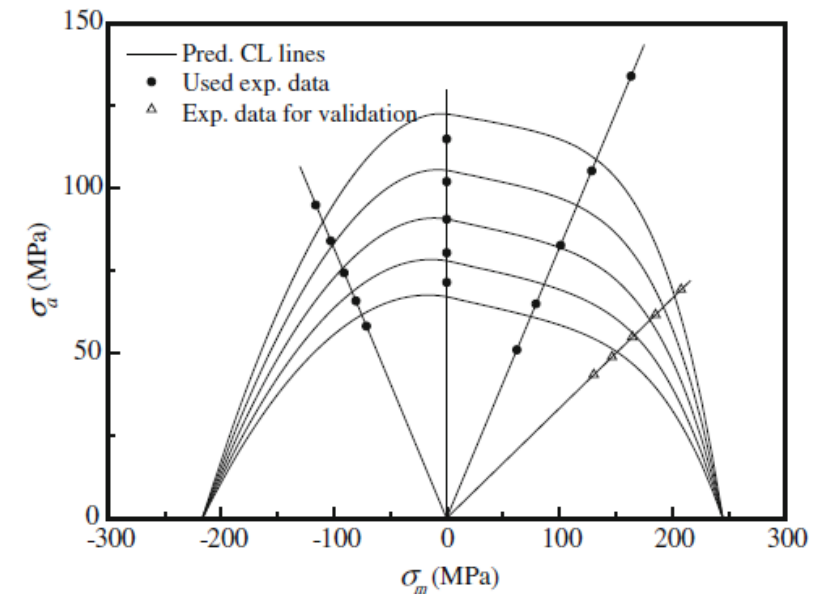


Boerstra

- An interesting formulation that does not need “complete” S-N curve data – a modified “Gerber” diagram
- The constant life lines are given by:

$$\text{For } \sigma_m > 0 : \quad \sigma_{ap} = \sigma_{Ap} (1 - (\sigma_m / UTS)^{\alpha_T})$$

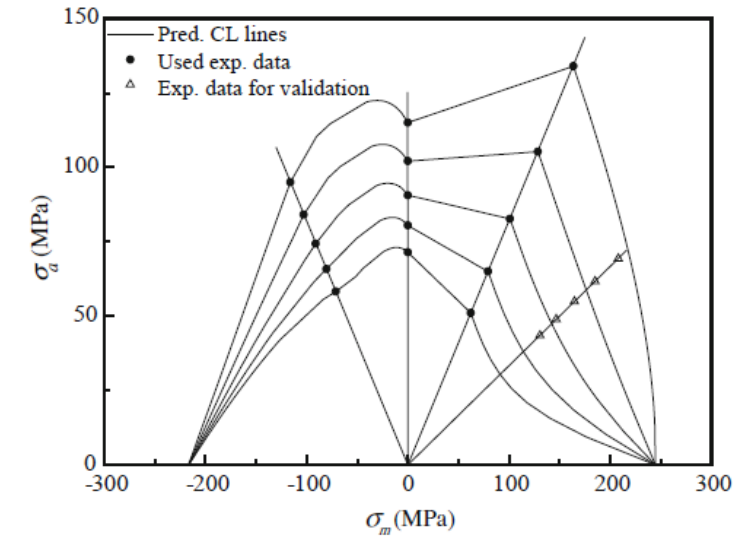
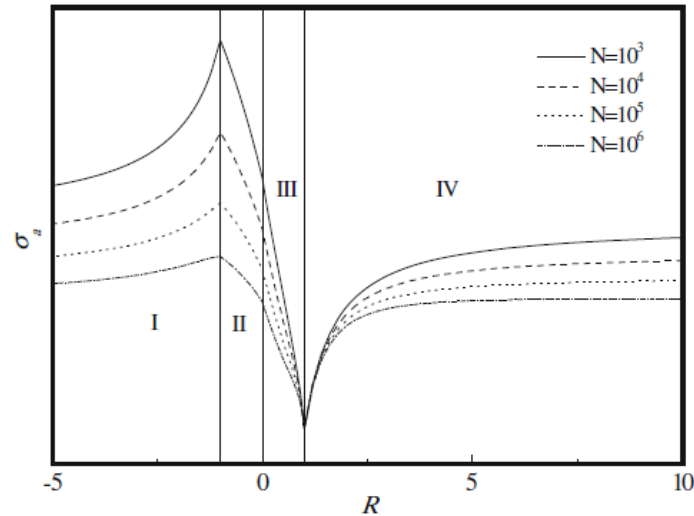
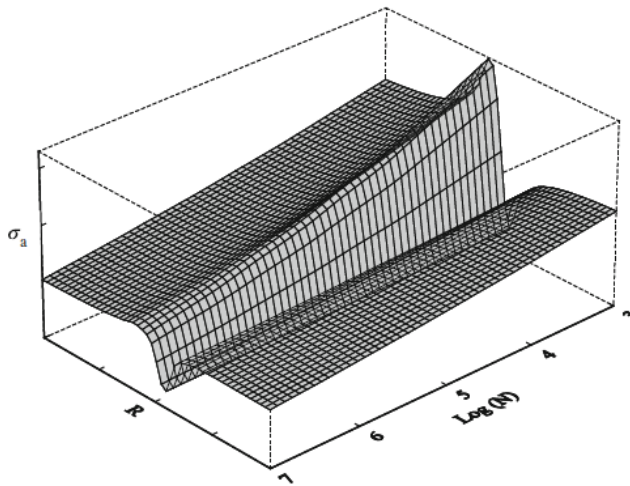
$$\text{For } \sigma_m < 0 : \quad \sigma_{ap} = \sigma_{Ap} (1 - (\sigma_m / UCS)^{\alpha_C})$$



- A multi objective optimization is needed to estimate the model parameters

Piece wise non-linear

- Suggests plotting stress amplitude vs. R-ratio for different cycle numbers



Stiffness degradation theories*

- Theory of Continuum Damage Mechanics

In 1958, Kachanov proposed to describe brittle creep rupture under uniaxial tension by a scalar field variable, the continuity ψ

- $\psi=1$ corresponds to a completely defect-free material
- $\psi=0$ corresponds to a completely destroyed material
- $D=1-\psi$ is therefore a measure of the state of deterioration or damage, $D=1$ failure, $D=0$ undamaged material!

* W. Van Paepegem "Fatigue damage modeling of composite materials with the phenomenological residual stiffness approach" in Fatigue and life prediction of composites and composite structures, 2010

Stiffness degradation theories

- Lemaitre introduced the concept of strain equivalence
- “A damaged volume of material under the nominal stress σ shows the same strain response as a comparable undamaged volume under the effective stress $\bar{\sigma}$ ”
- $\varepsilon_e = \frac{\bar{\sigma}}{E_o} = \frac{\sigma}{E_o(1-D)}$, with E_o the stiffness of the undamaged material.
- Then the damage variable D becomes a measure of stiffness degradation:
- $D = 1 - \frac{E}{E_o}$

Stiffness degradation – early investigations

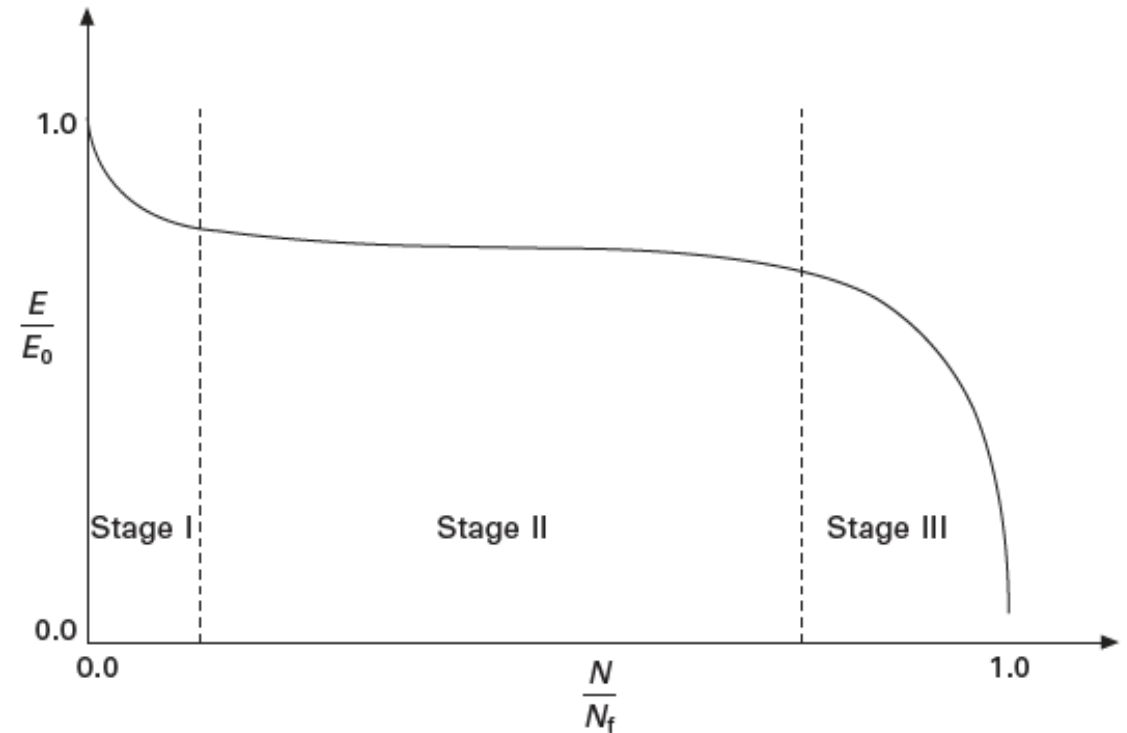
- Early investigations on composite materials were conducted by the groups of Karl Schulte and Kenneth Reifsnider in the 1980s.
- Schulte distinguished three distinctive stages in the stiffness reduction curve

Stiffness degradation diagram

- Schulte distinguished three distinctive stages in the stiffness reduction curve (for cross ply laminates)
- The initial region (stage I) with a rapid stiffness reduction
 - Mainly matrix cracking
- The intermediate region (stage II), approximately linear with cycles
 - Damage due to edge delaminations and additional longitudinal cracks along the 0°
- The final region (stage III), in which stiffness reduction occurs in abrupt steps ending in specimen fracture.
 - In stage III, a transition to local damage progression occurs, when the first initial fiber fractures lead to strand failures.

Stiffness degradation diagram

- Stiffness can be measured frequently or even continuously during fatigue experiments, and can be measured without further degrading the material
- The residual stiffness model may be **deterministic** (a single-valued stiffness property is predicted) or **statistical** (predictions are for stiffness distributions)

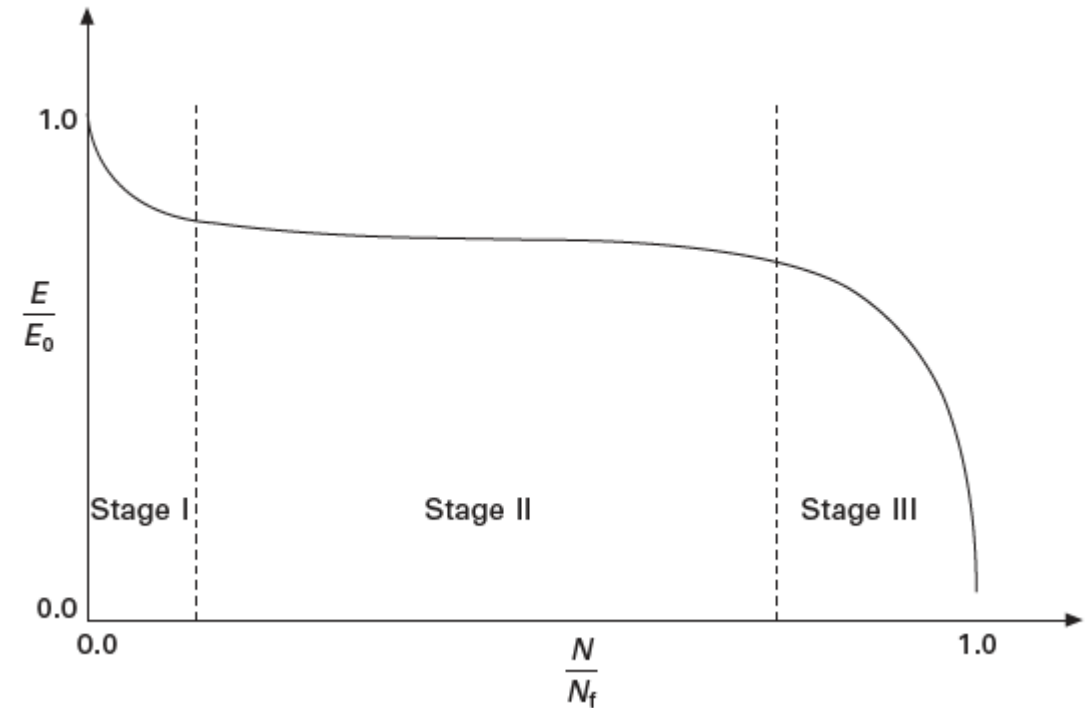


Summary: Stiffness degradation theories

- Stiffness degradation – decrease of the elastic properties (especially E_1) with cycles
- Residual stiffness = remaining stiffness after N cycles

$$D = 1 - \frac{E}{E_0}$$

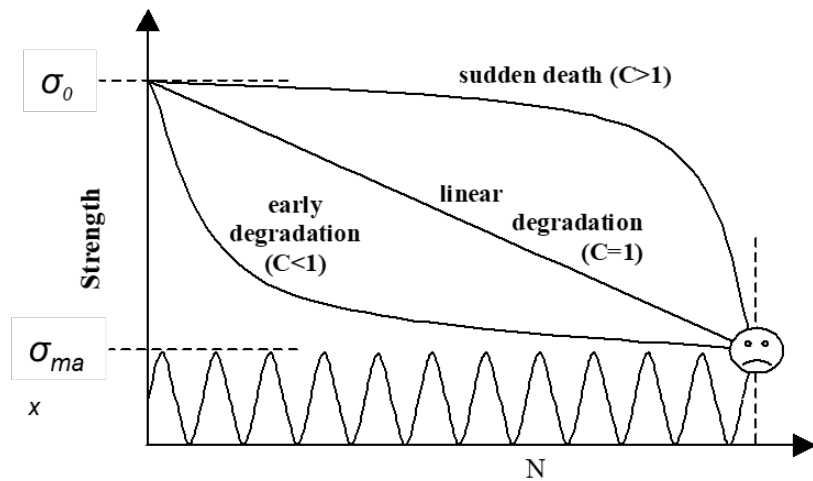
- With $D=0 \rightarrow$ undamaged (pure) material
- $D=1 \rightarrow$ complete loss of integrity



