ChE-403 Problem Set 3.2

Week 11

Problem 1

We determined that the diffusion of molecules in a single pore was governed by the following dimensionless equation:

$$\frac{d^2 C_A'}{d\chi^2} - \phi^2 C_A' = 0$$

With boundary conditions:

$C_A' = 1 \ @ \ \chi = 0$

$\frac{dC_A'}{d\chi} = 0 \ @ \ \chi = 1$ (no flux at the end of the pore)

Can you solve this equation?

Problem 2

The isothermal first order reaction of gaseous A occurs within the pores of a spherical catalyst pellet. The reactant concentration halfway between the external surface of the pellet and the center of the pellet is equal to ¼ of the concentration at the surface of the pellet (C_{AS}). What is the concentration of A at the center of the pellet?

Problem 3

Demonstrate that the solution for the following differential equation seen in class for diffusion in a spherical catalyst pellet):

$$\frac{d^2 C_A'}{d\bar{r}'^2} + \frac{2}{\bar{r}'}\frac{dC_A'}{d\bar{r}'} - \phi^2 C_A' = 0$$

With boundary conditions

$C_A' = 1 \ \ @ \ \bar{r}' = 1$

$\frac{dC_A'}{d\bar{r}'} = 0 \ \ @ \ \bar{r}' = 0$

has a solution:

$$C_A' = \frac{\sinh(\phi\,\bar{r}')}{\bar{r}'\sinh(\phi)}$$

With: $\sinh(x) = \frac{e^x - e^{-x}}{2}$

Hint: This is a second order differential equation with non-constant coefficients, which is quite difficult to solve. However, try doing a variable change (with a new variable *y*) where: $y = C_A'\bar{r}'$ or $C_A' = y/\bar{r}'$. You might get a familiar equation…

Annex:

A small translated portion from the "Formulaire et tables":

Linear 2nd order differential equations with constant coefficients:

An equation of the type: $ay'' + by' + cy = g(x)$ with $a \neq 0$

Case where $g(x) = 0$

The general solution depends on the *characteristic equation*: $ar^2 + br + c = 0$

If this equations has…	The solution to the differential eq. is…
2 real solutions r_1 and r_2	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
1 real solution r	$y = (c_1 x + c_2) e^{rx}$
2 complex solutions $p \pm q\,i$	$y = e^{px}[c_1 \cos(qx) + c_2 \sin(qx)]$