This is an open book exam but you are not allowed to discuss your work with anyone or accept anyone's help to solve the exam problems! Please show all your work, answers without supporting work will not be given credit. Write answers in spaces provided. You have from 12:15 until 15:15 (3 hrs) to complete this exam. You will get an additional 15 minutes until 15:30 to upload your answers
Total achievable points 100.

Name:________________________________

1. Ray Optics

   (a) Sketch setup and rays for pinhole, shadow, projection. Label your sketch (3 points)

   (b) For a reflection from a mirror, the angle of incidence equals the angle of reflection. Name Fermats principle. Sketch and argue this law based on Fermat's principle. (2 points)

   (c) In a solar energy farm you want to collect the parallel sunlight and focus it in a heat absorber. What optical element (mirror) do you use to collect and focus the light? Sketch. (2 points)

(d) At the interface between two media light is refracted. In Fig. 1 draw the rays on the other side of the interface. (2 points)

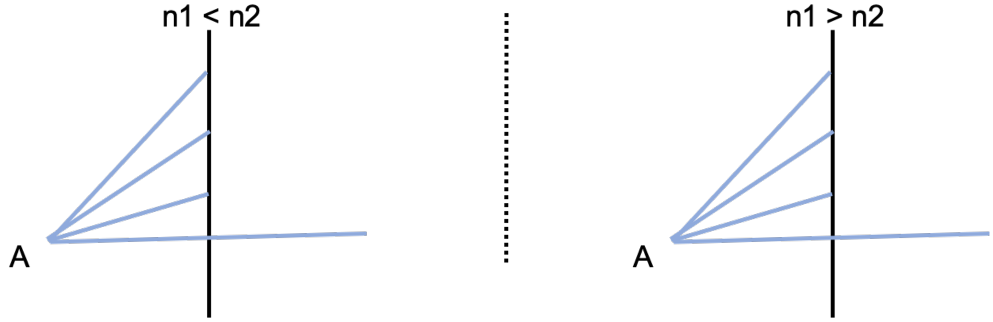

Figure 1: Refraction at interface

(e) The two optical setups In Fig. 2 are cut exactly in the middle. Draw the other side of the setup. (2 points)

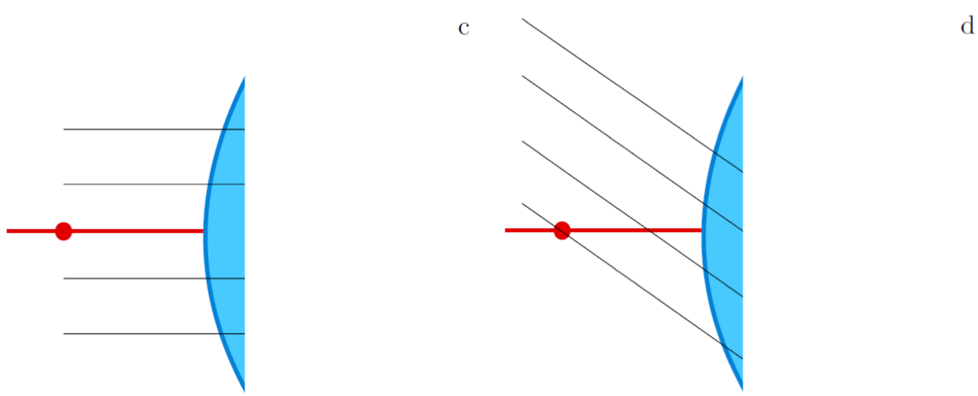

Figure 2: Two lens setups to complete

2. Beam Optics

A Gaussian beam is described by

$$U(r) = A_o \frac{\omega_o}{\omega(z)} exp[\frac{-\rho^2}{\omega(z)^2} exp[-ikz - ik\frac{\rho^2}{2R(z)} + i\xi(z)] \quad (1)$$

and $\rho = \sqrt{x^2 + y^2}$

(a) Describe the meaning of $\omega(z) = \omega_o\sqrt{1 + (\frac{z}{z_o})^2}$. (1 point)

