Optical methods in chemistry
or
Photon tools for chemical sciences

Session 3:

Course layout – contents overview and general structure

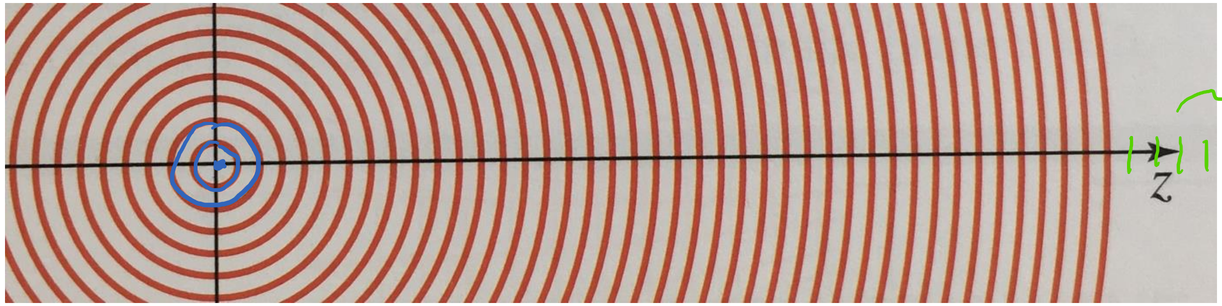
- Introduction and ray optics
- Wave optics
- Beams
- From cavities to lasers
- More lasers and optical tweezers
- From diffraction and Fourier optics
- Microscopy
- Spectroscopy
- Electromagnetic optics
- Absorption, dispersion, and non-linear optics
- Ultrafast lasers
- Introduction to x-rays
- X-ray diffraction and spectroscopy
- Summary

Todays learning goal:

Introduction to beam optics

Change the way you think about light propagation, foci, etc

Special case: Fresnel approximation



- Spherical wave close to z-axis but far away from origin

$$U(\vec{r}) = \frac{A}{r} \exp\left[-i \vec{k}\vec{r}\right]$$

(approx with Taylor expansion)

$$= \frac{A_0}{z} \underbrace{\exp\left[-ikz\right]}_{\text{Looks planar}} \underbrace{\exp\left[-ik\frac{x^2+y^2}{2z}\right]}_{\text{Looks parabolic}}$$

Beam optics

add today

- Paraxial beams with intensity distribution
- Gaussian beam is solution for Helmholtz equation

The Gaussian beam takes the name of the celebrated German mathematician **Carl Friedrich Gauss (1777–1855)**, who established what is now known as the Gaussian distribution.

Lord Rayleigh (John William Strutt) (1842–1919) contributed to many areas of optics. The Rayleigh range of a Gaussian beam characterizes its depth of focus.

Gaussian beam I

- Consider paraxial beam with (slowly) varying complex amplitude

We start with a plane wave $U(\vec{r}) = A(\vec{r}) \exp[ikz]$ ← plane wave travelling along z

slowly varying amplitude with z

Amplitude with paraboloidal wave nature $A(\vec{r}) = \frac{A}{z} \exp\left[-ik\frac{\rho^2}{2z}\right]$ $\rho^2 = x^2 + y^2$

Note if $U(\vec{r})$ satisfies Helmholtz equation, then $A(\vec{r})$ needs to do so, too!

- One possible solution for the complex envelope is $A(\vec{r})$

→ consider a "shifted" version $q(z) = z - \xi$

→ simple transformation → will also satisfy Helmholtz equation

→ paraboloidal wave centered around $z - \xi$ instead of $z = \emptyset$

→ use more generalized (→ complex) version $q(z) = z - \xi$ with $\xi = -iz_0$

$$\boxed{q(z) = z + iz_0}$$ q – parameter

$$A(\vec{r}) = \frac{A(\vec{r})}{q(z)} \exp\left[-ik\frac{\rho^2}{2q(z)}\right]$$

Gaussian beam II

- Separat phase and amplitude in terms of real and imaginary part
- Further use substitution:

$$\frac{1}{q(z)} = \frac{1}{(z - i z_0)}$$

introduce new $R(z)$ $w(z)$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w^2(z)}$$

- So that you obtain:

