Elasto-Plasticity via OptumG2 101

1 Drained triaxial test of a Mohr-Coulomb material

First, we model a simple a triaxial test of a Mohr-Coulomb material. We will simulate two stages: i) a hydrostatic confining stage, ii) a deviatoric stage. We model the core sample as a rectangle in axisymmetry. We will use the following material properties:

$$E = 20 \text{ GPa}, \nu = 0.3 \qquad C = 3\text{MPa}, \qquad \phi = 30^\circ \qquad \psi = 20^\circ$$

[OptumG2 does not account for softening of the MC model]. Use a load multiplier approach, and perform an elastoplastic analysis using 6-nodes triangular elements. Set an observation point on the top surface (where the axial load is applied), and tell OptumG2 to step until a maxium displacement of 0.02m.

We will use this exercise to show you basic features of Optum during the session.

2 Pressuremeter test

We will model in Optum G2 the pressuremeter test in a (associated perfectly plastic) Mohr-Coulomb material. The test consist of pressurizing a flat jack in a wellbore and record pressure-versus volume change of the flat-jack as the soil plastify. We will model the problem in plane-strain, and apply an internal load (P) to a wellbore. At infinity, we will assume the in-situ stress (P_0) to be isotropic and compressive.

An analytical solution exists for this problem, which relates wellbore pressure (P) to the size of the plastic region (b, see Figure 1). For a wellbore of radius a in an infinite medium of cohesionless soil ($c = 0$) with friction angle ϕ (perfectly plastic material), the solution takes the following form:

$$P = P_0(1 + \sin(\phi)) \left(\frac{b}{a}\right)^{(N-1)/N}$$

$$N = \frac{1 + \sin(\phi)}{1 - \sin(\phi)}$$

Note that when $b = a$, then $P = P_* = P_0(1 + \sin(\phi))$. This is the pressure when plastification first occured in the medium. Plastification starts right at the wellbore wall and then propagate through the infinite medium. Owing to the plastic region is always contained by the elastic infinite medium, we can also compute the evolution of plastic (radial) displacement at the wellbore wall ($r = a$) as function of the plastic region (b) as follows.

$$u_r = \frac{P_0 b \sin(\phi)}{2G} \left\{ 1 + \frac{BN}{N-1} \left[1 - \left(\frac{a}{b}\right)^{1/N} \right] - \frac{AM}{M+1} \left[1 - \left(\frac{b}{a}\right)^{1/M} \right] \right\}$$

$$M = \frac{1 + \sin(\psi)}{1 - \sin(\psi)}, \; B = \frac{2\xi}{M + N}, \; A = 1 + B, \text{ and } \xi = (1 - \nu)(1 + MN) - \nu(M + N)$$

Note that when $b = a$, then $u_r = u_r^* = P_0 b \sin(\phi)/2G$ (where G is the shear modulus, $G = E/(2(1 + \nu))$). This is the radial displacement at the wellbore wall when plastification first occured in the medium.

For the purpose of modeling, consider the following:

- material properties; $\phi = 25°$, $\psi = 5°$ (dilatation angle), $E = 25[MPa]$ and $\nu = 0.25$
- because of symmetry you just need to model a quarter of the problem with the proper boundary conditions in terms of displacement
- you have to solve first for the initial conditions (in-situ stress tensor, $\sigma_{xx} = \sigma_{yy} = P_0$ and $\sigma_{xy} = 0$) by imposing compressive stresses $P_0 = 0.1[MPa]$ at the free-displacement boundaries (the wellbore wall and the far-field boundary)
- use a medium of size at least 20 times the wellbore radius (for simplicity take the wellbore radius $a = 1[m]$).

Compare your results with the analytical solution, notably to the pressure P_* and displacement u_r^*.

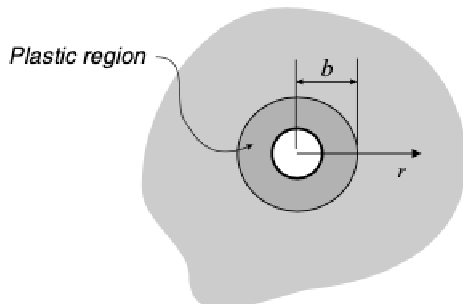

Figure 1: Plastic region surrounding a wellbore in the pressuremeter test. Taken from Davis and Selvadurai (2002).