Describe the meaning of $R(z) = z[1 + (\frac{z}{z_o})^2]$. (1 point)

Describe the meaning of $\xi(z) = tan^{-1}\frac{z}{z_o}$. (1 point)

Describe the meaning of $\omega_o = \frac{\lambda z_o}{\pi}$. (1 point)

(b) Sketch the intensity of a Gaussian beam as function of the radial distance in the focus ($z = 0$) and at the Rayleigh length ($z = z_0$). For the y axis use units normalized to the max intensity, i.e., I/I_0. (2 points)

(c) Now, sketch the intensity of a Gaussian beam on the beam axis around the focus. For the y axis use units normalized to the max intensity, i.e., I/I_0. Point out the Rayleigh length. (2 points)

(d) Sketch and describe the correlation between beam optics and Ray optics for z $\rightarrow \infty$? What is the Rayleigh length? (2 points)

(e) The confocal parameter of a Gaussian beam relates the Rayleigh length and beam width. It is defined as twice the Rayleigh length and is given by $2z_o = \frac{2\pi\omega_o^2}{\lambda}$. What is the relationship between depth of focus and focal spot size? Which parameters can you tweak to obtain the smallest possible focal spot? (2 points)

(f) When a Gaussian beam is focused with a lens the diameter of the spot can be described by $2w_o \approx \frac{4}{\pi}\lambda F_{\#}$ and $F_{\#} = f/D$ is the so called F-number. f is the focal length and D the diameter of your lens, i.e. maximum beam width $\omega(z)$ that your optical components can allow.
You want to focus your HeNe laser (640 nm, beam waist of laser = 2mm) as tightly as possible. You have three lens kits available. The first contains optics with a diameter of 1 cm and focal lengths of 4 cm, 6 cm, 10 cm. The second kit contains optics with a diameter of 2 cm and focal length of 1 cm, 2 cm, and 5 cm. The last kit contains larger optics with a diameter of 5 cm and focal lengths of 15 cm, 20 cm, and 30 cm. How do you setup your system? Argue. Sketch. What is the resulting spot size? (6 points)

Cont.

3. Microscopy

   (a) Sketch the real and virtual image in the scheme below. (4 points)

   (b) With very thin samples the absorption of your specimen might not yield sufficient contrast. What other microscopy technique could you apply? Sketch and explain. (3 points)

4. Diffraction and Fourier Optics

   (a) A plane wave impinges on a slit as laid out in Fig. 3. How is the wave transformed after the slit? What do you observe on the screen? Lay out the interference condition in Fig. 3 and describe. (4 points)

