Week #3 - Unconfined flow through an earth dam

Use the course notes to understand the solution of such a ***non-linear*** problem by the finite element method!

1 Flow through a homogeneous earth dam

In the script `unconfined_dam.ipynb` (located under exercise_week3/), you will have to find the solution of the flow through a homogeneous earth dam resting on an impervious base (see figure 1). Such type of problem has the particularity of owing an unknown boundary condition (the phreatic surface within the dam). The dimensions of the dam are: 8 m of crest, 40 m of height, and upstream and downstram slope of 2:1. The earth dam is retaining 35 m of water and there is no tail water ($h_0 = 0$, see figure 2).

As in past exercises, the script is partially written so that you have to finish coding up the remaining parts. Notably:

1. Impose the boundary conditions of the problem.
2. Compute the piezometric head at all the nodes and find the location of the phreatic surface within the dam, by using a relative *Tanh* permeability function (use the function ***`mesh_relative_permeability`***) and a fixed point iteration scheme.
3. **Compute averaged per element fluxes and plot them at the barycenter positions. For this you have to complete function '*compute_element_flux*' of '*Triangle*' class in Elements.py module.**

Then, compare the numerical solution of the phreatic surface to the Pavlovsky's solution —an approximated analytical solution— which gives the size of the seapage at the downstream of the dam, a_0, and the height of the phreatic surface h at $y = 0$ (see figure 2). This solution is obtained through the following system of two nonlinear equations (valid for the case $h_0 = 0$) that can be solved both graphically or iteratively, for instance, by fixed point iterations ;)

$$a_0 = \frac{b}{m_1} + h_d - \sqrt{\left(\frac{b}{m_1} + h_d\right)^2 - h^2}$$

$$\frac{a_0 m}{m_1} = (h_w - h)\ln\left(\frac{h_d}{h_d - h}\right)$$

In these equations, m is the upstream slope and m_1 is the downstream slope, so that $m = \cot(\beta)$ and $m_1 = \cot(\alpha)$ according to figure 2.

Note that both a_0 and h do not depend on the permeability of the material, so you can consider a permeability coefficient of unity for solving numerically this problem. Note also that the accuracy of the numerical solution for the phreatic surface location depends significantly on the size of the elements of the mesh, so consider a proper refinement of the mesh in order to do the comparison. Visualize your numerical results in terms of piezometric head and flux.

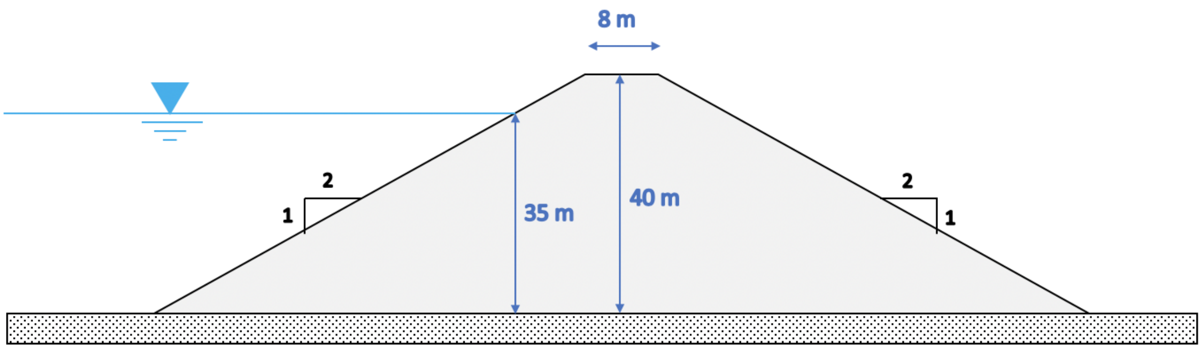

Figure 1: Sketch of a homogeneous earth dam resting on an impervious base.

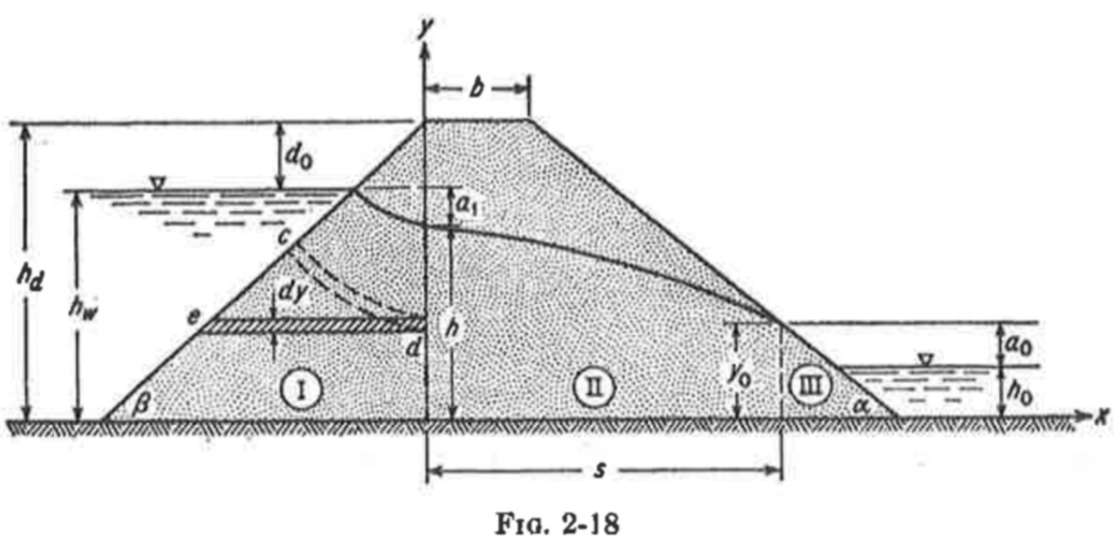

Figure 2: Geometry of the Pavlovsky's solution.