# Week #4 - Transient Flow in 1D

## 1 1D diffusion equation - Terzaghi's consolidation problem (transient response)

In the script `diffusion_1D.ipynb` (located under exercise_week4/), you will find the canvas of a finite difference solution of the problem described in Section 4.2.1 of the course notes. Note that this time we are using a finite volume / finite difference method in both space and time.

As in past exercises, the script is partially written so that you have to finish coding up the remaining parts. Notably:

1. Build the $\mathbb{L}$ matrix.
2. Implement the $\theta$-method for time integration via finite differences in time.

Compare the numerical solution to the analytical solution (see equation below) which is already coded up directly in the `diffusion_1D.ipynb` script. Use the graphics that are provided in the script `diffusion_1D.ipynb` in order to perform the comparison. Finally, select different values for $\theta$, $\Delta t$ and $h$, and see its effect in the convergence of the numerical scheme (especially explore the CFL condition).

**Note:** perform the simulation for a "dimensionless" case, i.e. with $L = 1$, $c = 1$ and $p_o = 1$.

The analytical solution in that case is given by:

$$p(x,t) = p_0 \sum_{k=1,3,\ldots}^{\infty} \frac{4}{k\pi} \sin\left(\frac{k\pi x^*}{2}\right) e^{-k^2\pi^2 t^*}$$

where $x^* = x/L$ and $t^* = ct/4L^2$. [Note the infinite series, which is of course must be truncated in practice. Its estimation at early time may suffer from oscillations if not enough terms are used (the convergence of the series is quite slow for very early time)].