Week #7 - Poroelastic Sphere

Isotropic compression of a poroelastic sphere drained on it surface is a well known, analytically solved, example of a response of a porous media. In order to model such a initial boundary value problem, we will apply the mechanical load via an initial stress field. As at this time, fluid is not allowed to escape, the response is undrained. At time $t = 0^+$ the surface of the sphere is free to drain. We will observe the poroelastic response of this medium: notably the pressure and deformations evolution in the form of drained response. You need to implement parts of the code in *poroelastic_ sphere.ipynb*.

1 Boundary and initial conditions

For this problem, boundary and initial conditions are needed. Due to the axis-symmetry of the problem, we will model a quarter of a circle. As a result, radial displacements will be fixed along $r = 0$ and vertical displacements along $z = 0$. This will be combined with an initial, hydrostatic stress field of unity. We recall the definition of the stress vector in axisymmetry:

$$\sigma = \begin{pmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \tau_{rz} \\ \sigma_{\theta\theta} \end{pmatrix}$$

As the poroelastic sphere is drained on its outer boundary, you'll also need to get the points on this boundary.

- Find the nodes corresponding to the boundary and block the relative degres of freedom (DOF's).
- Note that for the boundary on the arc, you'll need to account for the numerical precision and as well for the approximation of the circle by secants (the mid-point of a Tri6 edge will thus not lay exactly on the circular surface).

We set the initial stress field for you.

2 Assemble the different matrices

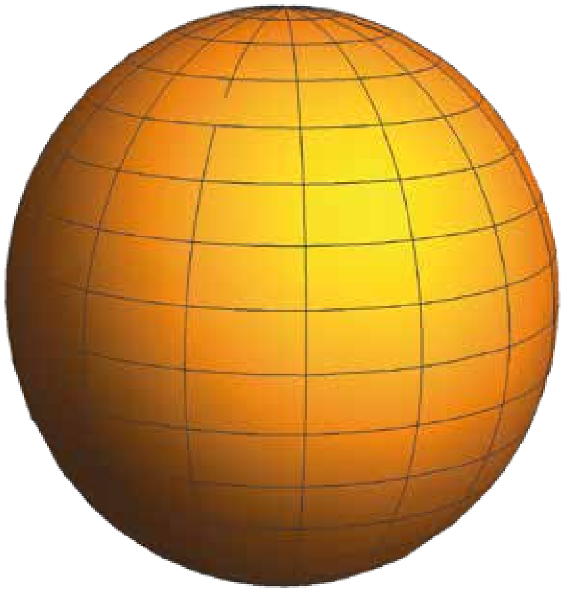
In a next step the different matrices need to be assembled. These are basically the Stiffness matrix $\mathbf{K}$, the mass matrix $\mathbf{M}$, the conductivity matrix $\mathbf{C}$ as well as the coupling matrix $\mathbf{A}$. As in the previous exercises, the first three matrix assemblies are already coded up. As it is commonly done, the interpolation order for the elasticity and the fluid flow is choosen differently (more on this subject in the course), the assembly varies slightly from the ones of the other matrices.

- Complete the function *assemble_ coupling_ matrix()* of MatrixAssembly class by looking at the assembly of the other matrices and adapt the code to this specific matrix.

3 Solution at $t = 0$

The solution at $t = 0$ corresponds to the undrained solution, ie. for a $\Delta t = 0$

- Assemble the full matrix and the force vector.
- Assess the free DOF's for the resolution of the problem.



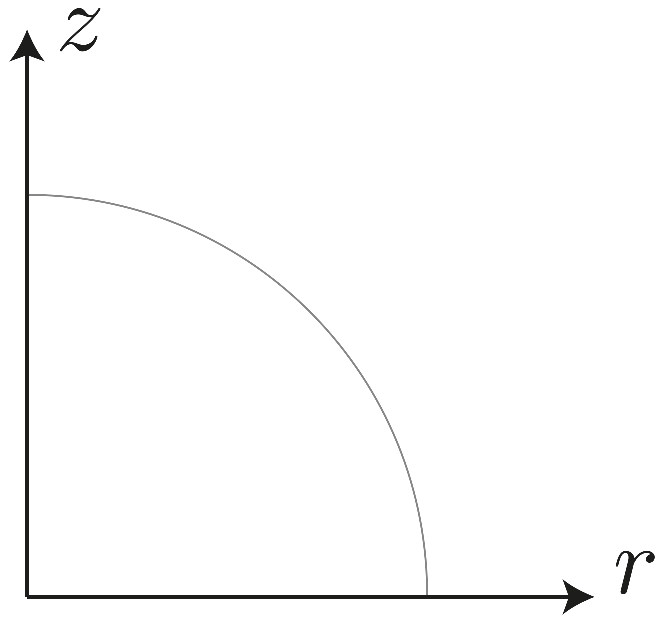


Figure 1: Sphere and coordinate system for the exercise.

- Note that for the undrained case, we will set the initial stress field but do not yet enforce the pore pressure boundary condition (this condition will be set for $t = 0^+$).

4 Solution for $t > 0$

The solution for $t > 0$ is defined by the boundary conditions in displacement and pore pressure. We will solve for a constant time step and a given number of steps (number of steps as loop end condition).

- Assemble the new total matrix
- Assess the free DOF's for the resolution

In the final part of the code, the pressure profile is compared to the analytical solution. Investigate the evolution of pore pressure with time.