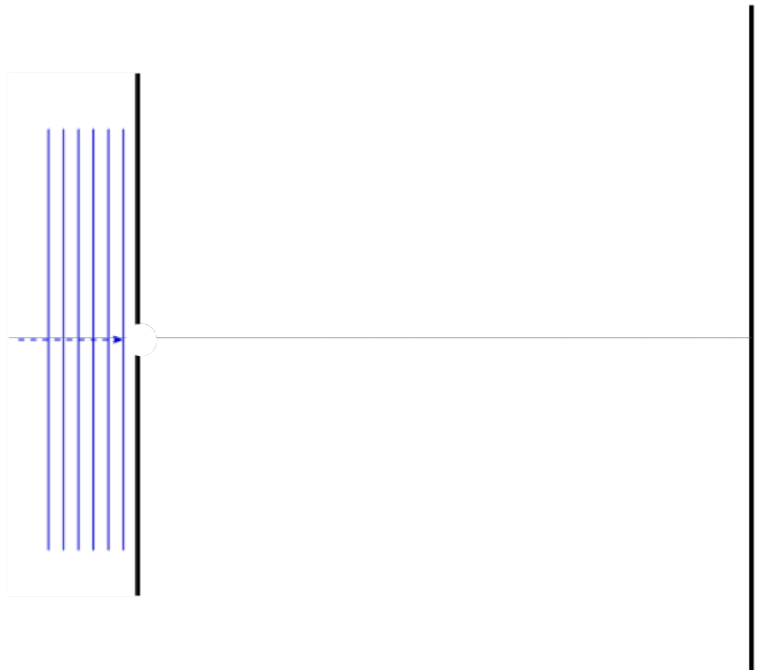

Figure 3: Diffraction at a slit

   (b) Sketch the interference pattern in the far field from (a) a single slit, (b) a double slit and (c) another double slit with different distance between the slits. Argue with the Convolution Theorem (6 points).

5. Optical trapping

   (a) Draw the force diagram of a transparent particle in an homogeneous light field. Which forces act in which direction. What is the net force. Use ray optics. (2 points)

   (b) Now the transparent particle is located in a focused Gaussian beam. Which forces act? Where does the particle go? Use ray optics to sketch and describe this optical trap. (6 points)

6. Spectroscopy

   (a) The vibrational modes of molecules represent a finger print of them. Measuring the vibrational modes is a common tool for chemical analysis. In which spectral regime are the vibrational modes located? (2 points)

   (b) Sketch and name the key concepts and key components of three different approaches to measure vibrational modes. Describe possible light sources, sketch the experimental setup, and describe the concept. (6 points)

7. Lasers

   (a) Describe (sketch) spontaneous emission, stimulated emission, and stimulated absorption. (3 points)

   (b) Describe (sketch) a two-level, three-level, and four-level system. Which of them is suitable for lasing? Why? What is the principle requirement for lasing? (3 points)

(c) The following pictures show energy states of three different lasers we discussed in class. For each of the three schematics (left, middle, right),

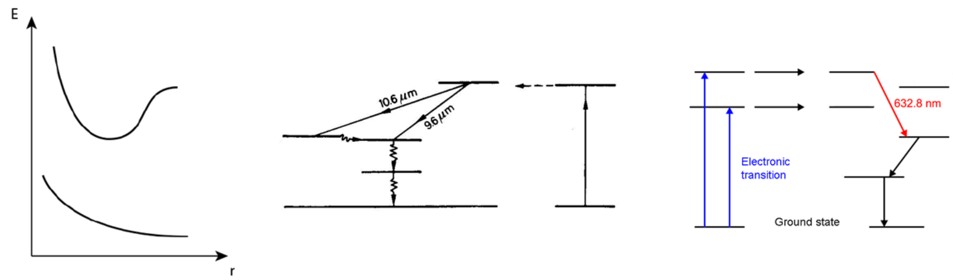

Figure 4: Lasing schemes

i. Name the participating actors (1.5 points)

ii. Describe the relevant transitions (1.5 points)

iii. Identify the number of levels in lasing process (1.5 points)

iv. Describe a possible use case (1.5 points)

(d) Use the following figure to show how an ultra short pulse can be amplified. Name the components in the black boxes, sketch beam profiles in the yellow boxes and sketch beam paths for red and blue wavelengths in the regions of the pink ovals. (7 points)