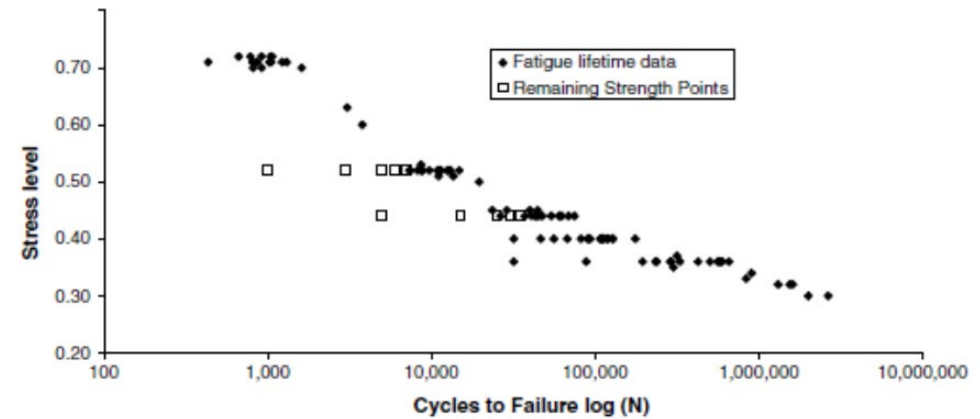
Typical stiffness degradation response

Residual strength degradation theories

- Basic assumptions
 - The residual strength is monotonically decreasing with cycles
 - The initial strength equals to the **static strength**
 - The residual strength at failure equals to the applied CA stress



Residual strength concepts



Residual strength data collection

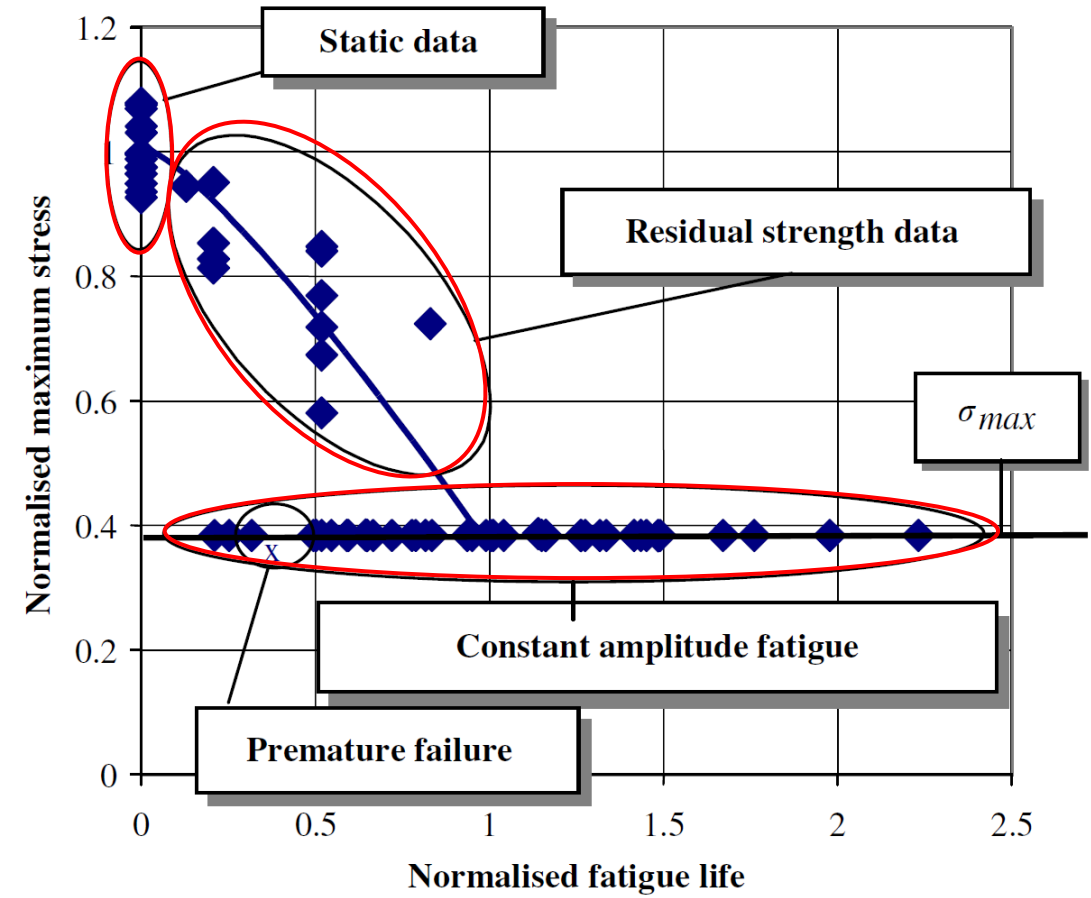
- Exp. data One of the most demanding theories

Strength degradation theories

- Strength degradation after fatigue can be described using residual strength tests. In these experiments, a specimen is subjected to fatigue loading and tested destructively in tension or compression at a series of predefined percentages of nominal fatigue life
- Experimental data and a line representing a commonly used one-parameter strength degradation model are shown in the next slide

Strength degradation data

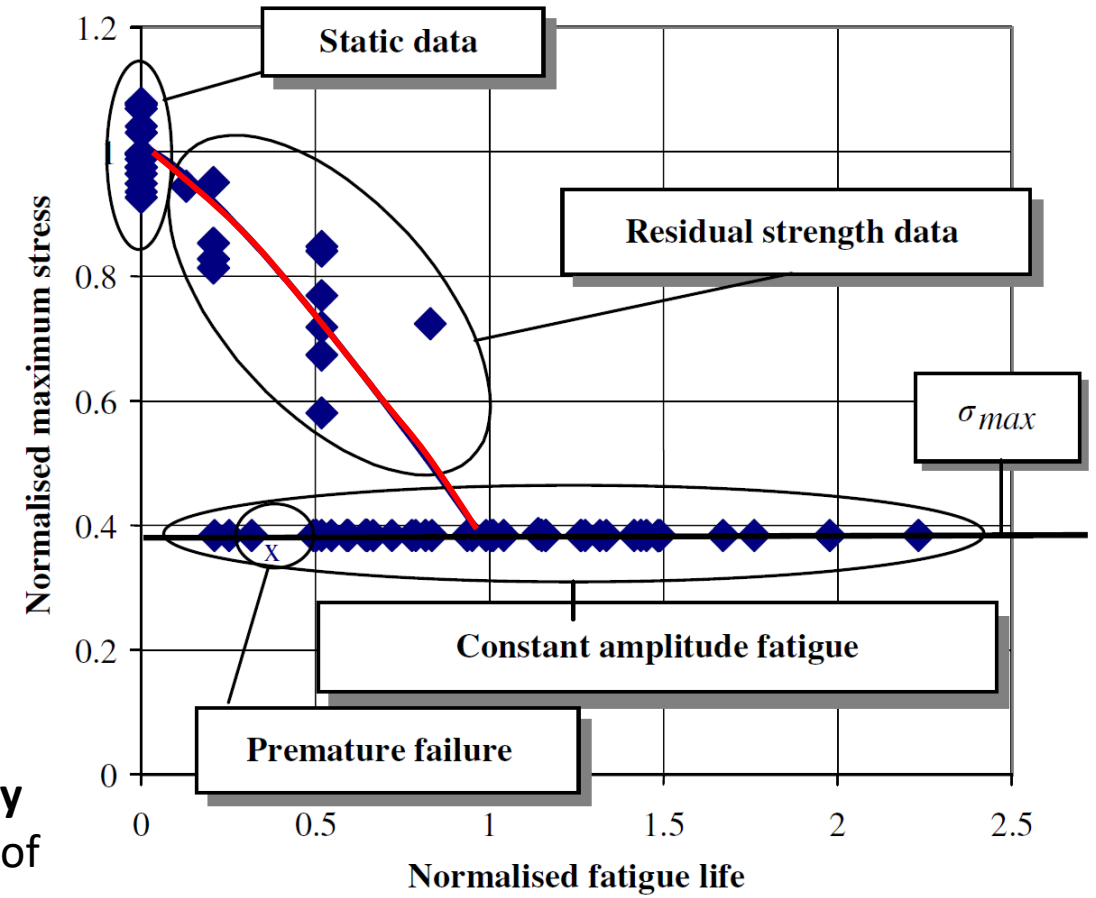
- Static strength data
- CA fatigue data
- Residual strength data. In this case, tensile strength tests were carried out at 20%, 50%, and 80% of average fatigue life; hence, the residual strength data are shown as vertical bands of data at these nominal life fractions



Strength degradation data

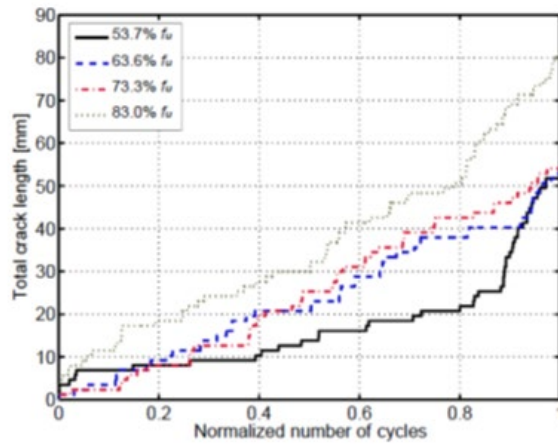
- A premature failure. This test was intended to be a residual strength test at a certain life fraction, but failed before the intended life fraction was attained
- A strength degradation curve, connecting the average static strength to the average fatigue life. Strength degradation for this dataset is almost linear.

In all strength degradation models, strength **degrades monotonously** from either tensile or compressive strength down to the maximum (of absolute values) applied cyclic load.

