École Polytechnique Fédérale de Lausanne
School of Architecture, Civil & Environmental Engineering

Traffic Engineering (CIVIL-349)
Nikolas Geroliminis
Revision Problems - Solution

Problem 1: Multiple Ramp Metering

Due to the increase in the number of vehicles between locations 4 and 2, which are heading to location 1 (see percentages of Output 1), we will prioritize ramp 4, then ramp 3, and ramp 2, respectively.

The first constraint that we must satisfy is the bottleneck between locations 4 and 3, called B1.

To do so, we must compute the flow of vehicles coming from location 5 that will exit in location 4. $O_{54} = 8,400 \times 0.01 = 84$ veh/h.

It means that only $8,400 - 84 = 8316$ vehicles remain on the freeway before on-ramp 4 and B1. This allows only for $9000 - 8316 = 684 < 800$ veh/h (demand on location 4 that must pass B1) from ramp 4 to enter in the freeway without exceeding the capacity of B1.

For ramp 3, we denote a variable x as the ratio of demand from location 3 (1,000 veh/h) allowed to continue towards locations 2 and 1. In other words, the served demand of ramp 3.

Again we compute the vehicles exiting at off-ramp 3: $O_{53} + O_{43} = 8,400 \times 0.02 + 684 \times 0.02 = 181.68 \approx 182$.

It means that, between ramp 3 and on-ramp 2, there will remain $9,000 - 182 + 1000x = 8818 + 1000x$ veh/h.

We can assume $x < 1$, that means that even if we prioritize ramp 3 (for the reason stated in the beginning) there is not enough space to serve all the demand on ramp 3 and, at the same time, ensure the minimum flow for ramp 2 (600 veh/h). That is the reason we consider to serve at the minimum rate ramp 2 (600 veh/h) and then find the maximum x of served demand for ramp 3.

With this assumption, the equation of the flow of vehicles passing entering the bottleneck at location 1 is:

$$\underbrace{8,818 + 1000x}_{\text{Enter in 3}} - \underbrace{0.03 \times 1000x}_{\substack{\text{Exit from}\\ \text{3 to 2}}} - \underbrace{13.68 - 168}_{\substack{\text{Exit from}\\ \text{5 and 4}}} + \underbrace{600}_{\substack{\text{Enter}\\ \text{in 2}}} = \underbrace{9900 \text{ veh/h}}_{\substack{\text{Capacity}\\ \text{of B2}}} \tag{1}$$

Therefore, $x = 663.68/970 \approx 0.684$

It means that we serve only $1000 \times x = 684$ veh/h in on-ramp 3. Consequentially, we serve just the minimum of 600 veh/h in ramp 2.

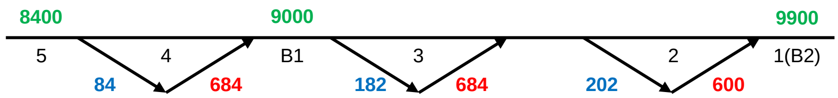

Figure 1: Solution for Problem 1. Served flows in the free-way and ramps.

Problem 2: Rail Crossings design

a. At first, we construct the Time-Space diagram that shows the movement of the first 6 trains doing round trips between points A and B (see figure 2). The headway between trains is 15 mins.

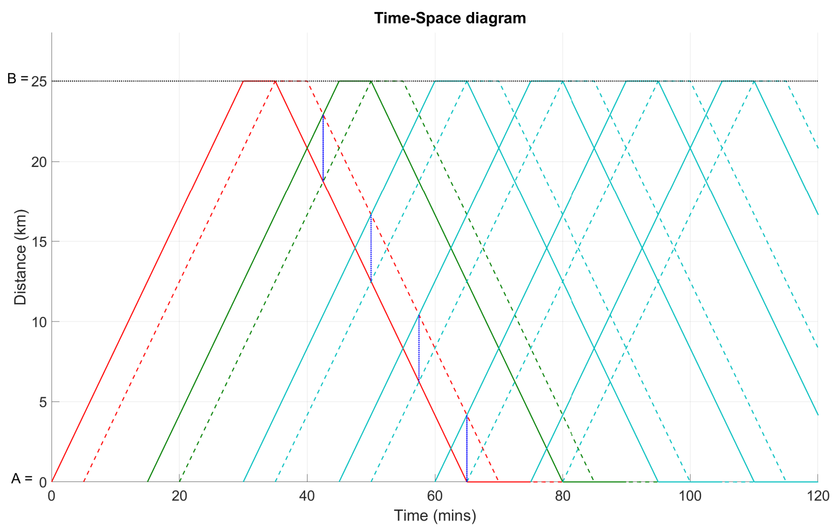

Figure 2: Time-Space diagram for the movement of the trains. In red we draw the 1st train, in green the 2nd and in cyan the rest. In dotted lines we see the maximum potential delay of 5 minutes for each train (question b).

It is easy to observe graphically from Figure 2 that in peak hour schedule, there are at least 4 points in the distance between A and B, where trains that move to opposite directions will meet. We can understand this by observing that before the first train completes its cycle, in the way back from B to A, it crosses with 4 trains moving towards A, the ones that departed 15 mins, 30 mins, 45 mins and 60 mins after the first one. Mathematically we think that during 65 mins (the time that a train needs to do a cycle and return in A), 4 other trains leave station A ($4 \times 15 = 60 < 65$) and none of them arrive before 35 mins (the second train in red in Figure 2 arrives at B at $15 + 30 = 45$ mins).