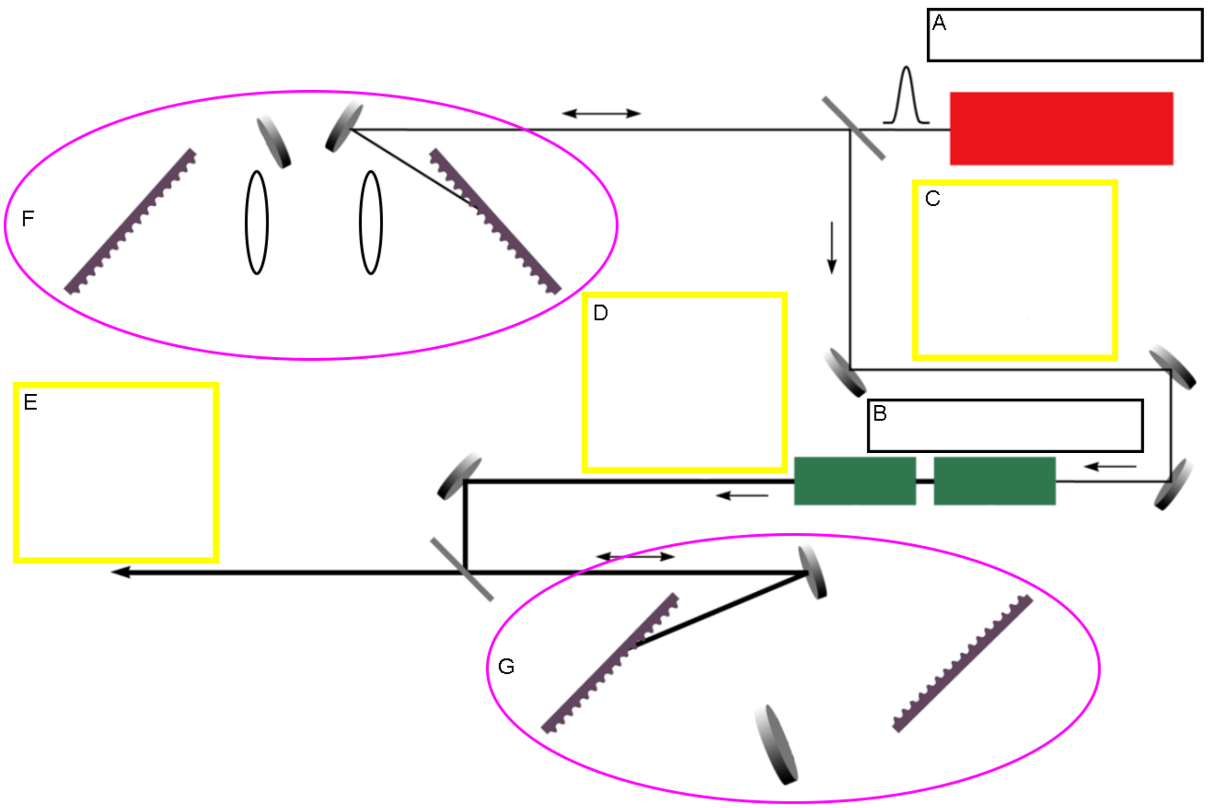

Figure 5: Ultrafast laser schematic

(e) You want to build a Laser with a 15 cm long cavity. The theoretical gain curve for your theoretical laser is 100% between 540 nm and 541 nm and 0 everywhere else (mathematical spoken it is a rectangular function). What is the frequency spacing of the system? Calculate how many longitudinal modes are present in the cavity. If you were to create an ultra short pulse, how short could the pulse be? Remember the minimum pulse duration from a laser is related to the number of modes it can support according to the following formula: $\Delta t = \frac{0.441}{N\Delta\nu}$ where $\Delta\nu$ is the frequency spacing, N is the number of modes and the $N\Delta\nu$ equals the bandwidth. (4 points)

(f) Now you do not want to create a short pulse, but rather select only one mode of the cavity for high-resolution spectroscopy experiments. Unfortunately, you do not have any gratings or coatings available, but you can get your hands on thin and flat pieces of glass. What can you do and how thin would the glass need to be able to select only one mode from your cavity? Where do you put it? (4 points)

(g) Sketch how the number of modes are affected by each component and develop throughout the system, similar as we discussed in class. (4 points)

8. Fundamental interactions in the x-ray regime

   (a) X-rays dominantly interact with core electrons. In Fig. 6, name the following processes (black boxes), define the emitted electrons (blue boxes) and label primary and secondary channels (yellow boxes) by filling in the boxes. (3 points)