$\boxed{\text{Gaussian Beam}}$

$$U(r) = \underbrace{A_0 \frac{w_0}{w(z)}}_{\text{Amplitude}} \underbrace{\exp\left[-\frac{\rho^2}{w^2(z)}\right]}_{\sim \rho} \exp\Big[\underbrace{-ikz}_{\text{planar}} \underbrace{- ik\frac{\rho^2}{2R(z)}}_{\text{parabolic}} \underbrace{+ i\xi(z)}_{\text{phase}}\Big]$$

Real

next slide

imaginary

Gaussian beam III

- U(r) is called a Gaussian beam

wave equation

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

- With the following parameters

we will focus on discussing meaning of parameters

→ A_0, z_0 from boundary conditions

→ all others related to z_0, λ

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$$ beam waist

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$ wave front radius

$$\zeta(z) = \tan^{-1}\frac{z}{z_0}$$ phase retardation

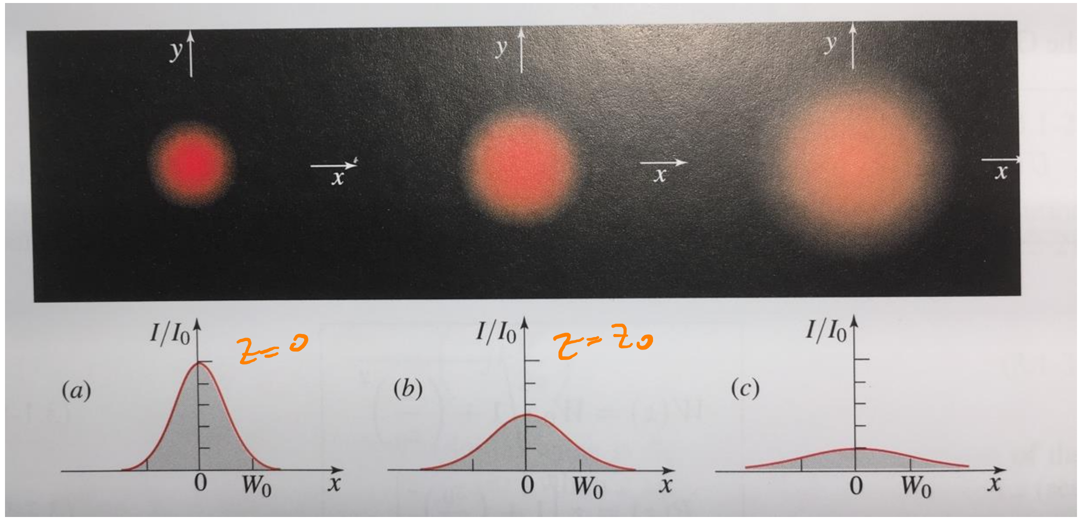
$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}.$$ waist radius

Parameter: Intensity as function of radial distance $\rightarrow r$

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

radial $\xrightarrow{\text{Gauss}}$

I/I_0 fct of z $\rightarrow$



Examples (c.f. slide 7)

$z = 0 \rightarrow W_0 = W_0\sqrt{1+0}$
$= W_0$
$\rightarrow I = I_0 \underbrace{\left(\frac{W}{W_0}\right)}_{=1} \exp[\ldots]$

$z = z_0 \rightarrow W = W_0\sqrt{1+1}$
$= W_0 \cdot \sqrt{2}$
$I = I_0 \cdot \frac{1}{2}$

Start: $I = |U(r)|^2 \rightarrow$ now function of axial and radial distance: z, $\rho = \sqrt{x^2+y^2}$

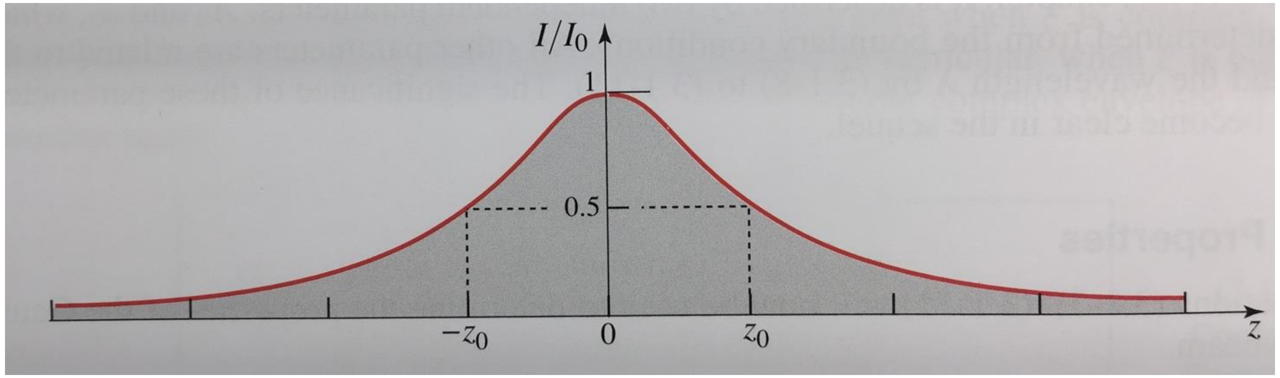
$$= \underset{\downarrow\; I_0}{|A_0|^2} \underbrace{\left(\frac{W_0}{W(z)}\right)^2}_{\downarrow} \underbrace{\exp\left[-\frac{\rho^2}{W^2(z)}\right]}_{} \underbrace{\exp[i \ldots]^2}_{=1}$$

