Week #5 - Transient Flow in 2D

1 2D diffusion equation - Fluid injection in a porous media

In the notebook `wellbore_injection.ipynb` (located under exercise_week5/), you will find the finite element solution for fluid injection at constant rate Q from a wellbore of radius r_w into a porous reservoir of radius R.

As in past exercises, the script is partially written: you have to finish coding up the remaining parts. Notably:

1. Create the mesh of the problem (get inspired from the one displayed in figure 1).
2. Impose the boundary conditions:
   - Constant flux (Q) at the wellbore. This means that you have to compute the force vector considering a constant specific discharge that is perpendicular to the perimeter of the wellbore.
   - No flow at the reservoir boundary (in the direction perpendicular to the boundary).
3. Compute the spatio-temporal evolution of fluid pressure by the finite element method (use the θ-method for time integration).
4. Compare the numerical solution to the analytical solutions (see description and equations below) which are already coded up in the notebook. In order to do this, create a graph showing the temporal evolution of pressure at 3 different distances from the wellbore (r): $r = r_w$ (right at the wellbore), $r = R$ (at the reservoir boundary), and $r = (r_w + R)/2$.

Note: perform the simulation for a "dimensionless" case and a reservoir radius 30 times larger than the wellbore radius, i.e., $r_w = 1$, $R = 30$, $k = 1$, $S = 1$ and $Q = 1$.

Analytical Solutions

- Line-source in an infinite reservoir: this solution approximates the wellbore as a line-source and considers the reservoir as an infinite medium. It is known as the Theis solution in hydrogeology. When the fluid pressure front reaches the domain boundary, this solution is no longer valid due to the zero flux boundary condition: in this sense we can consider this equation as an "early-time" solution for the global problem.

$$p(r,t) = \frac{Q}{4\pi k} Ei\left(\frac{r^2}{4ct}\right)$$

where, k is the permeability coefficient, $c = k/S$ and S is the storage coefficient. $Ei()$ is the exponential integral function.

- Cylindrical-source in a finite reservoir: this solution is more realistic and consider a finite size for both the wellbore and the reservoir. In this exercise we will use only the pseudo steady-state part of this analytical solution, then we will call this solution as "pseudo steady-state" solution.

$$p(r,t) = \frac{Q}{2\pi k}\left[\frac{2}{R_D^2-1}\left(\frac{r_D^2}{4}+t_D\right) - \frac{R_D^2\ln(r_D)}{R_D^2-1} - \frac{3R_D^4-4R_D^4\ln(r_D)-2R_D^2-1}{4\left(R^2-1\right)^2}\right]$$

Here, $R_D = R/r_w$, $r_D = r/r_w$, $t_D = t/(cr_w^2)$.

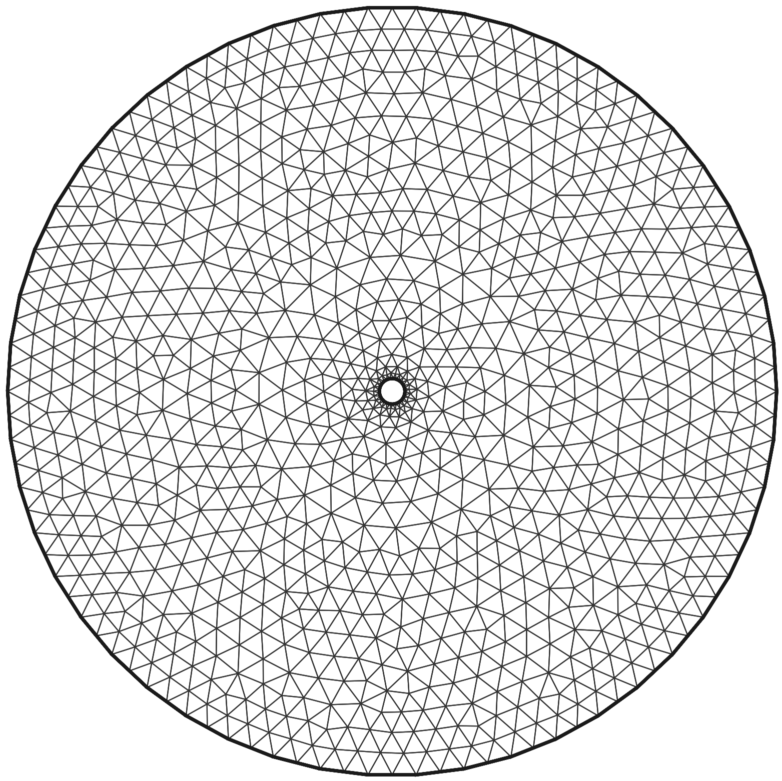

Figure 1: Example of a descent mesh for this problem.