Week #10 - Elastoplastic Punch Solution

Note: Some clarifications, details and important points to retain from the exercise are added below directly at every question. As a mark-up for you to recognize them as such, those are written in blue (same as this note).

We will model in Optum G2 the elastoplastic punch of a shallow foundation for various constitutive models and time scales (drained vs undrained loading).

1 Mesh and loading conditions

We will let you create the mesh for the problem given in figure 1 - Use the symmetry of the problem, and ensure to take a *large enough* domain as to limit the influence of the finite boundaries to a minimum but not too large. In order to compare side by side different constitutive model, create a single mesh with 600 elements, and with the following features: i) a mesh fan at the end of the footing, 2) enforce a mesh size of 0.1 along the footing. In what follows, always use 6-nodes element in the analysis. We will assume that the water table coincides with the ground surface.

The constitutive relation for the soil will change troughout the exercice. Note that for this, you need to use different type materials for different stage (recommended). The footing itself is assumed rigid. Set a vertical distributed load multiplier on the footing.

To simulate the punch, we will use an *elasto-plastic multiplier* analysis with the load stepping scheme as AUTO.

Soil properties Use the default elastic and weight properties for the Mohr-Coulomb (MC), Tresca and Modified Cam-Clay (MCC). Use the following strength properties for the MC and MCC:

$$c = 5\text{kPa} \qquad \phi = 25^\circ$$

Also use the following for the initial earth coefficient and over-consolidation ratio:

$$K_o = 0.5 \qquad OCR = 1$$

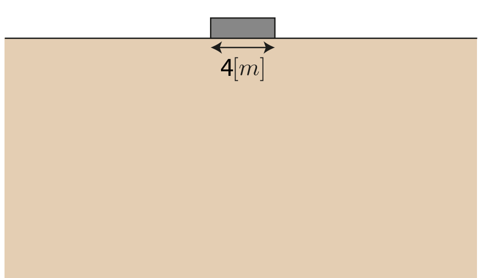

Figure 1: Sketch of the footing. The water table coincides with the ground surface.

2 Long-term analysis: Comparing Mohr-Coulomb and Cam-Clay

2.1 Mohr-Coulomb

Perform an elasto-plastic multiplier analysis for the MC material. The load at which unrestricted plastic flow occurs - i.e. the bearing capacity - can be estimated analytically using the well-known Terzaghi formula - which for a drained/long term analysis reads:

$$q_{ult} = N_c c + N_q q + \frac{1}{2}\gamma' b N_\gamma$$

with here $q = 0$ (footing at the ground level), and

$$N_q = e^{\pi \tan\phi} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$
$$N_c = (N_q - 1)\cot\phi$$
$$N_\gamma = 2(N_q - 1)\tan\phi$$

Compare the OptumG2 results with this solution. In Optum, use 10 elastic and 10 plastic steps. Perform another analysis allowing a refinement of the mesh.

We prepare a mesh using a meshing step. This stage is then used as the input for all other drained calculations to have the same starting (or constant) mesh.

The above calculations using the parameters specified in section 1, lead to an analytical ultimate drained bearing capacity of

$$q_{ult} = 283.8\text{kPa}.$$

The Optum calculation will actually yield slightly larger values (overestimation of bearing capacity)

$$q_{ult}^{\text{MC}} \approx \begin{cases} 328.5\text{kPa} & \text{for a constant mesh} \\ 303.0\text{kPa} & \text{using mesh adaption} \end{cases}.$$

In the optum file, the constant mesh corresponds to the *"MC drained"* stage and the mesh adaption to the *"MC drained adapt"* stage. The effect of the mesh adaptation is rather large as the resulting limit load is about 8% smaller. Compared to the analytical solution the difference of the mesh adaption calculation is thus as low as 7%. You can try to refine the mesh or use more steps of mesh adaptivity to see if you can get even closer to the analytical solution.

2.2 Modified Cam-Clay

Perform the same analysis with MCC constitutive law - ensuring you use the same strength (c, ϕ). Here because the constitutive relation is more non-linear, in order to estimate the ultimate load, it is suited to add a solution point at the center of the footing and target its displacement equal to the one obtained with the mohr-Coulomb analysis.

Discuss.

The target point on displacement serves to assure a convergence of the settlements and thus a "final" stage giving the ultimate bearing capacity. Using the MCC criterion you should get a result of approximately (stage *"MCC drained")*

$$q^{\text{MCC}} \approx 260.2\text{kPa}$$

which represents a clear underestimation. If you check the corresponding load-curve you will see that it is not yet strictly flattened. This means that you did not yet converge to a ultimate solution and need more elasto-plastic loading steps to get a valid solution. As a comparison we perform a limit analysis (with 6-nodes Gauss elements, stages *"MCC drained LA"* and *"MC drained LA"*) and obtain a bearing capacity of

$$q^{\text{MCC}} \approx \begin{cases} 296.4\text{kPa} & \text{MCC limit analysis} \\ 306.7\text{kPa} & \text{MC limit analysis} \end{cases}.$$

The two solutions are very close, which is not surprising because Optum actually switches to an equivalent Mohr-Coulomb for a limit analysis in MCC (see the documentation of Optum Materials section 15 for more details).

3 Short term analysis

3.1 Tresca material

It is usual to model soil strength using the Tresca model under undrained loading, using the undrained cohesion. Note that from an undrained (homogeneous stress loading e.g.) triaxial test, for a soil, an "equivalence" can be obtained between the drained Mohr Coulomb strength parameters and the undrained shear strength in geotechnical setting

$$c_u = C\cos\phi + \frac{1}{2}\left(K_o + 1\right)\sigma'_{v,o}\sin\phi$$

where $\sigma'_{v,o} = \gamma' z$ is the initial vertical effective stress. Such an equivalence typically over-estimate the undrained strength for a normally consolidated soil (our case here).