Therefore, in order to allow all trains that travel in conflicting directions to continue their trips without delay, we need 4 double-tracked sections in this connection between A and B. These parts should be placed in the positions corresponding to the intersection points of the trajectories of the opposite moving trains, having a minimum length equal to the maximum length of the trains travelling to this connection. If this minimum length for the double-tracked trains is considered, no train will have to stop and delay their trip, **provided that all trains are on time**.

b. To answer this question we have to think of the worst possible case(s) that can happen, in order to verify that the length of the double-tracked parts is such that, in any case, the trains can travel continuously, without stopping to wait for delayed trains moving in the opposite direction.

We will examine the problematic cases below, by knowing that any train can be delayed at most 5 minutes. Due to the symmetry of the problem we consider only the first two trains: the red (R) and the green (G).

There are essentially two worst cases that we have to consider in order to maintain safety:

Case 1: *R is* 5 *min late and G is on time*

In this case the two trains will cross at point B′ in figure 3 (extreme case of the maximum delay of 5 mins). Consequently, for any possible delay of R between 0 and 5 mins while G is on time, the two trains will meet at some point on the time-space segment OB′ (i.e. in the distance defined by B′ and C, notated in violet in Figure 3). In order to measure the necessary distance for the double-tracked part, we have to consider what distance (km) our trains can travel in 5 min with a speed of 50 km/h:

$$d = v \times t = 50 \ \frac{\mathrm{km}}{\mathrm{h}} \times \frac{1}{60}\frac{\mathrm{h}}{\mathrm{min}} \times \ 5 \ \mathrm{min} = \frac{25}{6} \ \mathrm{km} \tag{2}$$

Since the two trains of interest travel in opposite directions, this distance is divided by half. So the violet section will be $\frac{1}{2}\frac{25}{6} = \frac{25}{12}$ km. This fact is shown geometrically in Figure 3.

Case 2: *R is on time and G is 5 mins late*

This case is symmetric to the previous one but the $\frac{25}{12}$ km of double-tracked section will now be in the distance defined by A′ and C (or O), which is the initial crosspoint from question a, when all trains are on time.

Finally, the sum of the first and the second section will be:

$$CB' + CA' = 2 \times \frac{25}{12} = 4.166 \simeq 4.17\mathrm{km} \tag{3}$$

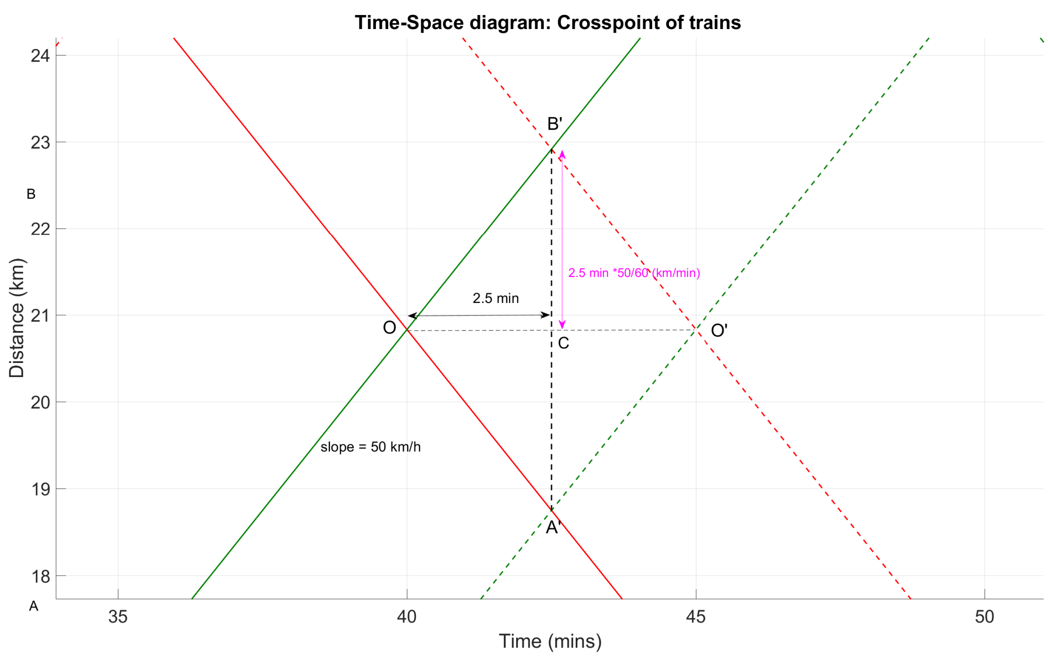

Figure 3: Detail of the Time-Space diagram around the 1st crosspoint between two trains (Red and Green) moving in opposite directions. All four cross points of the 25 km distance would have the same length due to the symmetry of the problem. The required length of each segment is found to be 4.17 km, and the midpoints of these segments should be the crossing points of the train trajectories moving to opposing directions in the case where all trains are on time, e.g. point O etc.