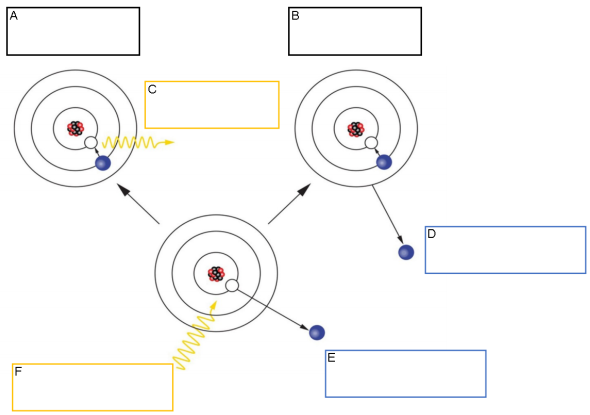

Figure 6: Fundamental x-ray processes

   (b) The atomic scattering factor is given by $f(Q, h\nu) = f^{\circ}(Q) + f'(h\nu) + if''(h\nu)$

   Which component is the atomic form factor? (1 point)

   What does the atomic form factor represent for $Q \longrightarrow 0$? (1 point)

   What components describe f1 and what is the meaning of f1 (1 point)?

   What components describe f2 and what is the meaning of f2 (1 point)?

   (c) Figure 7 shows the atomic scattering factors of an element from approx. 12 eV to 12 keV.

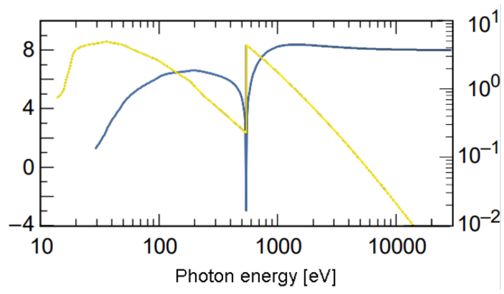

Figure 7: Atomic scattering factors

Which graph describes f1 and f2? Label in figure (1 points)

Which vertical axis describes f1 and f2? Label axis (1 points)

To which element does the graph belong? Explain your conclusion based on f1 and f2, respectively. Use table in appendix as reference. (2 points)

Appendix

Table 1-1. *Electron binding energies, in electron volts, for the elements in their natural forms.*

Element	K 1s	L_1 2s	L_2 $2p_{1/2}$	L_3 $2p_{3/2}$	M_1 3s	M_2 $3p_{1/2}$	M_3 $3p_{3/2}$	M_4 $3d_{3/2}$	M_5 $3d_{5/2}$	N_1 4s	N_2 $4p_{1/2}$
1 H	13.6										
2 He	24.6*										
3 Li	54.7*										
4 Be	111.5*										
5 B	188*										
6 C	284.2*										
7 N	409.9*	37.3*									
8 O	543.1*	41.6*									
9 F	696.7*										
10 Ne	870.2*	48.5*	21.7*	21.6*							
11 Na	1070.8†	63.5†	30.65	30.81							
12 Mg	1303.0†	88.7	49.78	49.50							
13 Al	1559.6	117.8	72.95	72.55							
14 Si	1839	149.7*b	99.82	99.42							
15 P	2145.5	189*	136*	135*							
16 S	2472	230.9	163.6*	162.5*							
17 Cl	2822.4	270*	202*	200*							
18 Ar	3205.9*	326.3*	250.6†	248.4*	29.3*	15.9*	15.7*				
19 K	3608.4*	378.6*	297.3*	294.6*	34.8*	18.3*	18.3*				
20 Ca	4038.5*	438.4†	349.7†	346.2†	44.3 †	25.4†	25.4†				
21 Sc	4492	498.0*	403.6*	398.7*	51.1*	28.3*	28.3*				
22 Ti	4966	560.9†	460.2†	453.8†	58.7†	32.6†	32.6†				

Figure 8: Electron binding energies for light elements

The End.