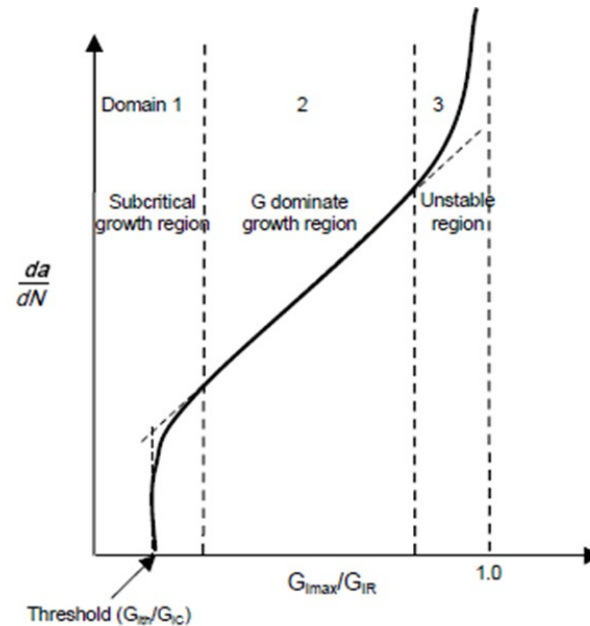


Fracture mechanics theories-PDM

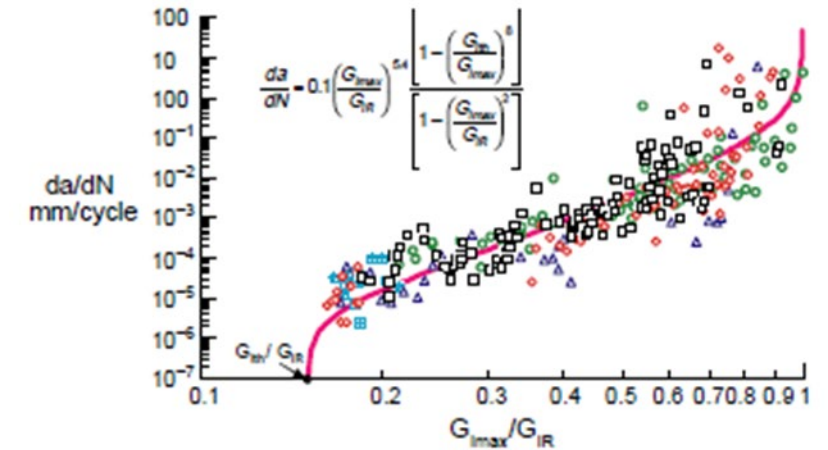
- Follow the development of crack (or cracks) and associate this to energy



Crack propagation vs stress level

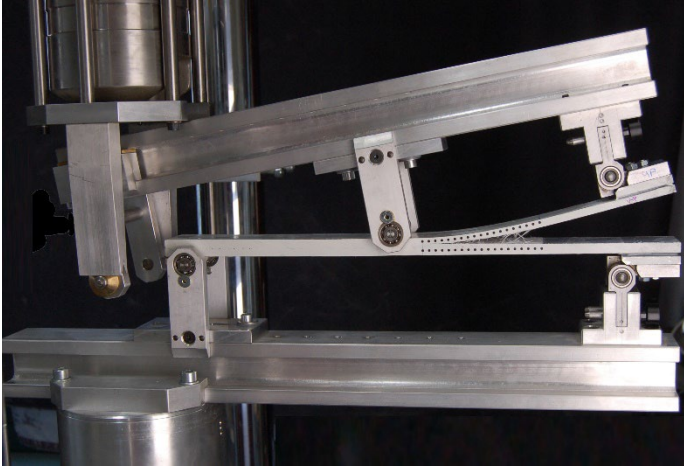


Fatigue crack growth curve concept

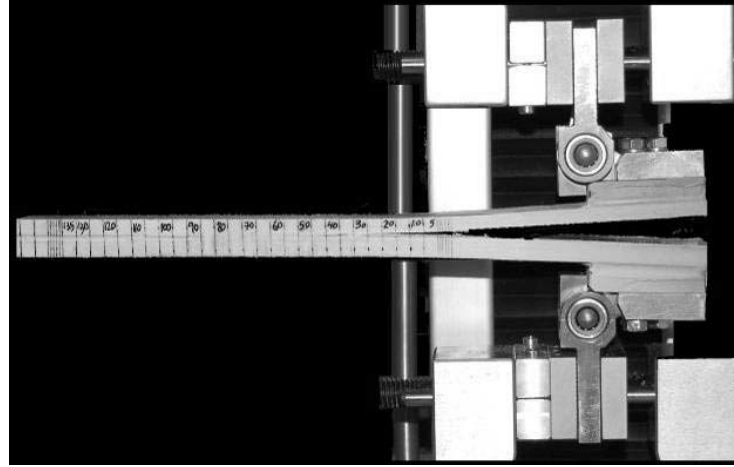


Fatigue crack growth curve

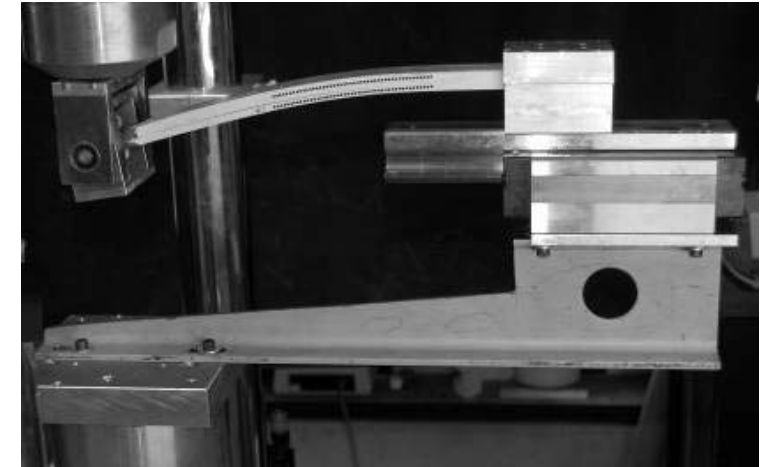
Mode I, Mode II, mixed-Mode I/II



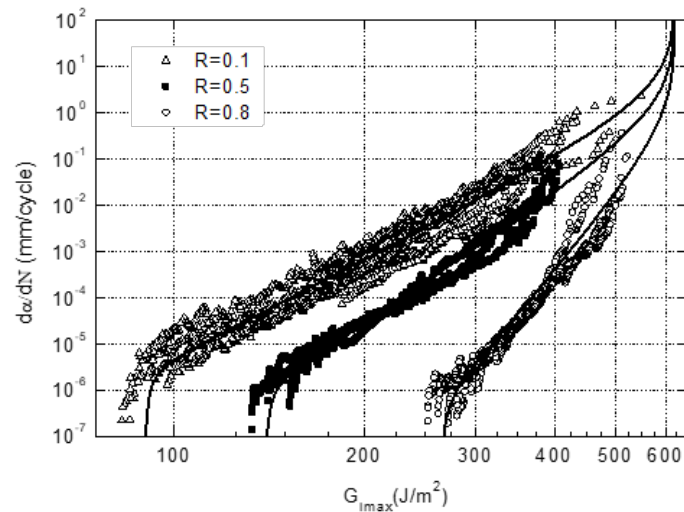
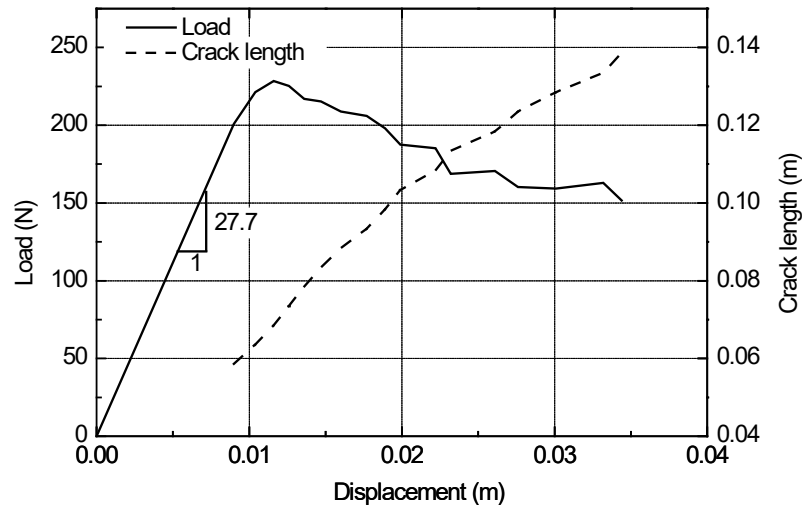
Mode I



Mode II



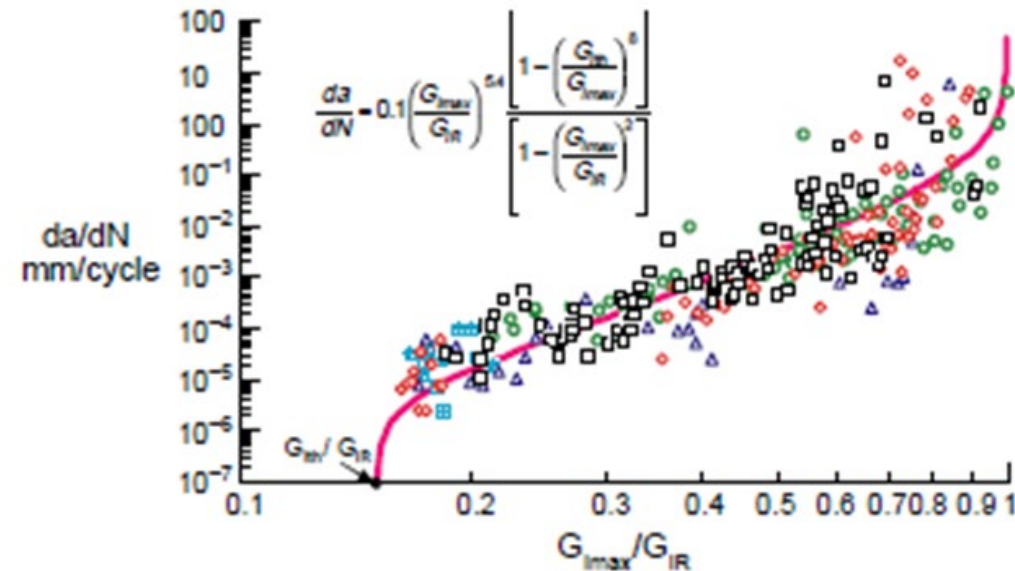
Mixed-Mode I/II



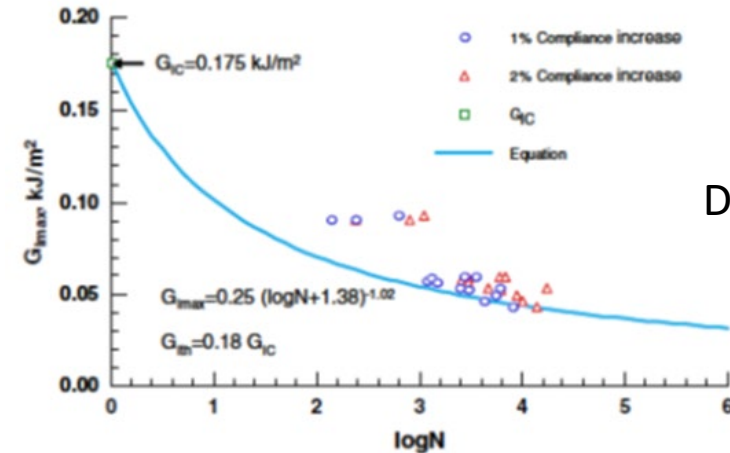
$$\frac{da}{dN} = D(G_{max})^m \frac{\left(1 - \left(\frac{G_{th}}{G_{max}}\right)^{Q_1}\right)}{\left(1 - \left(\frac{G_{max}}{G_C}\right)^{Q_2}\right)}$$

*M. Shahverdi, A. P. Vassilopoulos, T. Keller "A total fatigue life model for the prediction of the R-ratio effects on fatigue crack growth of adhesively-bonded pultruded GFRP DCB joints" Composites A, 2012;43(10):1783-1790