$$I_0 \underbrace{\left(\frac{W}{W(z)}\right)}_{\text{axial}} \underbrace{\exp\left[-\frac{\rho^2}{W^2(z)}\right]}_{\text{radial}} \longrightarrow \text{Gauss with peak } \rho = 0$$

Once a Gaussian – always a Gaussian!

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

Parameter: Intensity on beam axis



Now beam on axis, i.e. $\rho = 0$

$$I(0,z) = I_0 \left[\frac{W_0}{W(z)}\right]^2 = \frac{I_0}{1 + \left(\frac{z}{z_0}\right)^2} \quad \Big| \text{ use } W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$$

$\Rightarrow$ Greatest possible intensity at $z = 0$

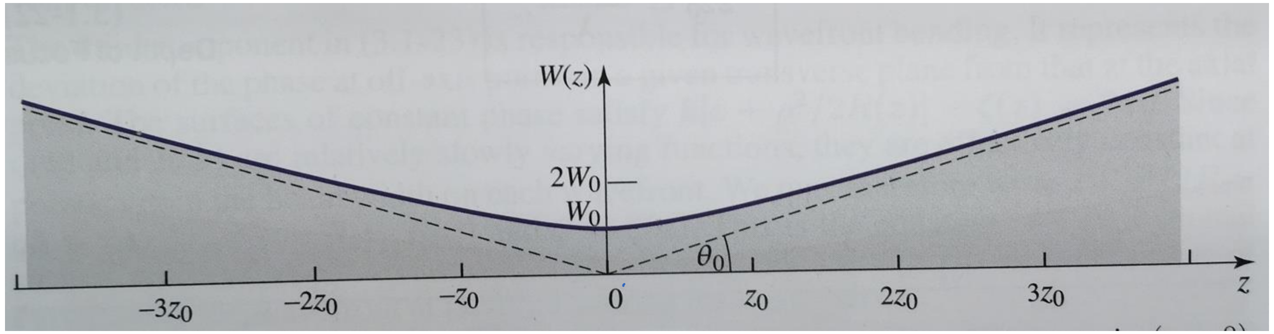
50% at $z = z_0$

Parameter: Beam waist

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

focus

↓



Note

Radius of beam

RMS / $\sigma = \frac{1}{2} W(z)$

86% of power in this circle

① Beam width: $w(z) = w_0 \sqrt{1+\left(\frac{z}{z_0}\right)^2}$

$w_0 \rightarrow$ smallest waist radius

$2w_0 \rightarrow$ waist diameter $\rightarrow$ spot size

② Beam divergence for $z \gg z_0 \rightarrow \sqrt{1+\left(\frac{z}{z_0}\right)^2} \rightarrow \sqrt{\left(\frac{z}{z_0}\right)^2} = z/z_0$

