

*Transportation Systems Engineering (CIVIL-349)*  
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*Exercise 7 - Solution*  
*Public Transit*

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*Problem: Bus Route Planning*

| Parameter                    | Symbol | Value | Unit  |
|------------------------------|--------|-------|-------|
| Length of circular bus route | $D$    | 20    | km    |
| No of bus stops              | $n$    | 24    | -     |
| Bus speed                    | $v$    | 20    | km/h  |
| Fixed time waste at stops    | $\tau$ | 0.016 | h     |
| Demand rate                  | $q$    | 0.25  | pax/h |
| Average service time         | $T$    | 0.8   | h     |

Table 1: Table of problem parameters

*Problem Parameters:*

**a. Number of buses:**

The commercial speed of the bus  $v'$  is the speed taking into account the time lost in each bus stop:

$$v' = \frac{D}{\frac{D}{v} + n\tau} = 14.29 \text{ km/h}$$

As defined in the statement of the exercise, the average service time of all passengers should be:

$$T = \text{waiting time} + \text{average riding time} = W + R = 0.8 \text{ h} \quad (1)$$

The average waiting time is equal to  $H/2$ , where  $H$  is the headway (time between two buses), because it reflects the average waiting time for all people waiting at the bus stop.

$$W = \frac{H}{2} \quad (2)$$

The average travel distance among all passengers is equal to half of the maximum distance (cycle) of the bus route, i.e.  $D/2$ . This is result from a weighted average of the distances traveled and the demands as weights (remembering the assumption of uniform demand between all Origin-Destination pairs and than stations are equidistant).

$$\bar{D} = \frac{\sum_{i=1}^{n-1} q_i \times \frac{i}{n} D}{\sum_{i=1}^{n-1} q_i} = \frac{\frac{D}{n} \sum_{i=1}^{n-1} q \times i}{\sum_{i=1}^{n-1} q} = \frac{\frac{Dq}{n} \sum_{i=1}^{n-1} i}{(n-1)q} = \frac{D \frac{n(n-1)}{2}}{n(n-1)} = \frac{D}{2} \quad (3)$$

Where  $q_i$  refers to the demand rate that will travel  $i$  stations and is used as 'weight', while  $\frac{i}{n}D$  is the respective traveled distance. Note that no demand travels zero stations ( $q_0 = 0$ ) nor there is demand that travels a complete cycle ( $q_n = 0$ ). For this reason the summation is bounded between 1 and  $n - 1$  stations.

Hence, the average riding time is:

$$R = \frac{D}{2v'} \quad (4)$$

By regrouping equations (1), (2) and (4), we obtain:

$$\frac{H}{2} + \frac{D}{2v'} = 0.8 \quad (5)$$

By solving equation (5) for  $H$ , we find  $H = 0.2$  h. Then, we can calculate the number of buses needed as:

$$N_b = \frac{1}{H} \frac{D}{v'} \quad (6)$$

The results are shown below:

| Parameter             | Symbol | Value | Unit  |
|-----------------------|--------|-------|-------|
| Commercial speed      | $v'$   | 14.29 | km/h  |
| Average service time  | $T$    | 0.8   | h     |
| Headway between buses | $H$    | 0.2   | h     |
| Number of buses       | $N_b$  | 7     | buses |

Table 2: Question a: Number of required buses

**b. Number of buses clockwise and counter-clockwise directions:**

We follow the same approach as in part (a) but in this case since we have both directions available, the average passenger riding time will change, as for distances larger than  $1/2$  of the total route  $D$ , passengers can indistinctly choose to travel in the both direction, so each direction will serve half of the demand.

In order to find the average riding time of the passengers, we will first calculate the average number of stops per passenger trip (for how many stops on average passengers stay in the bus). Since we have a demand  $q = 0.25$  pax/h for every O-D pair of stops, this means that starting at any bus stop  $i$ , there will be  $q$  pax/h going to the next stop (1 stop),  $q$  pax/h going to the second next (2 stops), ..., and  $q/2$  pax/h going to the  $n/2$ -th stop, i.e. the largest possible trip.

In this case, as we have  $n = 24$  stops and buses travelling in both directions, the maximum distance between any origin-destination is

12 stops ( $n/2$ ). Thus, we can calculate the average number of stops per passenger by the following weighted average (see Figure 1):

$$\text{Average number of stops} = \frac{\sum_{i=1}^{11} (q \times i) + 0.5q \times 12}{11.5q} \quad (7)$$

Then we can calculate the average riding time of passengers by taking a fraction of the riding time of the whole cycle (trip that includes all  $n$  stops), that is equal to the average number of stops visited over the total number of stops, so we can write the following equation:

$$\text{Average riding time} = R' = \frac{D}{v'} \frac{\text{Average number of stops}}{24} \quad (8)$$

