Computational geomechanics
Oral exam questions

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Note - work out these questions with the course notes. Some of the questions require some additional developments from the course notes. Answer time is 10 minutes, with 10 minutes of preparation.

For all answers, do not derive everything on the board, but state appropriely the steps of your reasoning.

1. Discuss the finite element discretization and matrix assembly for the following operator appearing for example in the weak form of the steady state flow problem (written in indices / differential operator form below)

$$\int_\Omega r_{,i} \kappa h_{,i} \mathrm{d}\Omega \equiv \int_\Omega \boldsymbol{\nabla} r \cdot \kappa \cdot \boldsymbol{\nabla} h \ \mathrm{d}\Omega$$

   where r is a scalar test function with the same regularity than h the scalar unknown function of the problem. Assume that the domain Ω is discretized via a tessalation of triangles, and that linear shape functions on triangular elements are used.

2. Derive the weak form of the following scalar transient diffusion partial differential equation:

$$S \frac{\partial p(\boldsymbol{x}, t)}{\partial t} + \nabla \cdot (\boldsymbol{q}(\boldsymbol{x}, t)) = 0 \qquad \text{for } \boldsymbol{x} \in \Omega,\ t \in [0, \infty[$$

$$\boldsymbol{q}(\boldsymbol{x}, t) = -\kappa \boldsymbol{\nabla} p(\boldsymbol{x}, t) \qquad \text{for } \boldsymbol{x} \in \Omega,\ t \in [0, \infty[$$

$$p(\boldsymbol{x}, 0) = p_o \qquad \text{for } \boldsymbol{x} \in \Omega$$

$$p(\boldsymbol{x}, t) = p_g \qquad \text{for } \boldsymbol{x} \in \Gamma_p$$

$$-\boldsymbol{q}(\boldsymbol{x}, t) \cdot \boldsymbol{n} = q_g \qquad \text{for } \boldsymbol{x} \in \Gamma_q$$

3. After spatial discretization via the finite element method, derive possible $\theta-$time integration schemes for the coupled system of poroelasticity with displacement and pore-pressure as the primary nodal variables:

$$\left[\begin{array}{cc} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{C} \end{array}\right]\left(\begin{array}{c} \mathbf{u}(t) \\ \mathbf{p}(t) \end{array}\right)+\left[\begin{array}{cc} \mathbf{K} & -\mathbf{A} \\ -\mathbf{A}^T & -\mathbf{S} \end{array}\right]\left(\begin{array}{c} \dot{\mathbf{u}}(t) \\ \dot{\mathbf{p}}(t) \end{array}\right)=\left(\begin{array}{c} \dot{\mathbf{f}}_t(t) \\ \mathbf{f}_q \end{array}\right)$$

   where $\dot{f}=\partial f/\partial t$ denotes the time-derivative.
   Express the final system either in term of increment (over a time-step) or total values at time t_{n+1} (as you wish). By analogy with pure flow problems, discuss algorithm stability (do not derive), accuracy and computational cost for integration performed either with a constant or a variable time-step size for the forward and backward Euler cases.

4. For a linear poroelasticity material, express the variation of fluid content ζ (with respect to the reference state) as function of the mean-stress & pore pressure changes. Then derive the expression for the Skempton B coefficient relating variation of mean-stress and pore-pressure under undrained conditions. Recalling that $\zeta=\varphi+\phi_o c_f(p-p_o)$, explore now the case of a poroelastoplastic material following a Cam-Clay associated yield function *under purely isotropic undrained loading* (no shear) $F=q+MP'(\chi+P')$ (P' is the mean effective stress $>$0 in tension), and the hardening rule $\dot{\chi}=\chi\beta\dot{\epsilon}^p_{kk}$.

5. Derive the coupled $\boldsymbol{u}-\boldsymbol{p}$ linear system to solve for an undrained poroelastic problem (discuss how one get the weak form of the coupled undrained problem). Express the final system to solve, where a set of nodes s_u have their u_1 & u_2 components fixed and s_p their pore pressure fixed to constant/given values.

6. Derive the Newton-raphson scheme for a non-linear mechanical quasi-static problem where the applied loads are parametrized with a load multiplier α. We use the arc-length method and control the load increment to target a given value of the absolute displacement at a node. Write the residuals of the weak form of the mechanical problem for a generic elasto-plastic material as function of nodal displacement and α. Express *schematically* the Jacobian matrix of the tangent system (global operator).

7. Write the constitutive stress update (performed at a Gauss-point) for a non-associated elasto-(perfectly) plastic material with a yield function $f(\sigma_{ij})$ and flow rule potential $g(\sigma_{ij})$ using the elastic predictor / plastic corrector scheme. The inputs of the constitutive stress update are the strain $\boldsymbol{\epsilon}_{n+1}$ (obtained from displacement at the nodes).The constitutive stress update returns the new stresses, plastic strain and the consistent tangent stiffness. Discuss the extension to the poroelasto-plastic case where in addition to $\boldsymbol{\epsilon}_{n+1}$, pore-pressure p_{n+1} is given to the constitutive update.

8. For a perfectly-plastic non-associated Mohr-Coulomb (with a constant dilation angle $\psi < \phi$ and cohesion C), for a (drained) triaxial test performed at confining pressure $\sigma'_{III} = -\sigma_c$ under a constant applied axial strain rate $\dot{\epsilon}_{zz}$, derive the evolution of axial and volumetric plastic strain as well as the resulting evolution of the axial stress.