Week #10 - Elastoplastic Punch

We will model in Optum G2 the elastoplastic punch of a shallow foundation for various constitutive models and time scales (drained vs undrained loading).

1 Mesh and loading conditions

We will let you create the mesh for the problem given in figure 1 - Use the symmetry of the problem, and ensure to take a *large enough* domain as to limit the influence of the finite boundaries to a minimum but not too large. In order to compare side by side different constitutive model, create a single mesh with 600 elements, and with the following features: i) a mesh fan at the end of the footing, 2) enforce a mesh size of 0.1 along the footing. In what follows, always use 6-nodes element in the analysis. We will assume that the water table coincides with the ground surface.

The constitutive relation for the soil will change troughout the exercice. Note that for this, you need to use different type materials for different stage (recommended). The footing itself is assumed rigid. Set a vertical distributed load multiplier on the footing.

To simulate the punch, we will use an *elasto-plastic multiplier* analysis with the load stepping scheme as AUTO.

Soil properties Use the default elastic and weight properties for the Mohr-Coulomb (MC), Tresca and Modified Cam-Clay (MCC). Use the following strength properties for the MC and MCC:

$$c = 5\text{kPa} \qquad \phi = 25°$$

Also use the following for the initial earth coefficient and over-consolidation ratio:

$$K_o = 0.5 \qquad OCR = 1$$

2 Long-term analysis: Comparing Mohr-Coulomb and Cam-Clay

2.1 Mohr-Coulomb

Perform an elasto-plastic multiplier analysis for the MC material. The load at which unrestricted plastic flow occurs - i.e. the bearing capacity - can be estimated analytically using the well-known Terzaghi formula -

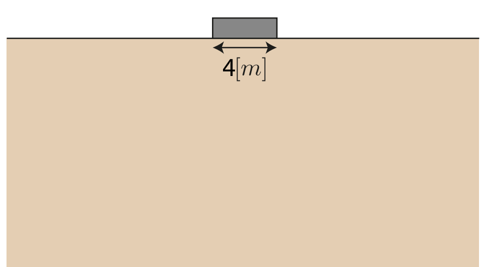

Figure 1: Sketch of the footing. The water table coincides with the ground surface.

which for a drained/long term analysis reads:

$$q_{ult} = N_c c + N_q q + \frac{1}{2}\gamma' b N_\gamma$$

with here $q = 0$ (footing at the ground level), and

$$N_q = e^{\pi \tan\phi} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 2(N_q - 1)\tan\phi$$

Compare the OptumG2 results with this solution. In Optum, use 10 elastic and 10 plastic steps. Perform another analysis allowing a refinement of the mesh.

2.2 Modified Cam-Clay

Perform the same analysis with MCC constitutive law - ensuring you use the same strength (c, ϕ). Here because the constitutive relation is more non-linear, in order to estimate the ultimate load, it is suited to add a solution point at the center of the footing and target its displacement equal to the one obtained with the mohr-Coulomb analysis.

Discuss.

3 Short term analysis

3.1 Tresca material

It is usual to model soil strength using the Tresca model under undrained loading, using the undrained cohesion. Note that from an undrained (homogeneous stress loading e.g.) triaxial test, for a soil, an "equivalence" can be obtained between the drained Mohr Coulomb strength parameters and the undrained shear strength in geotechnical setting

$$c_u = C\cos\phi + \frac{1}{2}(K_o + 1)\,\sigma'_{v,o}\sin\phi$$

where $\sigma'_{v,o} = \gamma' z$ is the initial vertical effective stress. Such an equivalence typically over-estimate the undrained strength for a normally consolidated soil (our case here).

Nevertheless, if we use it to estimate c_u, taking an average between $z = 0$ and $z = b/2$, we obtain for the parameter used here:

$$c_u = 7.7\text{kPa}$$

Perform a short-term (undrained) elasto-plastic multiplier analysis using such a value of undrained shear strength, and compare the results obtained with the well-known Prandtl solution for the bearing capacity of a Tresca material

$$q_{ult} = (2 + \pi)c_u \tag{1}$$

[You can play with mesh adaptativity to explore the localization of the plastic deformation into well known shear bands].

3.2 Mohr-Coulomb

Revert to the Mohr-Coulomb material and perform a short-term analysis. What do you observe ? Explore the excess pore-pressure, the plastic volumetric strain. Explain the results that you obtain.

3.3 Non-associated Mohr-Coulomb

Now, switch to a non-associated MC constitutive relation with a zero dilatancy angle. Re-perform the analysis, compare (with Tresca, associated MC) and discuss.

3.4 Modified Cam-Clay

The modified Cam-Clay model is a better choice to model a soil undrained response. Perform a elasto-plastic analysis using the strength parameters chosen above (and the default non-linear elastic parameters of Optum). Explore the results.

Note that here also, the undrained shear strength of a MCC soil can be estimated (assuming homogeneous stress) from the model parameter. For a geotechnical application, it can be written as - (in plane-strain) -

$$c_u \approx M \times \frac{1}{3}(2K_o + 1) \times \left(\sigma'_{v,o} + c/\tan\phi\right) \times \left(\frac{(2K_o + OCR^{\sin\phi})OCR}{2(1+2K_o)OCR^{\sin\phi}}\right)^{1-\kappa/\lambda}$$

$$M = \frac{6\sin\phi}{3+\sin\phi}$$

again here for an estimate, average between between a depth $z = 0$ and $z = b/2$.

Compare the results with the Prandtl solution (1) again.