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Try to keep your books closed. Discuss with your fellow students to come to an answer. Show your work.

Name:____________________

1. Beam optics
   A Gaussian beam is described by

$$U(r) = A_o \frac{\omega_o}{\omega(z)} exp\left[\frac{-\rho^2}{\omega(z)^2}\right] exp[-ikz - ik\frac{\rho^2}{2R(z)} + i\xi(z)] \tag{1}$$

   and $\rho = \sqrt{x^2 + y^2}$

   (a) Describe the meaning of $\omega(z) = \omega_o\sqrt{1 + (\frac{z}{z_o})^2}$.

   Beam width as a function of z, depending on beam waist w_o and Rayleigh length z_o

   Describe the meaning of $R(z) = z[1 + (\frac{z}{z_o})^2]$.

   The curvature of wave front at a distance z

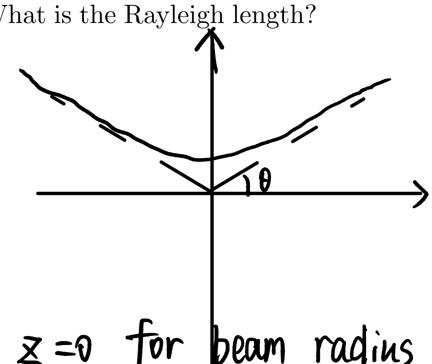
   Describe the meaning of $\xi(z) = tan^{-1}\frac{z}{z_o}$.

   Phase retardation of a Gaussian beam.

   Describe the meaning of $\omega_o = \sqrt{\frac{\lambda z_o}{\pi}}$.

   Beam waist, which is the beam radius at $z=0$. The tightest focus of the beam.

2. What is the correlation between beam optics and Ray optics for z → ∞? What is the Rayleigh length?



z small: Beam Optics $z \to \infty$: Ray Optics

$w(z) = w_0 \sqrt{1+\left(\frac{z}{z_0}\right)^2}$, $\lim_{z\to\infty} w(z) = w_0 \cdot \frac{z}{z_0}$

$\tan\theta = \frac{w_0}{z_0}$

Rayleigh length z_0 is the distance that takes from $z=0$ for beam radius to increase from w_0 to $\sqrt{2}\,w_0$; beam cross section to increase from πw_0^2 to $2\pi w_0^2$

3. The confocal parameter of a Gaussian beam relates the Rayleigh length and beam width. It is defined as twice the Rayleigh length and is given by $2z_o = \frac{2\pi\omega_o^2}{\lambda}$.
What is the relationship between depth of focus and focal spot size? How do you obtain the smallest possible focal spot?

Depth of focus: $d = 2z_0 = \frac{2\pi}{\lambda} w_0^2$

proportional to w_0^2 and inverse proportional to λ.

Obtain smallest possible focal spot:

According to next question (Q.4), focal size $2w_0 \approx \frac{4}{\pi}\lambda\frac{f}{D}$

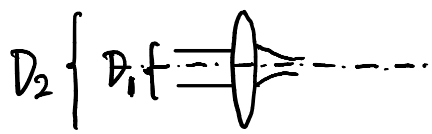
So $\lambda\downarrow$ $f\downarrow$ $D\uparrow$ for a smaller focal size.

(Optional) Remark: focal point size is limited by diffraction, so the minimum of $2w_0$ is on the scale of λ or $\min\{2w_0\} \sim \lambda$, the approximation equation $2w_0 \approx \frac{4}{\pi}\lambda\frac{f}{D}$ will fail when $2w_0$ is comparable to λ.
Search "Airy Disk Radius" if you are interested.

4. When a Gaussian beam is focussed with a lens the diameter of the spot can be described by $2w_o \approx \frac{4}{\pi}\lambda F_{\#}$ and $F_{\#} = f/D$ is the so called F-number. f is the focal length and D the diameter of your lens, i.e. maximum beam width $\omega(z)$ that your optical components can allow.
You want to focus your HeNe laser (640 nm, beam waist of laser = 1mm) as tightly as possible.
You have two lens kits available. One contains optics with a diameter of 1 cm and focal lengths of 2 cm, 5 cm, 10 cm. The other kit contains larger optics with a diameter of 5 cm and focal lengths of 10 cm, 20 cm, and 50 cm.

