FORMULAE – LECTURE 3

Triaxial elastic stress-strain relations

$$p = \frac{(\sigma_1 + 2\sigma_3)}{3} \qquad q = \sigma_1 - \sigma_3$$

$$p' = p - p_w \qquad q' = q$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}$$

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 = \frac{1}{K}p' \qquad \varepsilon_d = \frac{2(\varepsilon_1 - \varepsilon_3)}{3} = \frac{1}{3G}q$$

General effective stress-strain equation

$$d\sigma'_{ij} = \mathrm{C}_{ijkl} d\varepsilon_{kl}$$

Work

$$\delta W = \sigma'_{xx}\delta\varepsilon_{xx} + \sigma'_{yy}\delta\varepsilon_{yy} + \sigma'_{zz}\delta\varepsilon_{zz} + \sigma'_{xy}\delta\varepsilon_{xy} + \sigma'_{yz}\delta\varepsilon_{yz} + \sigma'_{zx}\delta\varepsilon_{zx}$$

$$\delta W = \sigma'_1\delta\varepsilon_1 + \sigma'_2\delta\varepsilon_2 + \sigma'_3\delta\varepsilon_3 = \sigma'_a\delta\varepsilon_a + 2\sigma'_r\delta\varepsilon_r$$

$$\delta W = \frac{(\sigma'_a + 2\sigma'_r)(\delta\varepsilon_a + 2\delta\varepsilon_r)}{3} + \frac{2(\sigma'_a - \sigma'_r)(\delta\varepsilon_a - \delta\varepsilon_r)}{3}$$

$$\delta W = p'\delta\varepsilon_p + q\delta\varepsilon_q = \delta W_v + \delta W_d$$

Isotropic linear elasticity

$$\sigma_{ij} = \left(K - \frac{2G}{3}\right)\varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij} \qquad \lambda = \frac{E\nu}{(1+\nu)\cdot(1-2\nu)} = K - \frac{2G}{3}$$

$$E = \frac{9KG}{3K+G} \qquad \nu = \frac{3K-2G}{6K+2G} \qquad \varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$K = \frac{E}{3(1-2\nu)} \qquad \tau_{xy} = G\gamma_{xy} \qquad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\mu = G = \frac{E}{2(1+\nu)} \qquad \varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Non-linear elasticity - some models examples

$$\bar{\sigma} = \frac{\bar{\varepsilon}}{a_1 + b_1\bar{\varepsilon}} \qquad E_t = \frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{a_1}{(a_1 + b_1\bar{\varepsilon})^2} \qquad E_i = E_0 \cdot \left(\frac{p}{p_0}\right)^n$$

$$V_s = \sqrt{\frac{G}{\rho}} \qquad V_p = \sqrt{\frac{\lambda + 2G}{\rho}} = \sqrt{\frac{K + \frac{4}{3}G}{\rho}}$$