Traffic Engineering (CIVIL-349)
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Exercise 1
Queueing systems

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Consider a fast-food drive-thru. On average, 10 cars arrive per hour and are served on a FCFS (*first-come-first-served*) basis.

Problem 1: If the average time to serve each client is 4 minutes, and both the time interval to serve a client and the time between successive arrivals follow exponential distributions:

a. How can one describe this queueing system using Kendall's notation?

b. What is the probability that the system is empty?

c. What is the average number of cars in the drive-thru and the average number of cars in the queue?

d. What is the average time a car spends in the drive-thru and the average waiting time?

e. What is the probability of having 2 or more cars in the system (in service and queue)?

Problem 2: Following **Problem 1**, use the same performance indicators from parts **b**, **c**, **d** to answer the following questions:

a. How will the performance indicators change if the service time is halved (service rate is doubled)?

This case is often referred to as the 'super-server'.

b. How will the performance indicators change if the restaurant opens a new drive-thru on the other side of the road (such that the queues form independently in each drive-thru)? Assume that the demand is perfectly split between both drive-thrus.

Two independent M/M/1 systems each serving half of the demand.

Problem 3: Following **Problem 1**, consider now that there is not enough space for more than 5 cars in the system (in-service and waiting), otherwise, the queue grows onto the street and incoming passengers leave the system without joining the queue.

a. What is the rate at which the restaurant loses customers, and what is the restaurant's average utilization rate?

b. How will the average number of cars in the queue and average waiting time change?

Problem 4: Consider that we add a second server to the same drive-thru from **Problem 1**, such that customers form a single queue which is served by the two servers. How will the performance indicators change compared to both situations in **Problem 2**? (Use the same performance indicators as in **Problem 1.b–1.d**)

Problem 5: Consider **Problem 1**. The restaurant is considering replacing the human operated server by an automated service device. The automated device no longer serves customers in exponential times but in deterministic times.

a. How can we describe this new system using Kendall's notation?

b. How will the performance indicators change? (Use the same performance indicators as in **Problem 1.b–1.d**)