How do you setup your system? Argue. Sketch.

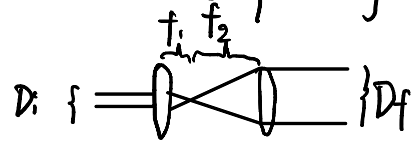
Remark: D in equation $2w_0 \approx \frac{4}{\pi}\lambda\frac{f}{D}$ actually represents "Effective" Lens Diameter instead of Lens Diameter.



focal spot size in this case $= \frac{4}{\pi}\lambda\frac{f}{D_1}$ instead of $\frac{4}{\pi}\lambda\frac{f}{D_2}$.

Thus, the idea is to $\min\{\frac{f}{D}\}$. $\Rightarrow$ ① f = 2cm, D = 1cm ② f = 10cm, D = 5cm

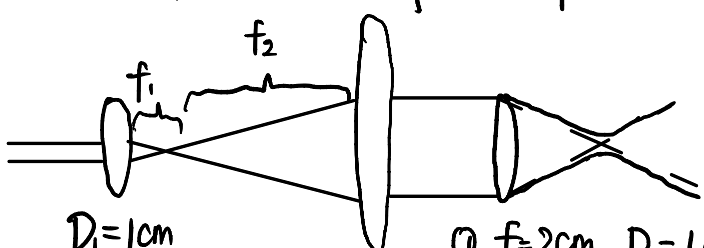
Original beam waist = 1mm, we need to use following optical lens group to extend the beam waist:



What is the resulting spot size? According to geometry we have $\frac{D_f}{D_i} = \frac{f_2}{f_1}$

for case ①, we need final $D_f = 1\text{cm}$ and initial $D_i = 2w_0 = 2\text{mm}$

$\therefore \frac{D_f}{D_i} = 5 = \frac{f_2}{f_1}$



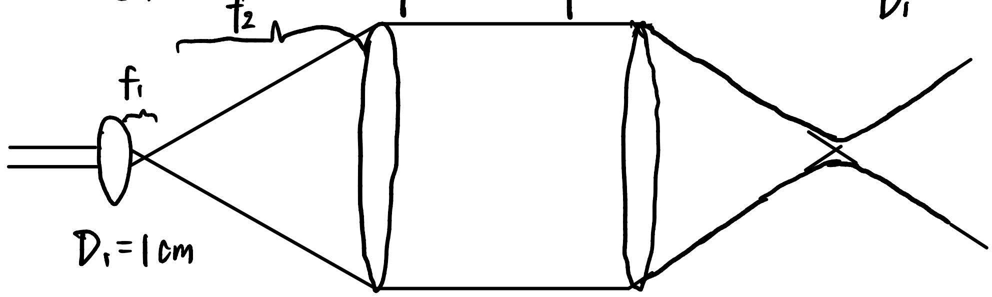
$D_1 = 1\text{cm}$ ① f=2cm, D=1cm.

$D_2 = 5\text{cm}$

two possible (f_1, f_2) sets: (1.1) $f_1 = 2\text{cm}$ $f_2 = 10\text{cm}$ (1.2) $f_1 = 10\text{cm}$, $f_2 = 50\text{cm}$

(1.1) uses one Lens twice, so only $(f_1, f_2) = (10\text{cm}, 50\text{cm})$

for case ②, we need final $D_f = 5\text{cm}$ with $\frac{D_f}{D_i} = 50 = \frac{f_2}{f_1}$



$D_1 = 1\text{cm}$

$D_2 = 5\text{cm}$ ② f=10cm, D=5cm

only 1 possible (f_1, f_2) set: ② $f_1 = 2\text{cm}$, $f_2 = 100\text{cm}$.

Focal spot size $2w_0 = \frac{4}{\pi}\lambda \cdot 2 \approx 1.63\,\mu\text{m}$.

The End.