Nevertheless, if we use it to estimate c_u, taking an average between $z = 0$ and $z = b/2$, we obtain for the parameter used here:

$$c_u = 7.7\text{kPa}$$

Perform a short-term (undrained) elasto-plastic multiplier analysis using such a value of undrained shear strength, and compare the results obtained with the well-known Prandtl solution for the bearing capacity of a Tresca material

$$q_{ult} = (2+\pi)c_u \tag{1}$$

[You can play with mesh adaptativity to explore the localization of the plastic deformation into well known shear bands].

The "equivalence" between the drained Mohr-Coulomb and the undrained shear strength can be derived using $\psi = 0$ and thus implying a zero change of effective mean stress. Note that those exact derivations are only valid for plane strain conditions. You are welcome to recast it on your own or consult the documentation of Optum for a derivation (Optum Materials Section 8.4.2). The analytical Prandtl solution (derivation in course "ouvrages géotechniques") yields a limiting capacity of

$$q_{ult} = (2+\pi)7.7\text{kPa} = 39.60\text{kPa} \tag{2}$$

and the solution with Optum using our standard mesh and parameters *("Tresca Undrained")* yields a value of

$$q^{\text{tresca}} \approx 43.2\text{kPa}.$$

We thus overestimate the resistance by about 9%. Again you can play with mesh-refinement and adaptive remeshing to get more accurate results.

3.2 Mohr-Coulomb

Revert to the Mohr-Coulomb material and perform a short-term analysis. What do you observe ? Explore the excess pore-pressure, the plastic volumetric strain. Explain the results that you obtain.

What you should observe is that you get an infinite bearing capacity if you do an infinite number of elastoplastic steps. When plotting displacement at a results points vs load multiplier, the curve never stabilizes. This phenomenon is related to an overestimation of dilatancy due to the associated Mohr-Coulomb model which results in negative excess pore-pressure, and leads to an infinite undrained hardening of the material such that the bearing capacity becomes infinite. In other words, the more you load the harder gets the material! An associated Mohr-Coulomb analysis is not suited for a short term analysis and you should NEVER use it (TAKE-HOME MESSAGE!). Actually when observing the notes during the calculation Optum tells us exactly this *("MC undrained")*.

3.3 Non-associated Mohr-Coulomb

Now, switch to a non-associated MC constitutive relation with a zero dilatancy angle. Re-perform the analysis, compare (with Tresca, associated MC) and discuss.

As the associated case gives rise to an unlimited hardenig (and thus an infinite bearing capacity) the only way to use Mohr-Coulomb is in a non-associated way - in order to switch off all possible plastic dilatancy and hence the appearance of negative excess pore-pressure. The solution in this case using our standard mesh yields a result of *("MCC Undrained - n.a.")*

$$q^{\mathrm{MC}\,na} \approx 40.82\mathrm{kPa}$$

which is pretty close to the Prandtl solution (only 3% difference) or the one estimated with Tresca.

Important Note that in a number of geo-materials, a degree of plastic dilatancy is often observed and as such undrained dilatant hardening can be observed - Nevertheless, it is always lower than the one for an associated Mohr-Coulomb. The proper way to model dilatancy is to allow the dilatancy angle ψ to drop from a peak value to zero as plastic strain accumulates. This option is available in Optum for a non-associated Mohr-Coulomn model. The Cam-Clay model has "built-in" this effect of limited dilatancy - in Cam-Clay, once the "critical state" is reached no volumetric plastic strain occurs anymore. Note that Cam-Clay also allows for the observed effect of compaction at the start of plastic deformation in the case of an under-consolidated material (a case often observed for some sands).

3.4 Modified Cam-Clay

The modified Cam-Clay model is a better choice to model a soil undrained response. Perform a elasto-plastic analysis using the strength parameters chosen above (and the default non-linear elastic parameters of Optum). Explore the results.

Note that here also, the undrained shear strength of a MCC soil can be estimated (assuming homogeneous stress) from the model parameter. For a geotechnical application, it can be written as - (in plane-strain) -

$$\begin{aligned} c_u &\approx M \times \frac{1}{3}\left(2K_o + 1\right) \times \left(\sigma'_{v,o} + c/\tan\phi\right) \times \left(\frac{(2K_o + OCR^{\sin\phi})OCR}{2(1+2K_o)OCR^{\sin\phi}}\right)^{1-\kappa/\lambda} \\ M &= \frac{6\sin\phi}{3+\sin\phi} \end{aligned} \tag{3}$$

again here for an estimate, average between between a depth $z = 0$ and $z = b/2$.

Compare the results with the Prandtl solution (1) again.

The derivation of the "equivalent" undrained shear strength for the MCC model can be found in the manual of Optum. We will not reproduce the derivation here and simply refer to the corresponding section (Optum manual, materials, chapter 15). note that the powers and "special" definitions are rooted in the definitions of the OCR and an equivalent preconsolidation pressure $\tilde{p}'_o$ which are specific to Optum. The resulting undrained shear strength is

$$c_u \approx 5.874\mathrm{kPa}$$

such that the exact value of the analytical Prandtl solution becomes

$$q_{ult} = (2+\pi)\, c_u \approx 30.2\mathrm{kPa}.$$

For the standard mesh we have used so far for this exercises, the result is about *("MCC Undrained")*

$$q^{\mathrm{MCC}} \approx 24.2\mathrm{kPa}$$

which is a rather significant underestimation (about 20%). Even though this is on the safe side, an under-estimation is often related to strong reinforcement efforts which might lead to high costs. Economically, a strong under-estimation is undesirable.