linear

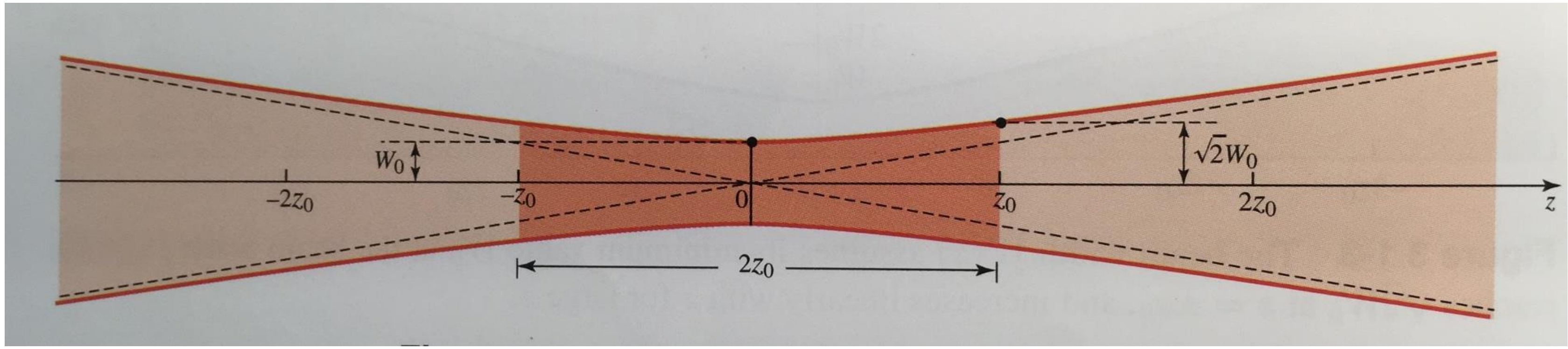
waist $w(z) \approx \frac{w_0}{z_0} z = \Theta_0 z \implies$ linear $\implies$ Ray optics!

trig $\Theta_0 = \frac{w_0}{z_0} = \frac{w_0^2}{w_0 z_0} = \frac{\lambda z_0}{w_0 \pi z_0} = \frac{\lambda}{\pi w_0}$ | with $w_0 = \sqrt{\frac{\lambda z_0}{\pi}}$

$\rightarrow$ in spot size $2\Theta = \frac{4}{\pi} \frac{\lambda}{2w_0} \longrightarrow$ opening cone of beam

Parameter: Depth of focus

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$



$$\boxed{2z_0 = \frac{2\pi w_0^2}{\lambda}} \qquad \text{c.f. slide 7} \quad w_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

depth of focus $\approx 2\times$ Rayleigh range z_0

⇝ length of constant power

Example (good) He Ne $\quad \lambda_0 = 633\,\text{nm}, \quad 2w_0 \approx 2\,\text{cm}$

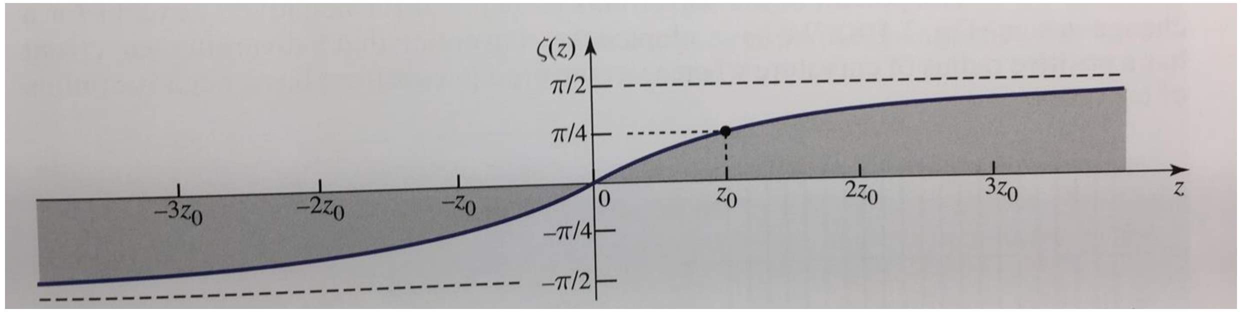
$$2z_0 = \frac{2\pi w_0^2}{\lambda} \approx \frac{2\pi\,(10^{-2})^2\,m^2}{633\cdot 10^{-9}\,m} \approx \ldots \approx 10^3\,m \approx 1\,km$$

now try with $2w_0 = 20\,\mu m$ … → 1mm

Parameter: Phase

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

φ



$$\varphi_{(\rho,z)} = kz - \zeta_{(z)} + \frac{k\rho^2}{2R_{(z)}}$$

$$\zeta_{(z)} = \tan^{-1}\frac{z}{z_0}$$

$\tan^{-1} -\infty$

$\tan^{-1} +\infty$

On axis $\varphi_{(0,z)} = kz - \zeta_{(z)}$

plane wave

phase retardation

$\longrightarrow$ a wave travelling from $-\infty$ to $+\infty$ accumulates phase retardation of π