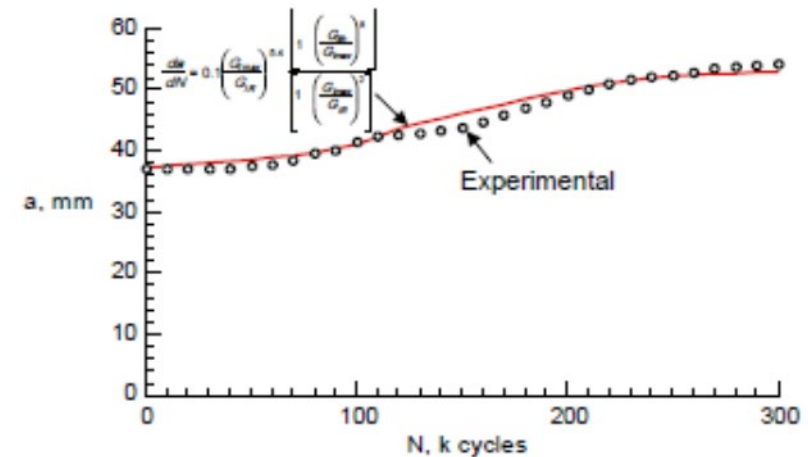
Fracture mechanics data interpretation



Total life fatigue model



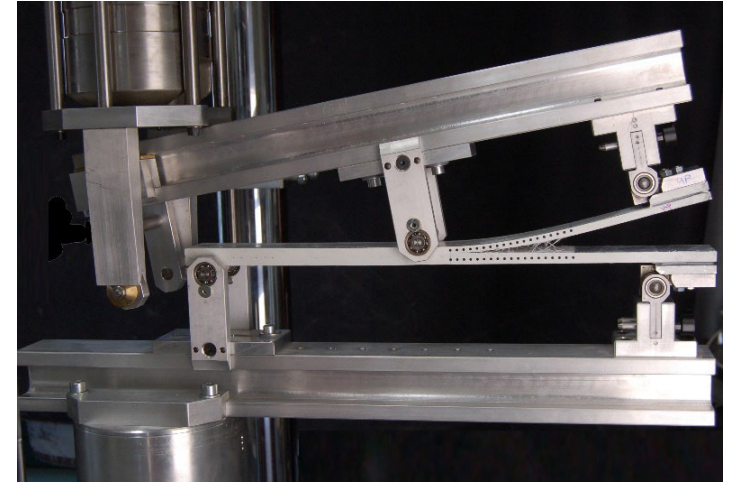
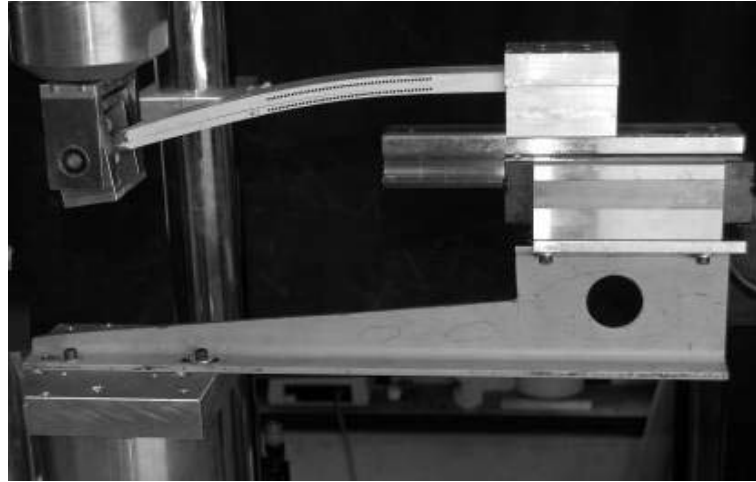
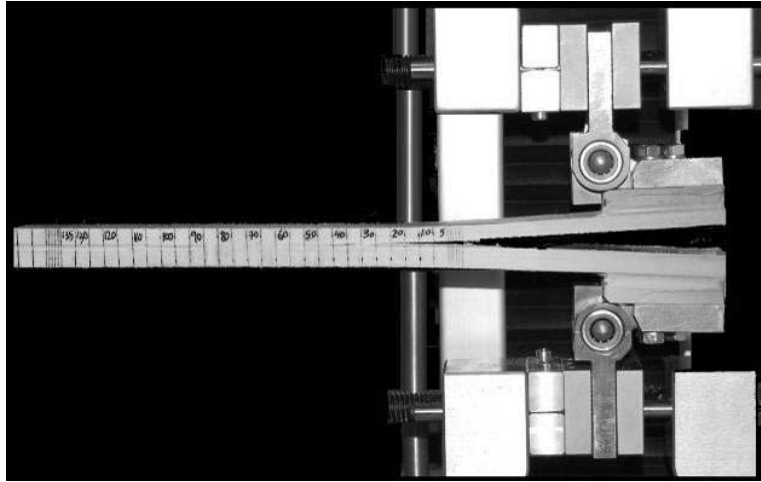
G_{max} vs N curve



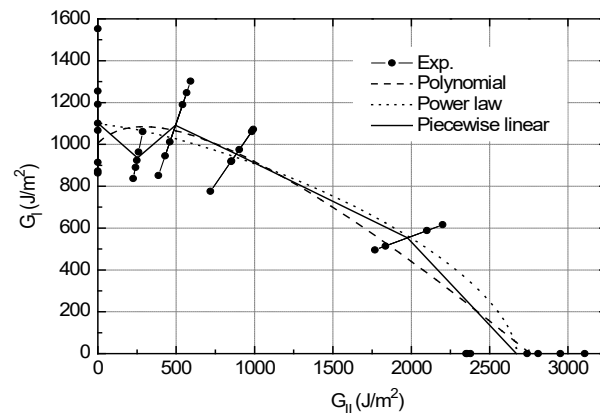
Crack vs N curve – simulation vs. experiments

Displacement control

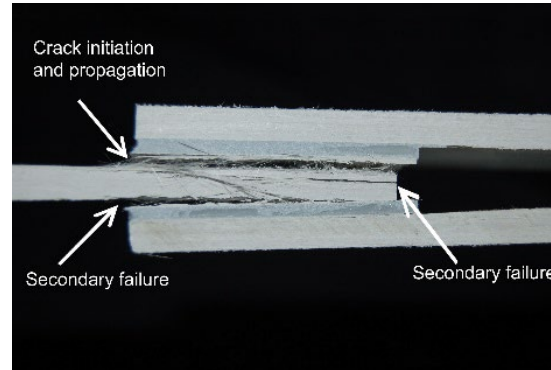
What we know about composites' fatigue



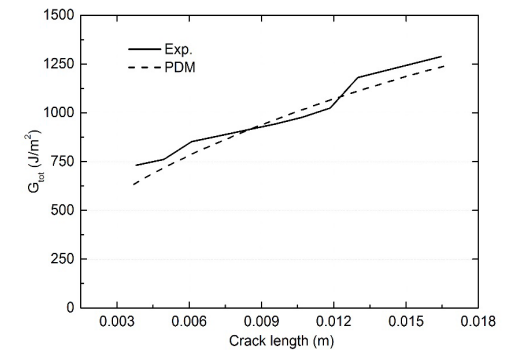
Mode I



Mode II



Mixed-Mode I/II



*A. Cameselle-Molares, R. Sarfaraz, M. Shahverdi, A. P. Vassilopoulos, T. Keller "Fracture mechanics-based progressive damage modelling of adhesively bonded fibre-reinforced polymer joints" FFEMS, 2017;40(12):2183-2193

Machine learning in fatigue analysis

- **Computational engineering** involves the design, development, and application of computational systems for solving physical problems
- Novel computational methods such as **neural networks** and **GAs** have initially been used as tools for optimization of design methods, and damage characterization of composite materials
- Initially intended for optimization, such methods were later used for fatigue life modeling



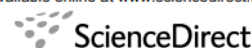
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International Journal of Fatigue 29 (2005) 1634–1645



Available online at www.sciencedirect.com



International Journal of Fatigue 30 (2008) 1634–1645



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Artificial neural networks in spectral analysis of composite materials

Anastasios P. Vassilopoulos^{a,*}, Efstratios F. Georgopoulos^a

Comparison of genetic programming with conventional methods for fatigue life modeling of FRP composite materials

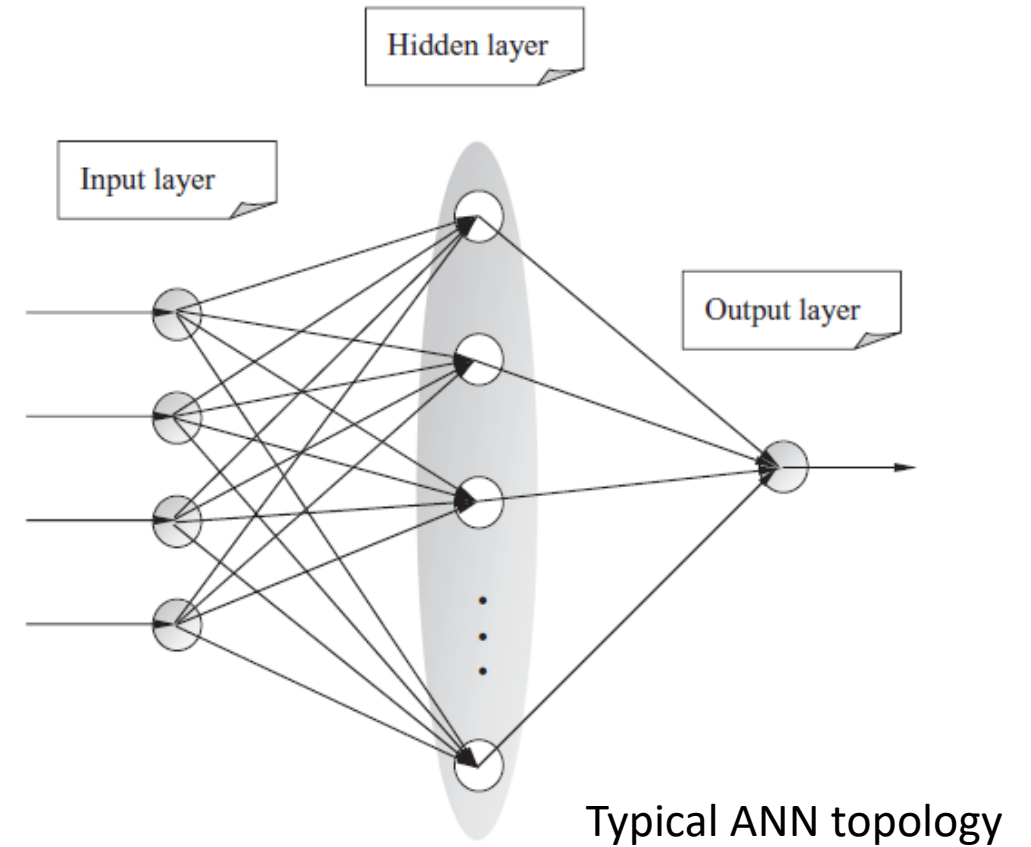
Anastasios P. Vassilopoulos^{a,*}, Efstratios F. Georgopoulos^b, Thorvaldur Elfridsson^c

Adaptive neuro-fuzzy inference system in modelling fatigue life of multidirectional composite laminates

Anastasios P. Vassilopoulos^{a,*}, Raman Bedi^b

Theoretical background-implementation

- Artificial neural networks
- Genetic programming
- Adaptive neuro-fuzzy inference systems
- ...



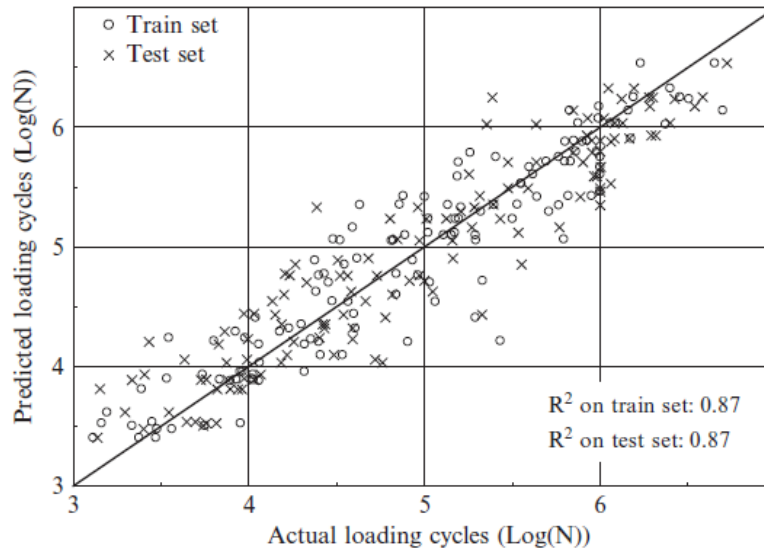
Modeling examples

- Various ML techniques were **successfully implemented** for representation of the fatigue data of composite materials and structures under constant amplitude and spectrum loading
- ML methods need (big) data. Databases with extensive material datasets for the characterization of entire categories of materials primarily used in specific applications, such as databases **DOE/MSU** and **Optidat** for materials used in the wind turbine rotor blade industry. Another fatigue database has been released by **Virginia Tech** containing experimental data from axial loading on pseudo-quasi-isotropic glass/vinyl ester specimens fabricated using the vacuum-assisted resin transfer molding (VARTM) technique. Additional databases released by **UPatras** for GFRP multidirectional composite laminates (Mat#1 of this presentation)

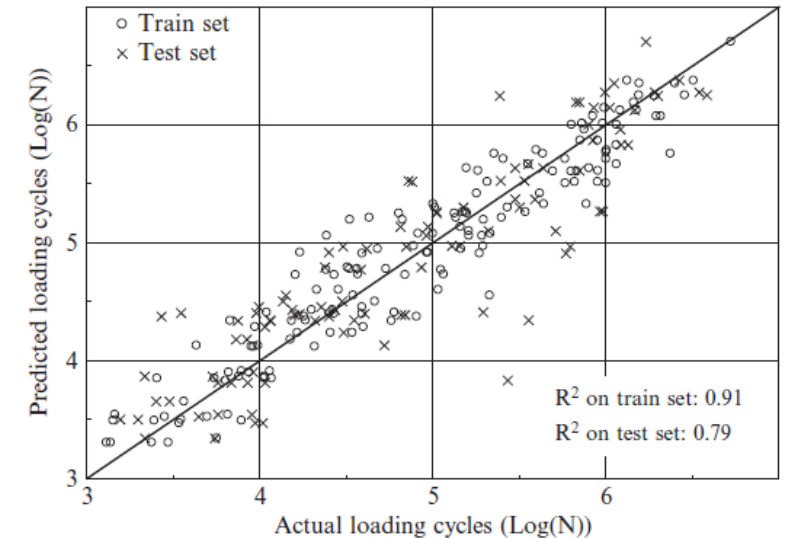
Data for training

- Previous results have shown that 50%–60% of available experimental data suffices for the derivation of reliable S-N curves and CLDs.
- An arbitrarily selected proportion of 50% of the data points proved sufficient for the derivation of reliable S-N curves by implementing an ANN model (left) or an ANFIS model (right).