Then, we just recalculate the new headway,  $H'$  as we did in equation (5) and we obtain the new number of buses needed (for both directions) which would be:

$$N'_{b,tot} = 2 \times N'_b = 2 \times \frac{1}{H'} \frac{D}{v'} \quad (9)$$

In equation (9), we multiply by 2 the whole expression in order to take into account both directions. The results are shown in the following table.

| Parameter                       | Symbol       | Value                | Unit  |
|---------------------------------|--------------|----------------------|-------|
| Commercial speed                | $v'$         | 14.29                | km/h  |
| Average service time            | $T$          | 0.8                  | h     |
| Headway between buses           | $H'$         | 0.87                 | h     |
| Number of buses for 1 direction | $N'_b$       | 1.61                 | buses |
| Total number of buses           | $N'_{b,tot}$ | $3.22 \rightarrow 4$ | buses |

### c. Minimum bus capacity:

In order to find the minimum bus capacity, we first need to calculate the critical demand, i.e. the highest number of passengers that we can possibly have on-board. We calculate this demand by constructing the following formula:

$$q_{cr} = \underbrace{(n-1)q}_{\text{boarding passengers}} + \underbrace{(n-2)q + \dots + q}_{\text{passengers from previous stops}} = q \frac{n(n-1)}{2} \quad (10)$$

The demand from any single stop to any other stop is  $q$  (as stated in the exercise description). If we take as point of reference one random stop of our circular, one-directional route (case (a)), we can

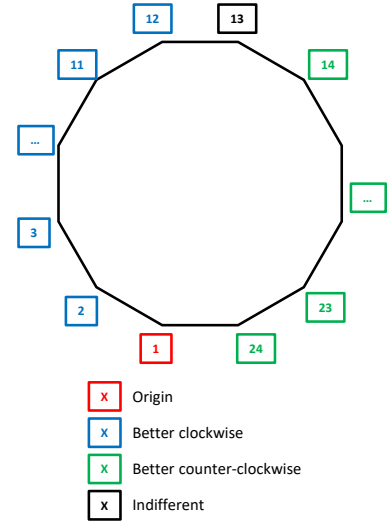


Figure 1: Reasoning behind equation (7) using Stop 1, as illustration. The  $0.5q \times 12$  in the equation refers to the stop that passengers are indifferent between clockwise and counter-clockwise directions because the distance is the same.

Table 3: Question b: Number of required buses clockwise and counter-clockwise

calculate the demand for all the other stops. Since the demand is uniform for all O-D pairs and the route circular, in any trip between two consecutive stops we will have the same number of passengers, which in this case will be out critical demand  $q_{cr}$ .

Let us explain the formula of equation (10). Let us consider station 1 and the colored segment in Figure 2. For our assumption on the demand, we have  $(n - 1)q$  pax that will take the bus at station 1, that is  $q$  pax for each of the following stations (e.g. from 2 to 24 =  $n - 1$  stations). BUT in the same bus there were already other passengers from the previous stations. In particular, from station 24,  $(n - 1)q$  pax entered in bus but  $q$  pax got off at station 1. So it remains  $(n - 2)q$  pax in the bus. Similar discussion for the passengers that get into the bus at station 23,  $q$  of these get off in station 24 and other  $q$  in station 1, that is  $(n - 3)q$  from station 23 are still in the bus from station 23 when the bus leaves station 1 to station 2. Continuing with similar argument, we arrive until considering the  $q$  pax that took the same bus at station 3, did all the round trip until station 2, and there are still in the bus. We need passengers that were already inside the bus at station 1, and we have to sum to the new  $(n - 1)q$  that entered in station 1 to know the critical  $q_{cr}$ . Formula (10) is the mathematical equation that calculates the sum that we just described.

Therefore, we find:

$$q_{cr} = q + 2q + 3q + \dots + 23q = q \frac{24(24 - 1)}{2} \quad (11)$$

Finally, we calculate our minimum capacity as the minimum required number of seats (assuming all passengers must be seated),  $N_{seats}$ , per bus.

$$N_{seats} \geq q_{cr} \frac{D}{v'} = q_{cr} H \quad (12)$$

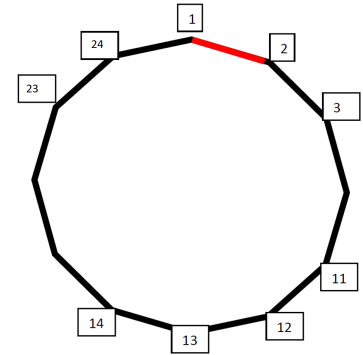


Figure 2: Schematic description of the circular bus route

| Parameter           | Symbol      | Value                 | Unit  |
|---------------------|-------------|-----------------------|-------|
| Demand rate         | $q$         | 0.25                  | pax/h |
| Critical demand     | $q_{cr}$    | 69                    | pax/h |
| Minimum No of seats | $N_{seats}$ | $13.8 \rightarrow 14$ | seats |

Table 4: Question c: Minimum bus capacity