$\boxed{\text{Gouy effect}}$

Parameter: Curvature of wavefront

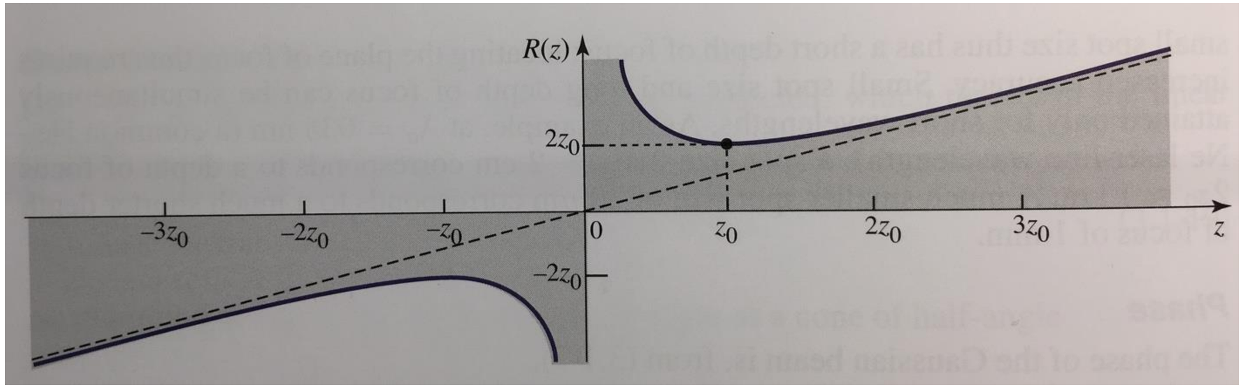
$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

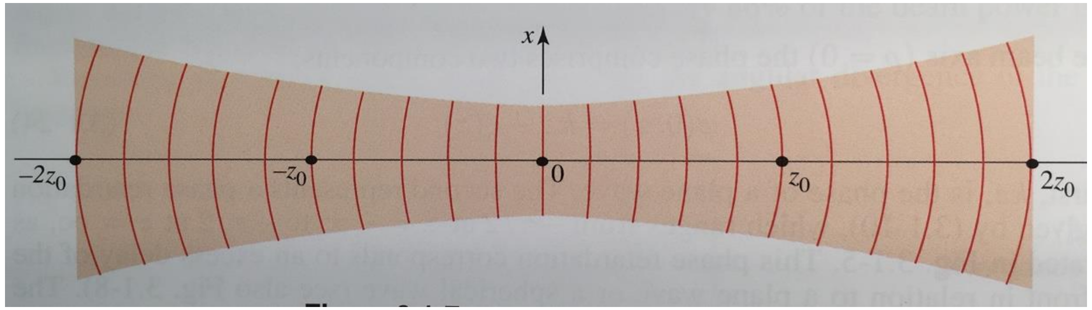
$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