Modeling ability
of ANN for 50%
training set, 50%
test set



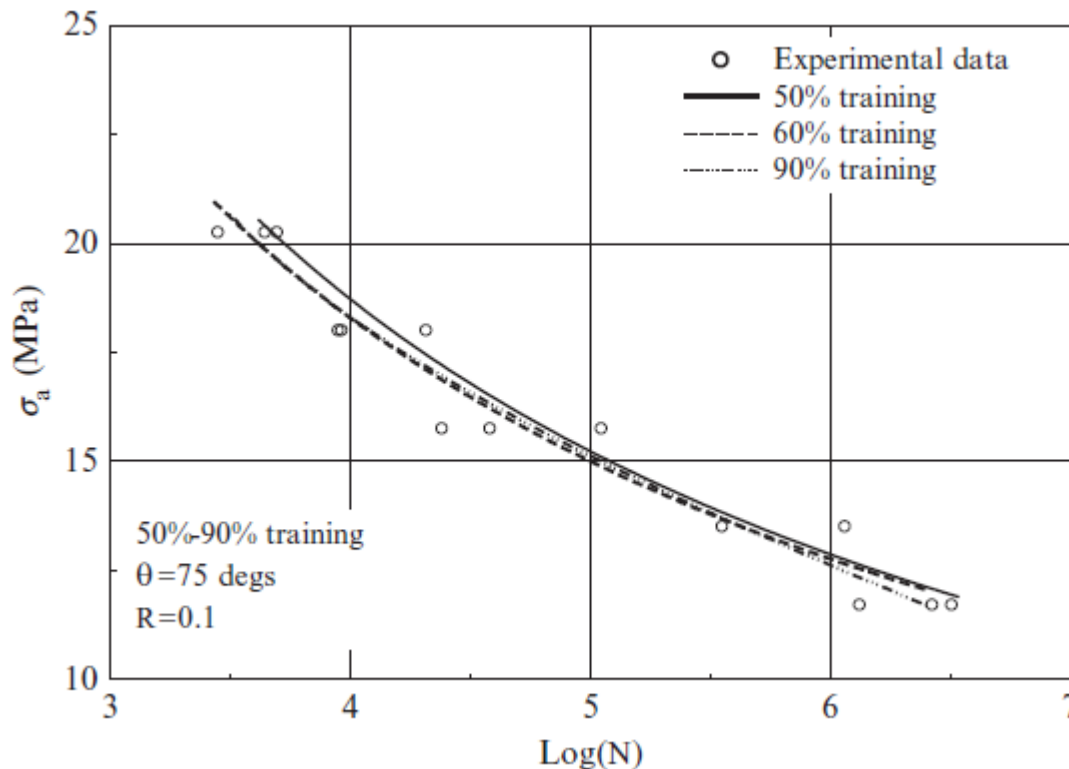
Modeling ability
of ANFIS for 60%
training set, 40%
test set



Data for training

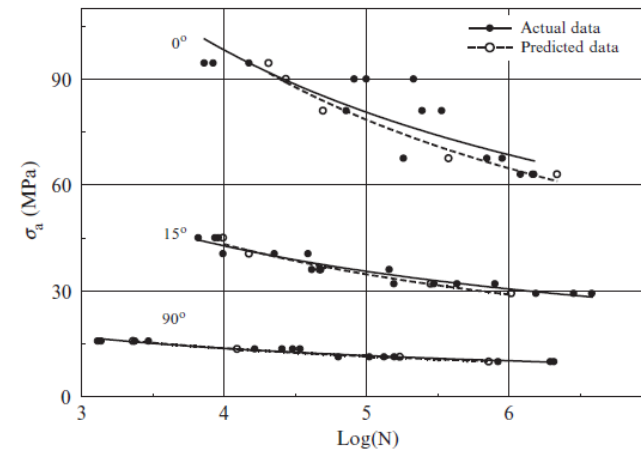
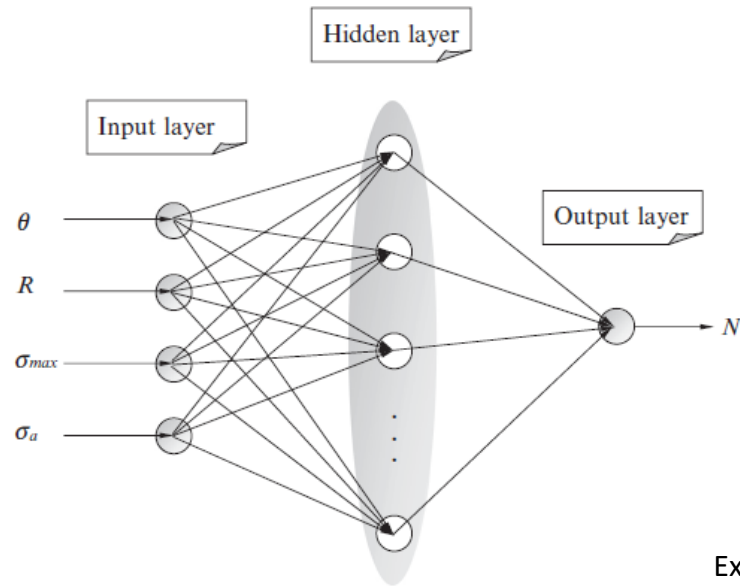
- An increase in the data used in the training set does not seem to significantly influence the accuracy of the ANN or ANFIS models

Modeling accuracy of ANFIS
according to data points
used to train the network

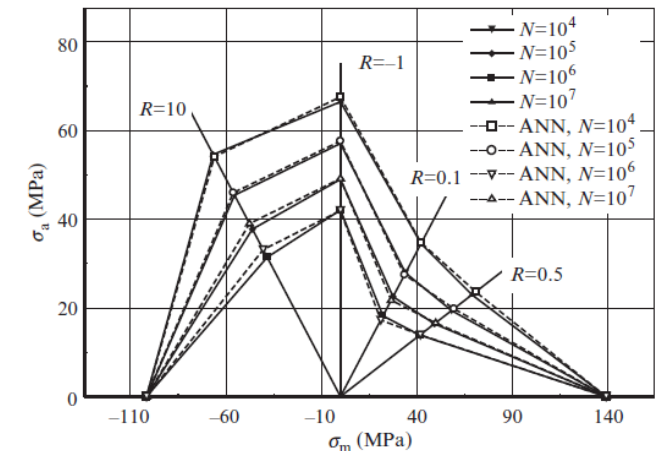


Application of methods - ANN

- After a certain pre-treatment of the data (normalization, shorting etc) an ANN was built using four input parameters for an output (N) with very accurate results for S-N and CLD modeling!



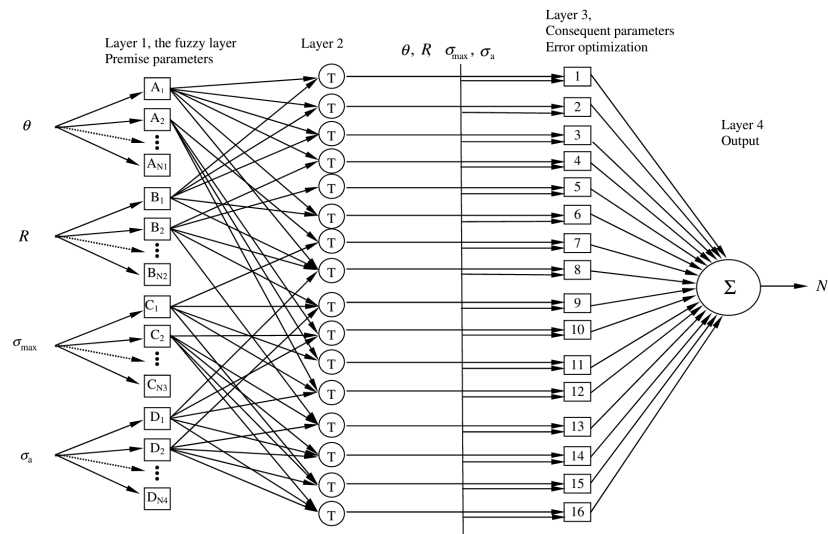
Experimental data vs. ANN modeling. Tension-tension fatigue, $R=0.1$. Coupons cut at 0, 15, 45, 75, and 90 degree; 30% used for training set, 70% for test set.



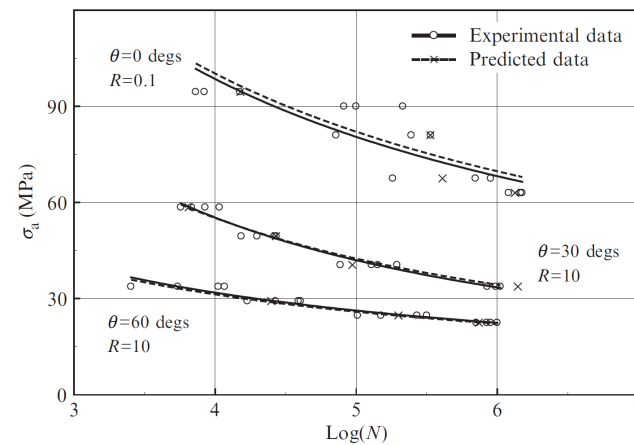
CLD for 45 degree off-axis coupons. ANN modeling vs. experimental data in range 10^4 – 10^7 loading cycles.

Application of methods - ANFIS

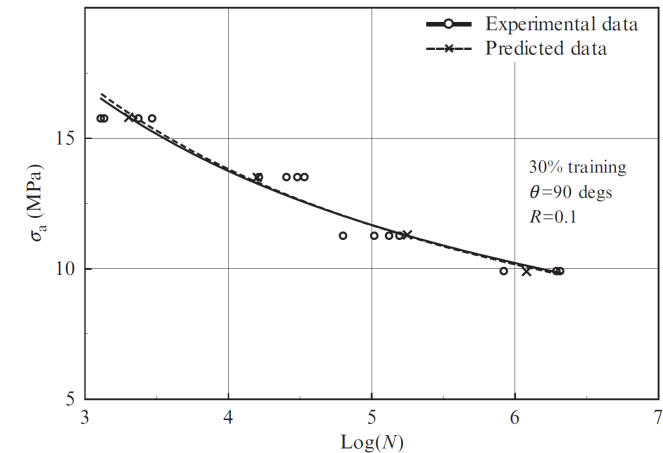
- The Model creates fuzzified linguistic rules (e.g., “If fiber orientation angle is near to zero and stress ratio is low and stress amplitude is low and maximum stress is low, then specimen life is long”)
- The output must be “de-fuzzified” by selected defuzzification methods.



Developed ANFIS topology based on the first order Takagi and Sugeno model



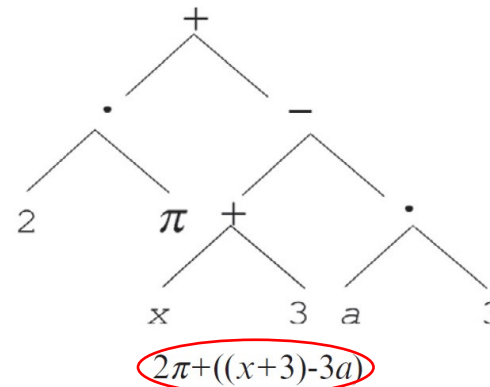
ANFIS modeling accuracy for 50% of training data



ANFIS modeling accuracy for only 30% of training data

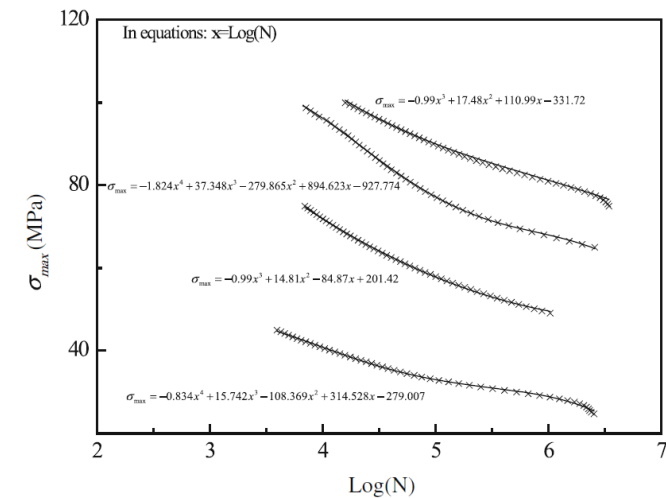
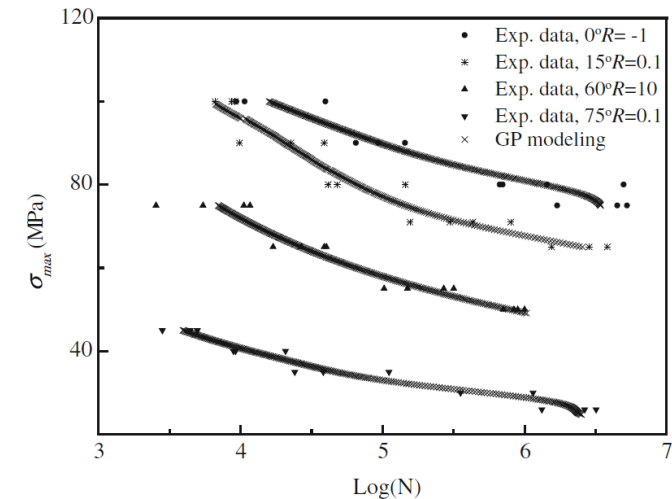
Application of methods - GP