$R(z \to 0) = \infty$ planar

$R(z = z_0) = 2z_0$ minimum $\to$ greatest curvature

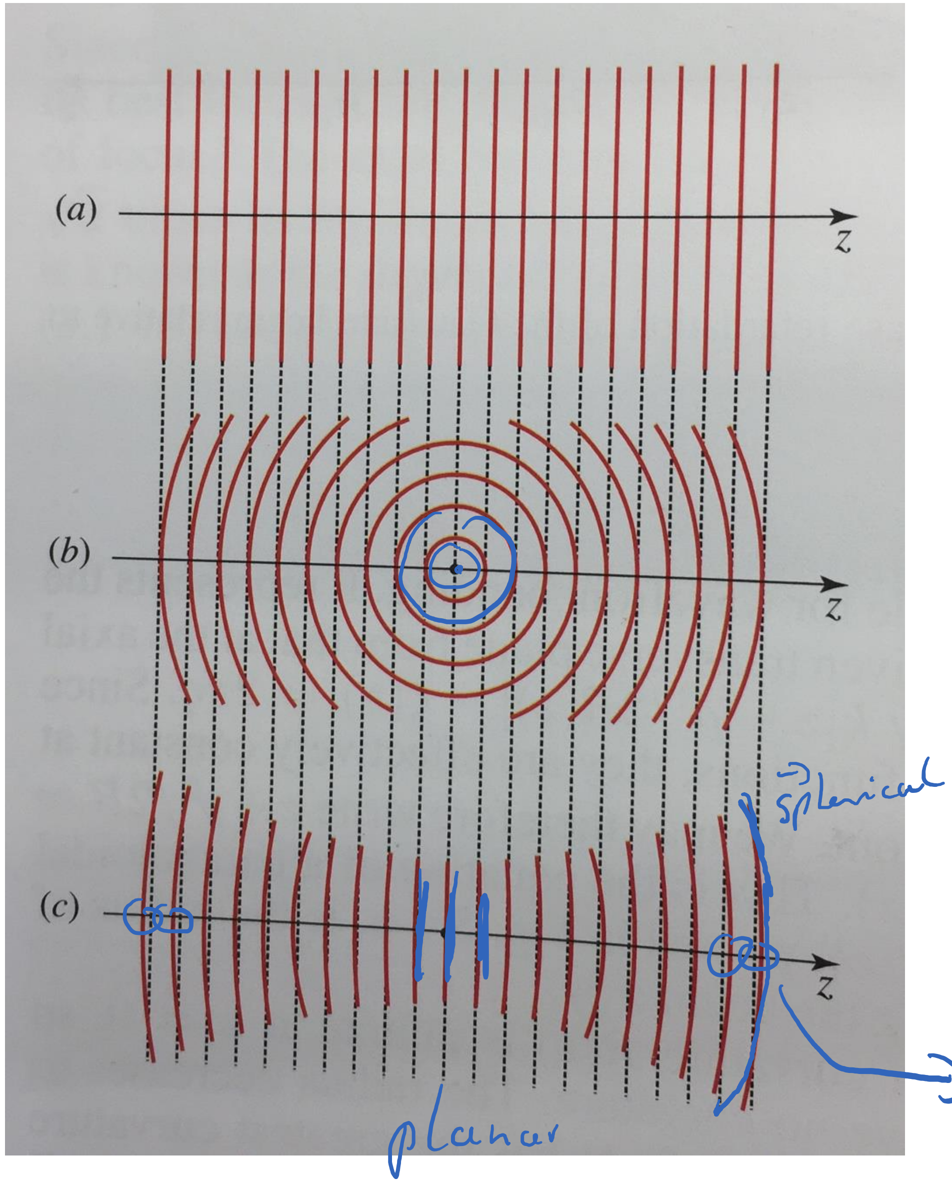
$R(z \gg z_0) \simeq z$ ~ spherical





Comparison: plance wave, spherical wave, Gaussian beam

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$



→ planar

→ spherical

→ Gaussian

→ phase shifted

Some statements to remember

- A Gaussian beam transmitted through a circularly symmetric optical component remains a Gaussian beam

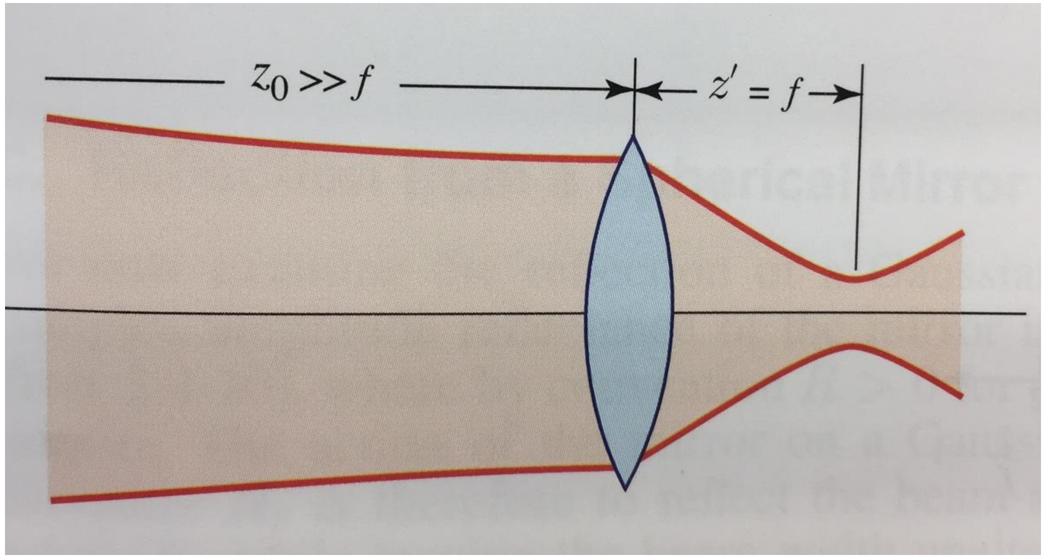
- Such optical components reshape the beam, i.e., its waist and curvature

$$2w_0' = \frac{4}{\pi}\lambda \frac{f}{D} \rightarrow \ominus$$

$$= \frac{4}{\pi}\lambda F_{\#}$$

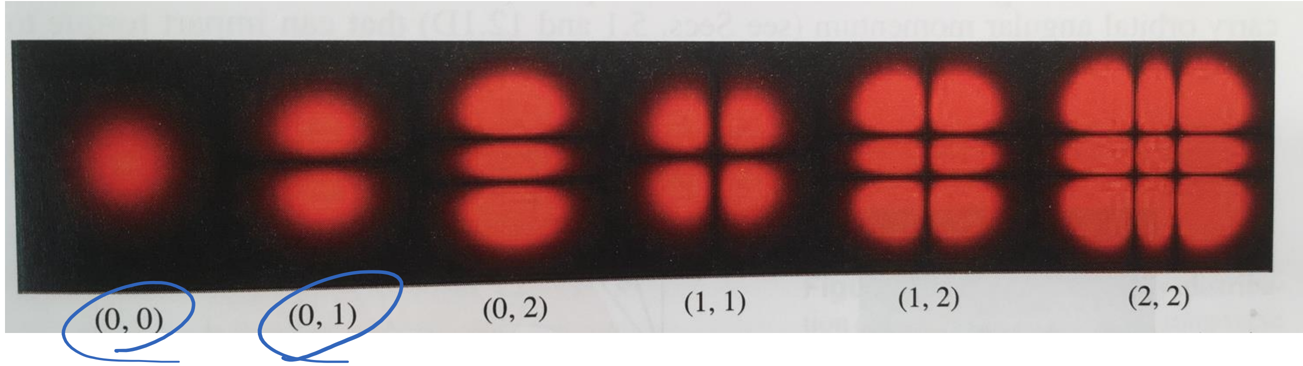
"F-number"

- Focusing a collimated Gaussian beam:



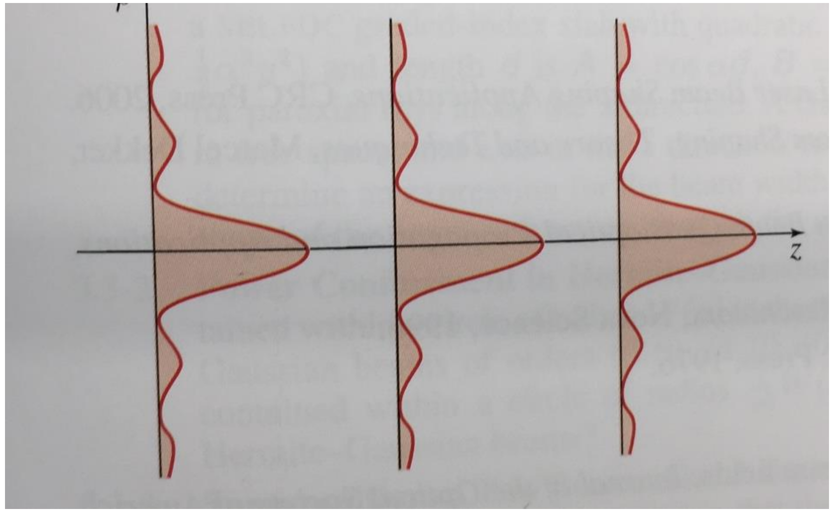
Spot size $\longleftrightarrow$ $F_{\#}$, λ

Note: There exist other solutions for the paraxial Helmholtz equation



- Hermite Gaussian beams
  - Non-Gaussian intensity distribution
  - Same wavefront as Gaussian beams
  - Can match curvature of spherical mirrors, ideal for building resonators

- Laguerre Gaussian beams
  - Solution of Helmholtz equation in cylindrical coordinates
  - Separation of variables in r and f (not x and y)



- Bessel beams
  - Solution of 2D Helmholtz equation in polar coordinates
  - For Bessel beams the intensity distribution is independent of z
  - Asymptotic behavior different from Gaussian beam

The end.