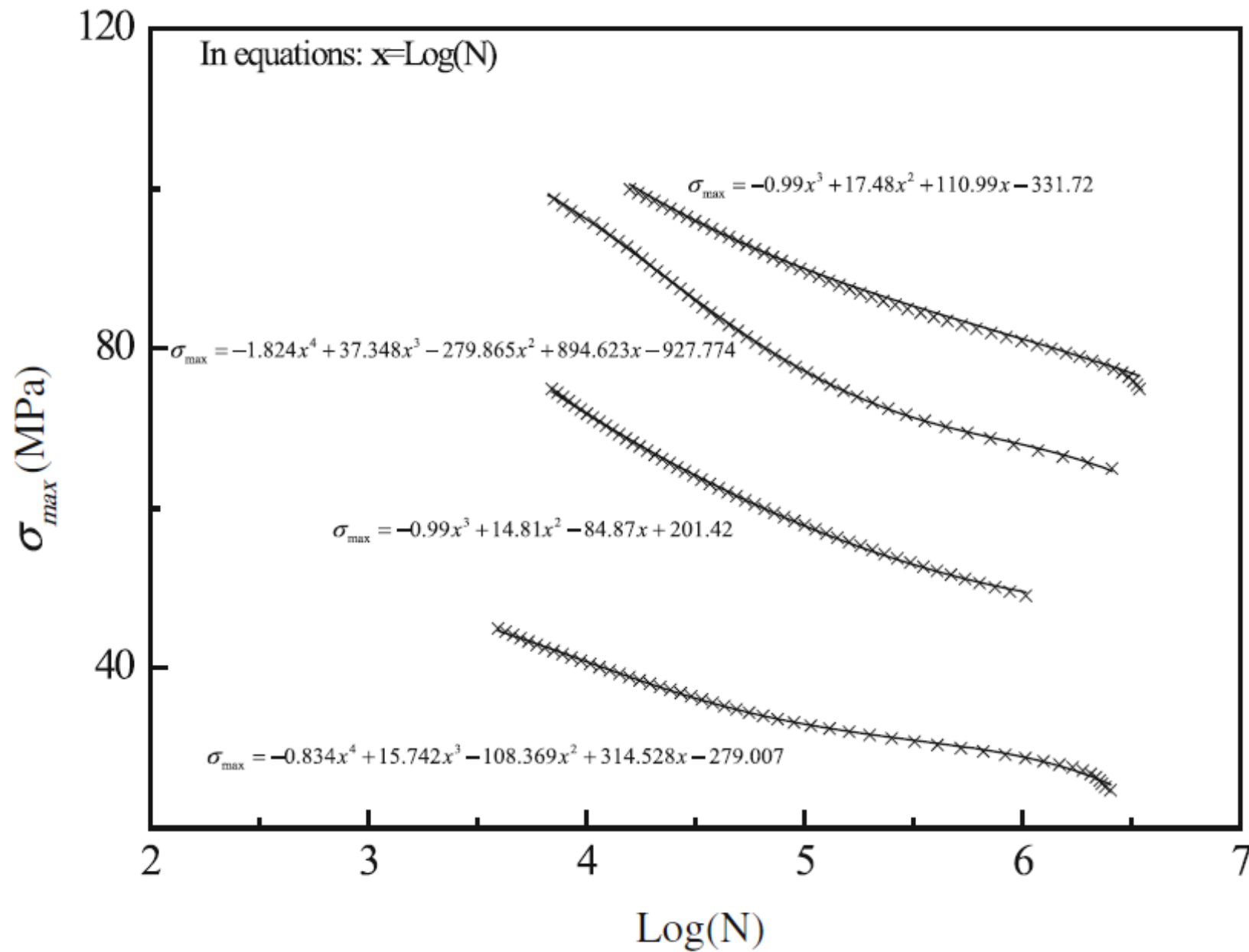
- A domain-independent problem-solving technique in which computer programs are evolved to solve, or approximately solve, problems.
- GP is a member of the broad family of techniques called **evolutionary algorithms** that are based on the **Darwinian principle of reproduction and survival** of the fittest and are similar to biological genetic operations.
- The evolution operates on a population of computer programs of varying shapes and sizes represented as trees.



Application of methods - GP

- Like other methods, GP needs to “learn” using a data subset “training data” in order to evolve programs.
- A termination criterion is set to define when an evolved program is terminated (when a desired accuracy is achieved)
- Another data subset is used to validate the evolved programs.
- A test data set follows for the model output.





Failure criteria/multiaxial fatigue

- **Macroscopic** fatigue failure theories for multiaxial stress states

Neglecting some of the stress components of plane-stress states is misleading and overestimates substantially the fatigue life of composite structural components



International Journal of Fatigue 24 (2002) 825–830

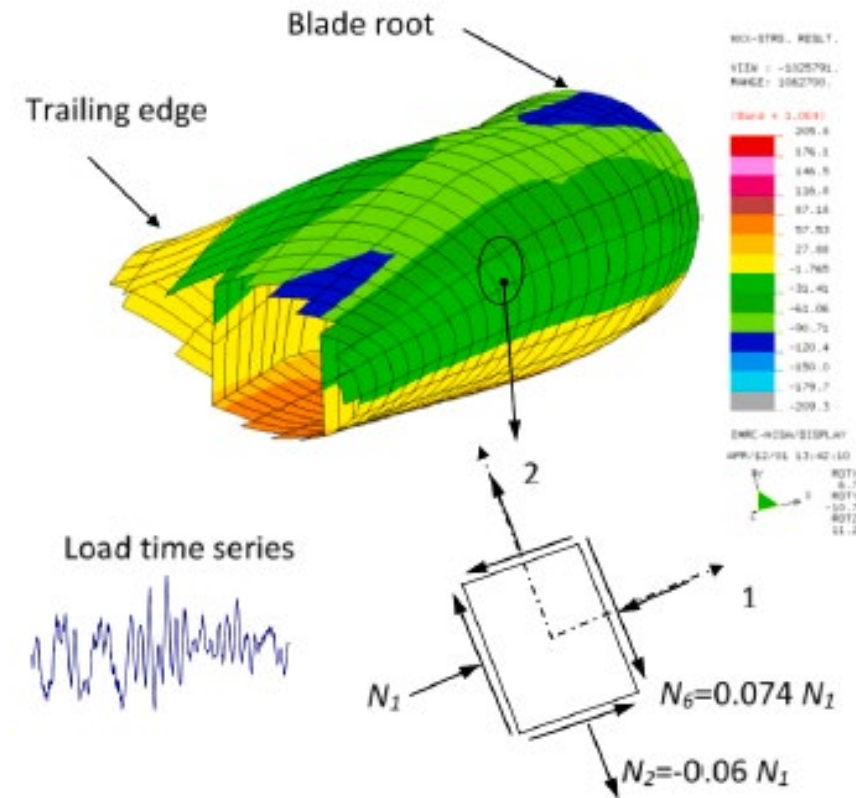


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Complex stress state effect on fatigue life of GRP laminates.
Part II, Theoretical formulation

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Macroscopic fatigue failure theories for multiaxial stress states

- Theories that do not consider the **interaction** of the various stress tensor parameters, like the maximum stress theory (Rankine) or the maximum normal strain theory (Saint Venant) are **reliable only** for specific cases of isotropic materials
- Maximum stress theory is the most appropriate for isotropic materials that fail due to a brittle fracture
- Maximum shear stress theory or the **maximum distortional energy** theory seems the most appropriate for the prediction of ductile material behavior

Strength of (isotropic) materials

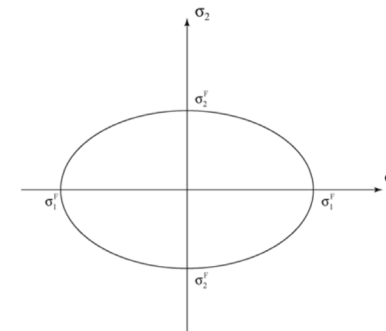
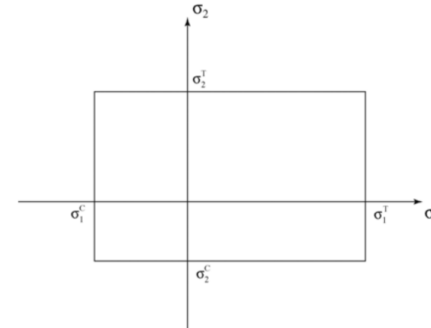
- Estimate the stress at the time and location when failure occurs
- For conventional materials:
 - Maximum tensile, compressive or shear stress and observation about the failure is enough
 - Usually, one strength constant is enough...
 - Young's modulus is enough since Poisson's ratio equals to 0.3 and the uniaxial tensile strength suffice since shear strength is taken to be about 50-60% of the tensile.

Strength of (composite) materials

- For composite materials:
 - The “**one-constant**” approach for strength or for stiffness is (usually) no longer adequate
 - (For the simplest case of the unidirectional laminate) four elastic constants are needed for the stiffness and six constants for the strength.
 - Unidirectional composites have highly directionally dependent strengths.
 - Therefore, for any state of applied stress, all stress components must be examined before judgment on the cause of failure can be made.

Macroscopic fatigue failure theories for (composites)?

- a) Maximum stress and strain criteria
 - Not interactive failure criteria associated with failure modes
- b) Quadratic interaction criteria
 - Interactive failure criteria that can be associated or not with failure modes



Maximum stress and strain criteria

Related to the maximum normal stress theory by Rankine and the maximum shear stress theory by Tresca, this theory is like those applied to isotropic materials.

The stresses acting on a **lamina** are resolved into the normal and shear stresses in the **local axes**. Failure is predicted in a lamina, if any of the stress components is equal to or exceeds the corresponding ultimate strengths of the material

$$-X' < \sigma_1 < X$$

$$-Y' < \sigma_2 < Y$$

$$-S < \sigma_6 < S$$

OR for the maximum strain theory:

$$-\left(\varepsilon_1^c\right)_{ult} < \varepsilon_1 < \varepsilon_{1ult}^t$$

$$-\left(\varepsilon_2^c\right)_{ult} < \varepsilon_2 < \varepsilon_{2ult}^t$$

$$-\left(\varepsilon_6\right)_{ult} < \varepsilon_6 < \varepsilon_{6ult}$$

For both cases: Each component of stress (strain) **has its own criterion**, and it is not affected by the other components, i.e., **there is no interaction**

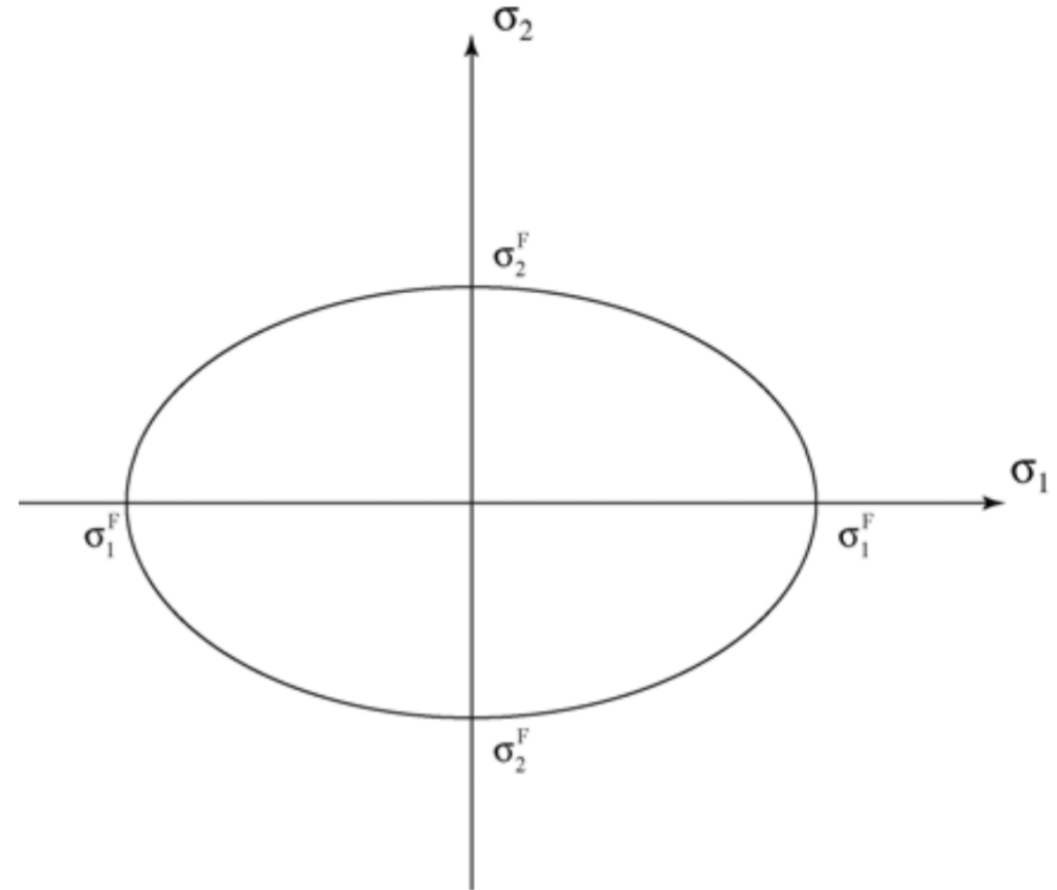
Quadratic interaction criteria

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$$

- And for the strain components:

$$G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$$

- Where the F 's and G 's are strength parameters and to the constants in equations of maximum stress/stress criteria. Failure occurs when equation is met.

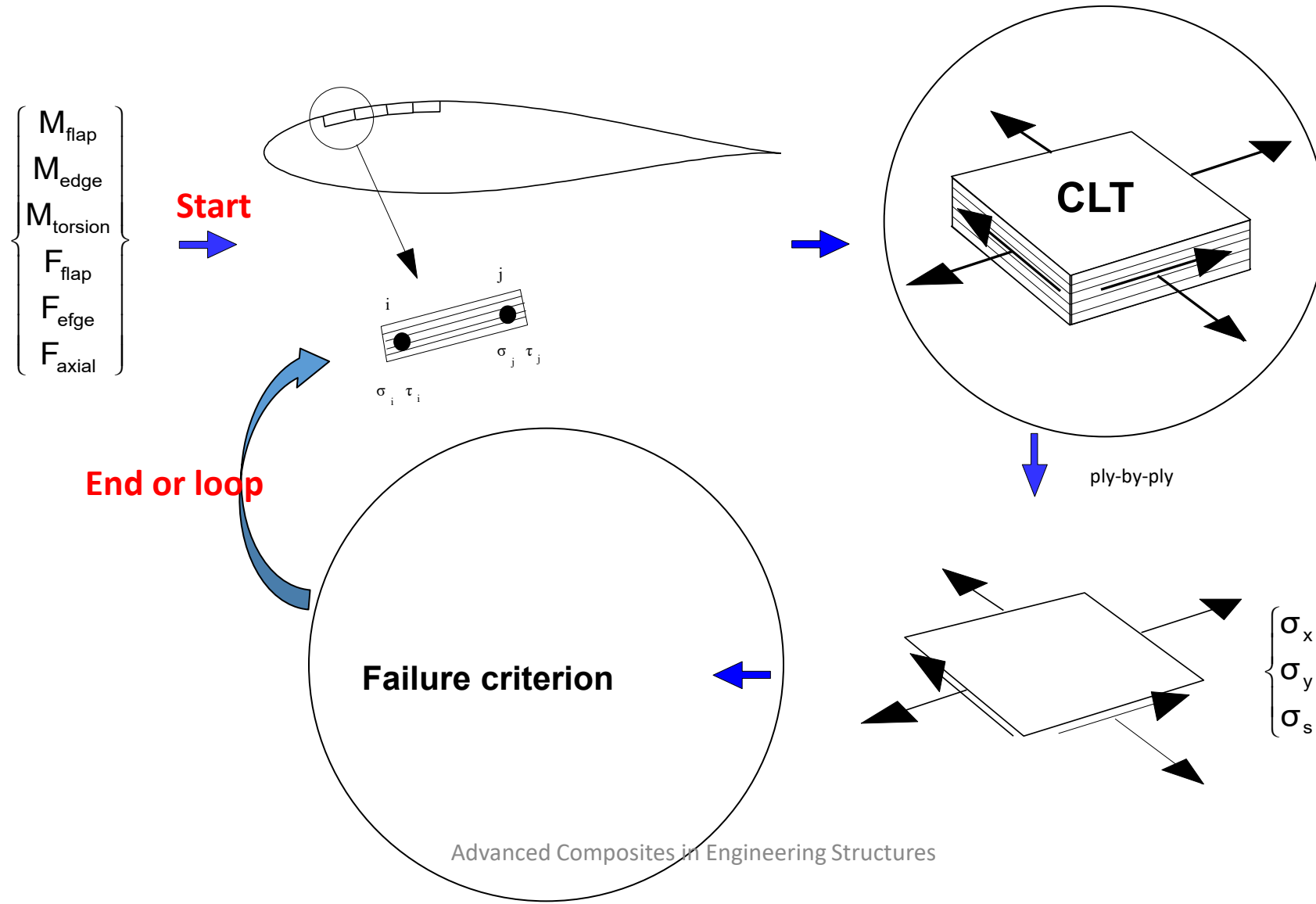


Use of failure criteria

- Failure criteria constitute the **analytical mathematical expressions** for the calculation of the failure under complex loading conditions as a function of the “basic” material properties, that can be determined through “simple” tests.
- Failure criteria serve important functions in the design and sizing of composite laminates. **However, the criteria are not intended to explain the mechanics of failure (although...)**. The framework should remain the same for different definitions of failure, such as the ultimate strength, the yield point the endurance limit or even a working stress based on reliability considerations.
- The quadratic criterion will be analyzed here. It is simple and versatile. It considers interaction between the stress or strain components analogous to the von Mises criterion for isotropic materials.

Implementation of failure criteria in a design cycle

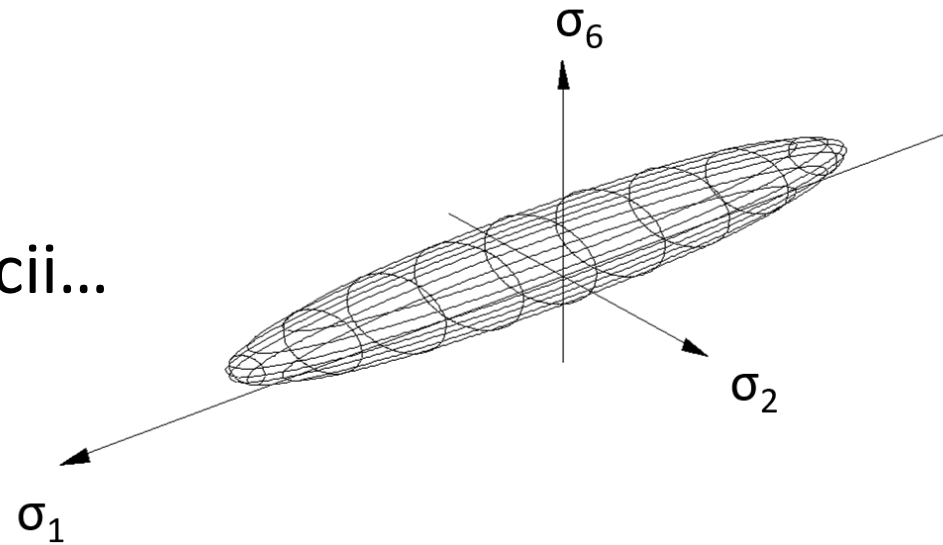
Or - How and where do we use a failure criterion



Fatigue failure criteria

- Predictive formulations (failure criteria) implemented for composites since the 1970s assuming uniaxial and multiaxial loading conditions

- “embedded” Locii...



Hashin-Rotem

- Assume two main modes in the case of unidirectional materials: the fiber failure mode and the matrix failure mode
- Failure prediction depends on failure mode and the critical angle of transition between failure modes is defined by:

$$\sigma_A = \sigma_A^u$$

$$\left(\frac{\sigma_T}{\sigma_T^u}\right)^2 + \left(\frac{\tau}{\tau^u}\right)^2 = 1$$

$$\tan \theta_c = \frac{\tau^s}{\sigma_A^s} \frac{f_\tau(R, N, fr)}{f_A(R, N, fr)}$$

- S-N curves for off-axis directions can then be calculated by:

$$f''(R, N, fr) = f_\tau \sqrt{\frac{1 + \left(\frac{\tau^s}{\sigma_T^s}\right)^2 \tan^2 \theta}{1 + \left(\frac{\tau^s}{\sigma_T^s} \frac{f_\tau}{f_T}\right)^2 \tan^2 \theta}}$$

Fawaz-Ellyin

- A very simple assumption that all the on- and off-axis S–N curves of the laminate, when normalized by the corresponding static strengths, lie in a narrow band on the S–N plane
- According to this criterion, if a reference S–N curve exists, e.g.:

$$S_r = m_r \log(N) + b_r,$$

- The S–N curve under any off-axis angle or biaxial ratio can be calculated by:

$$S(a_1, a_2, \theta, R, N) = f(a_1, a_2, \theta)(b_r + g(R)m_r \log(N))$$

$$f(a_1, a_2, \theta) = \frac{\sigma_x(\alpha_1, \alpha_2, \theta)}{X_r}$$

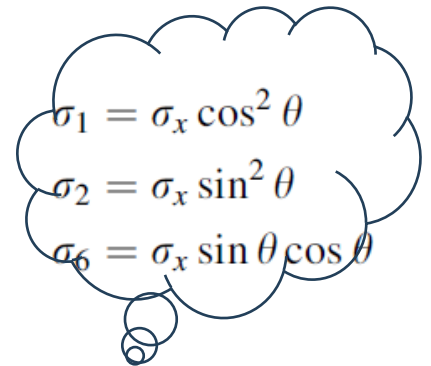
$$g(R) = \frac{\sigma_{\max}(1 - R)}{\sigma_{\max_r} - \sigma_{\min_r}}$$

Failure tensor polynomial in fatigue

- Modification of Tsai-Hahn failure criterion to consider fatigue cycles.
- **Simple idea:** Substitute the failure tensor components by the corresponding S-N curves!
- FTPF can be expressed in the material symmetry axes (1 and 2), under plane stress by:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 + F_{66}\sigma_6^2 - 1 = 0$$

$$F_{11} = \frac{1}{XX'}, F_{22} = \frac{1}{YY'}, F_{66} = \frac{1}{S^2}, F_1 = \frac{1}{X} - \frac{1}{X'}, F_2 = \frac{1}{Y} - \frac{1}{Y'}$$



$$\begin{aligned}\sigma_1 &= \sigma_x \cos^2 \theta \\ \sigma_2 &= \sigma_x \sin^2 \theta \\ \sigma_6 &= \sigma_x \sin \theta \cos \theta\end{aligned}$$

- And therefore, any off axis “S-N” curve can be estimated by:

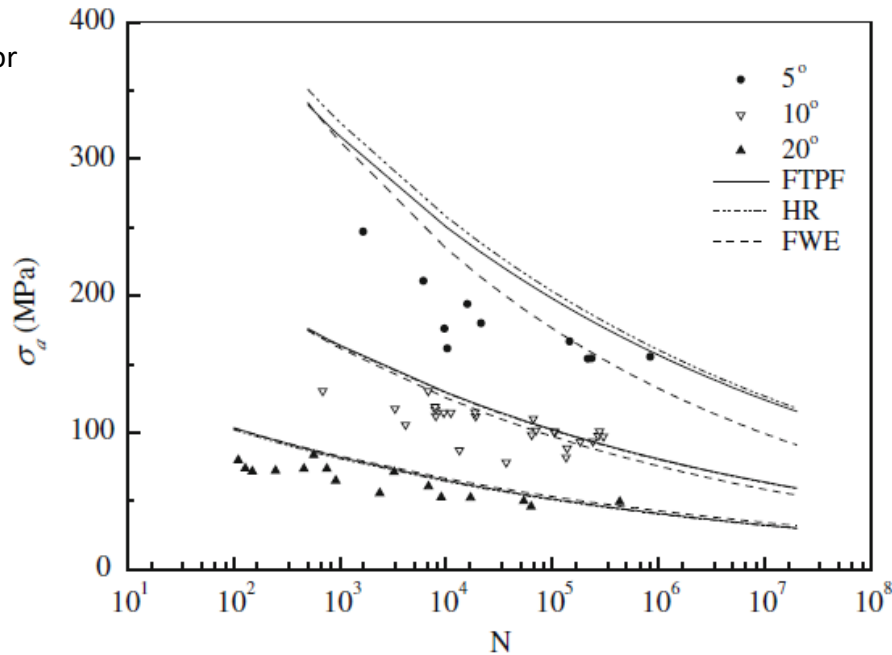
$$\frac{\sigma_1^2}{XX'} + \frac{\sigma_2^2}{YY'} - \frac{\sigma_1\sigma_2}{XY} + \frac{\sigma_6^2}{S^2} - 1 = 0$$

$$\sigma_x = \sqrt{\frac{1}{\left(\frac{\cos^4 \theta}{XX'} + \frac{\sin^4 \theta}{YY'} - \frac{\cos^2 \theta \sin^2 \theta}{XY} + \frac{\cos^2 \theta \sin^2 \theta}{S^2}\right)}}$$

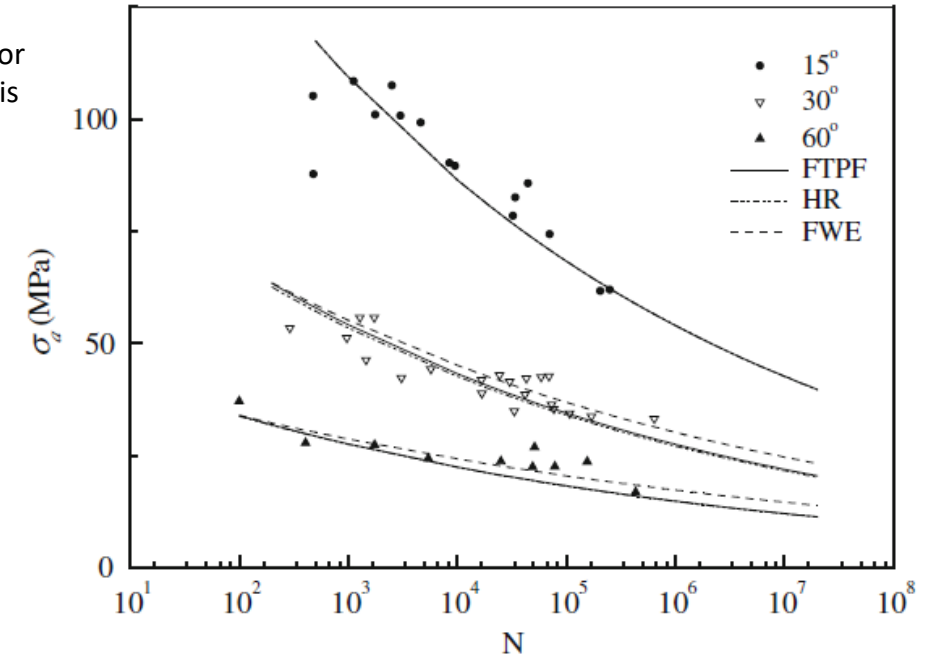
Application of fatigue failure criteria - uniaxial

- Data from Hashin-Rotem - Glass/Epoxy, UD, Flat Specimens

Predicted S-N curves for 5°, 10° and 20° off-axis specimens



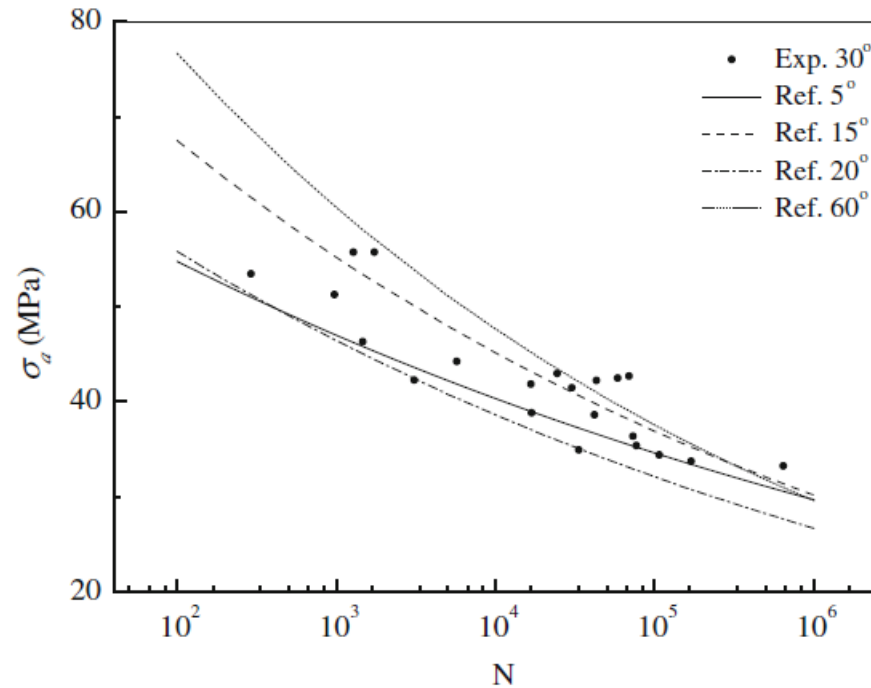
Predicted S-N curves for 15°, 30° and 60° off-axis specimens



Sensitivity analysis

- FWE sensitivity to reference angle selection
 - (only 15° ref seems to work well)

Predicted S-N curves for
30° off-axis specimens using
various reference curves

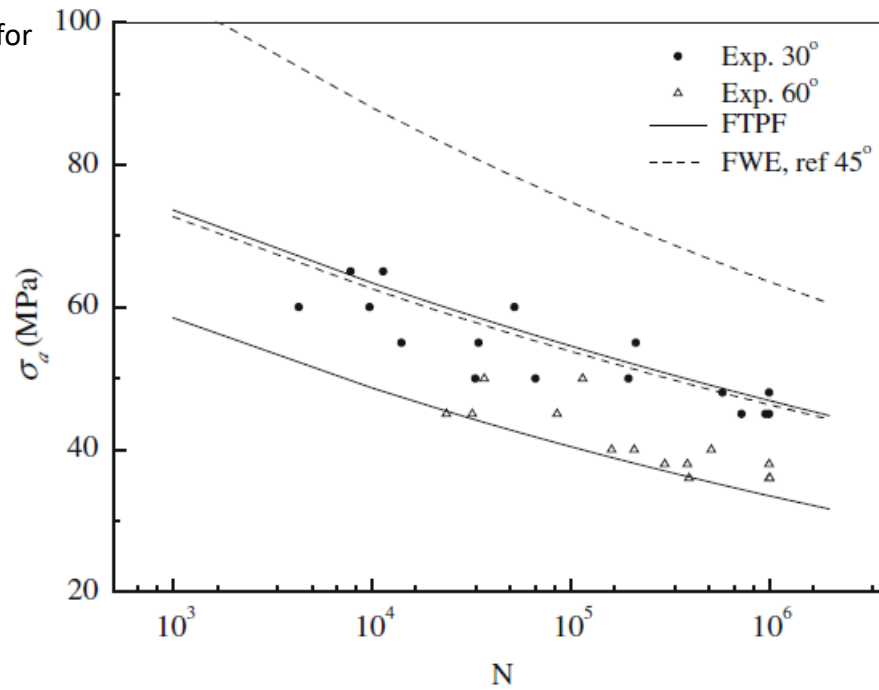


Application of fatigue failure criteria - uniaxial

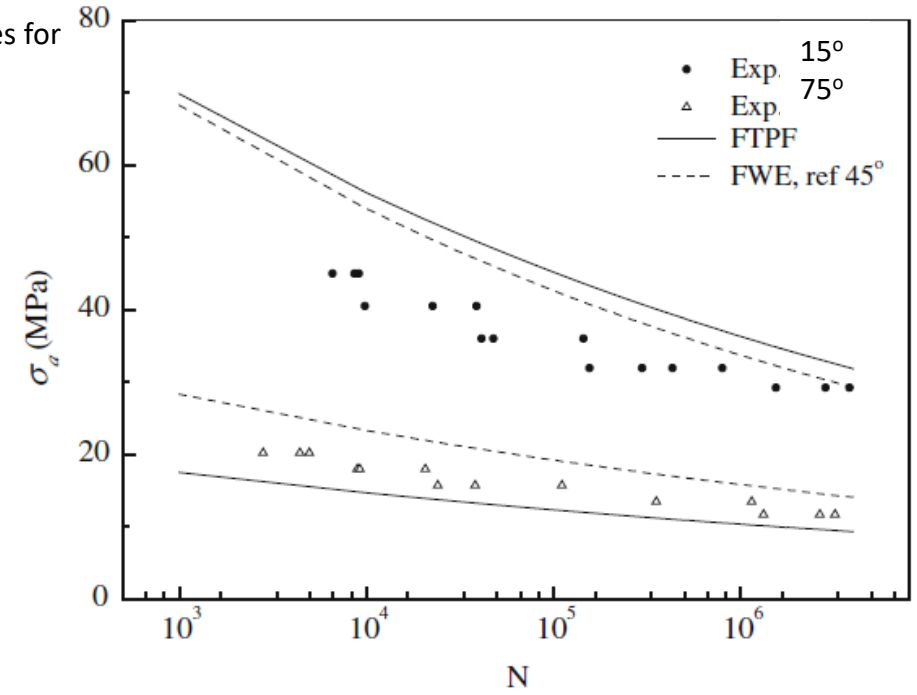
- Data from Mat #1 Glass/Polyester, Multidirectional laminate, Flat Specimens

15°

Predicted S–N curves for 30° and 60° off-axis specimens, $R=-1$



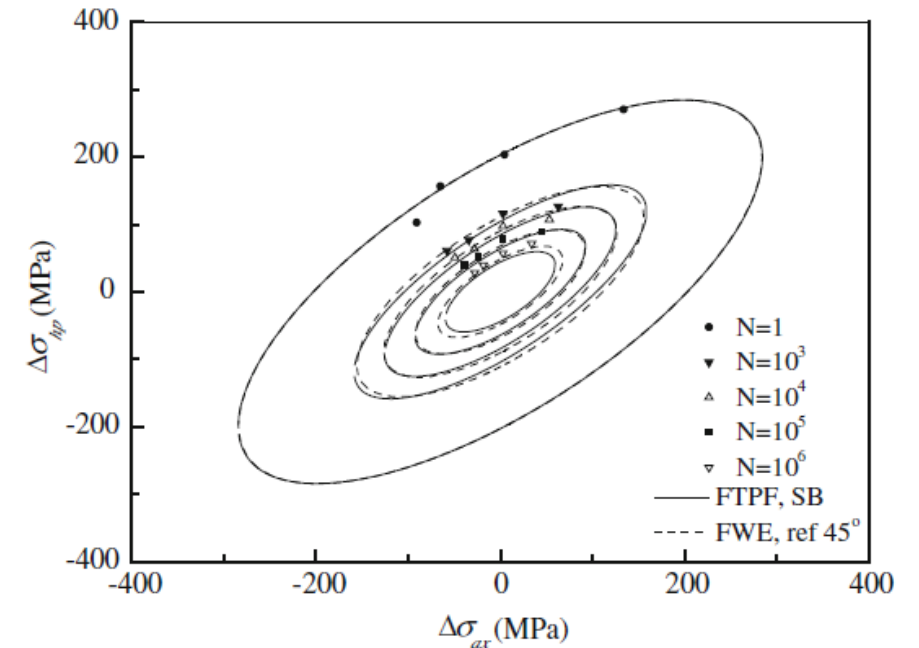
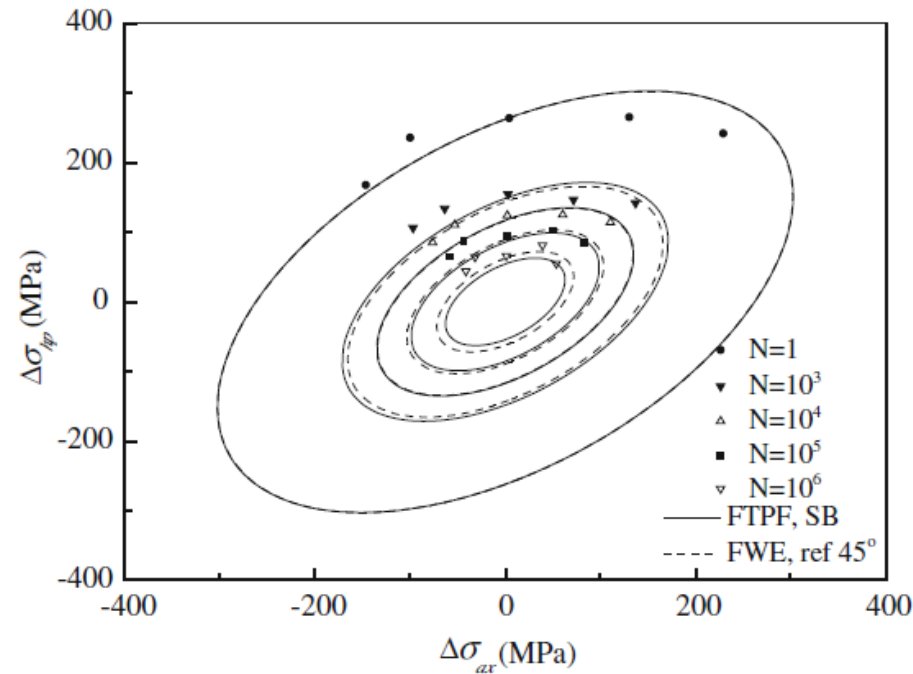
Predicted S–N curves for 15°, 75° off-axis specimens, $R=0.1$



Application of fatigue failure criteria - Biaxial

- Data from Owen-Griffiths* glass/polyester thin-walled tubes under combined axial loading and internal pressure under the stress ratio of $R = 0$

Comparison of theoretical (predicted by FTPF, and FWE) failure loci vs. experimental data of cylindrical specimens loaded at 0° and 45° . FWE, ref. 45°



M.J. Owen, J.R. Griffiths, Evaluation of biaxial failure surfaces for a glass fabric reinforced polyester resin under static and fatigue loading. J. Mater. Sci. 13(7), 1521–1537 (1978)

Conclusions #2

- Modeling $\neq$ prediction
- Methods (for both) exist
 - Different types – no unique theory
- Prediction capacity limited due to the lack of exp. data for validation
- “Hybrid” fatigue theories should (could) be appropriate

Where are we standing today?

- Theoretical approaches exist and data are gathered in various labs
No commonly accepted theory exists
- Challenges in sharing composites' fatigue data and methods
Homogenized protocols and standards for data sharing are missing
- We miss open access data bases
- We miss open access theory/model “databases”