

Upload your finished worksheet as a single pdf file on moodle before the next class session in order to get participation credit.
 Try to keep your books closed. Discuss with your fellow students to come to an answer.
 Show your work.

Name: _____

On moodle you will find some example images, work with them in the following order

1. Test image line
2. Test image cross
3. Test image ladder
4. Test image lattice

For each of the test images, with a software tool such as imagej (see moodle)

1.

1. Predict the main feature of the FFT and sketch / describe. (No fixed answer)

Mathematics is optional: Line: $f(x, y) = A \delta(y) \text{StepFunc}(|x| - x_0)$



$$FT = A \int_x \int_y \delta(y) \text{StepFunc}(|x| - x_0) e^{-ik_x x} e^{-iky y} dx dy$$

$$= A \int \text{StepFunc}(|x| - x_0) e^{-ik_x x} dx = A \int_{x=-x_0}^{x=x_0} e^{-ik_x x} dx = \frac{A}{-ik_x} e^{-ik_x x} \Big|_{x=-x_0}^{x=x_0} = \frac{A}{k_x} \sin x_0 k_x$$

Hence: Peak at $k_x = 0$ (center) and wiggle as a function of k_x (Basically single slit diffraction)

Cross: Overlap of two lines $FT = \frac{A}{k_x} \sin k_x x_0 + \frac{A}{k_y} \sin k_y y_0$

Hence: Peak at $k_x = 0$ and $k_y = 0$, wiggle along k_x direction with period $\frac{2\pi}{x_0}$ and wiggle along k_y direction with period $2\pi/y_0$.

2. Fourier transform the image and compare with your prediction.

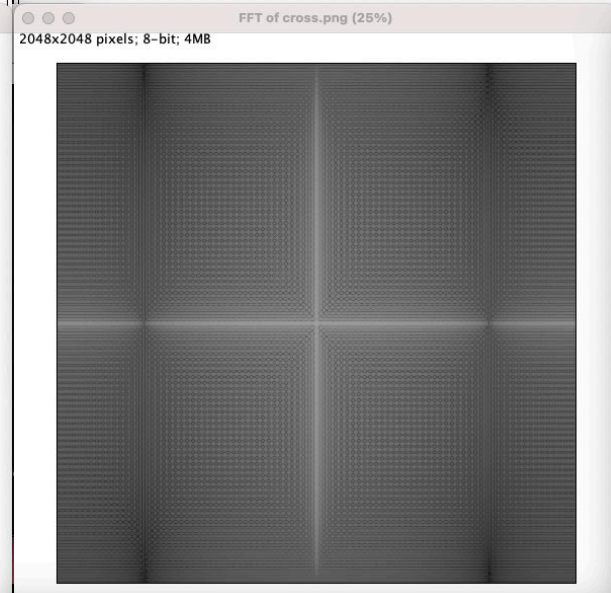
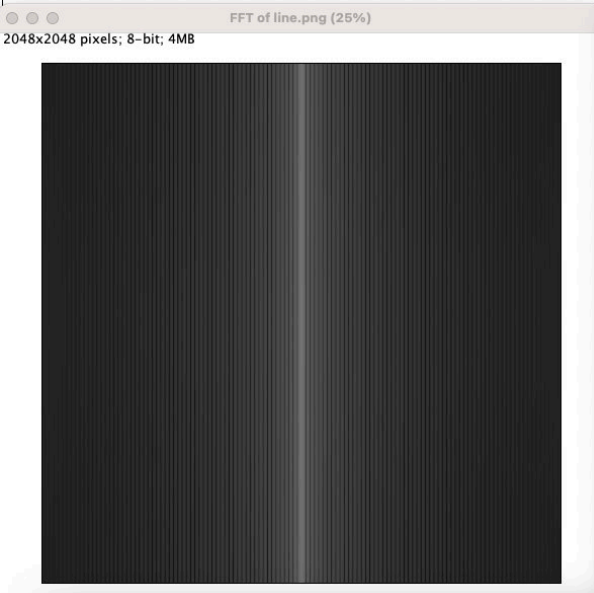
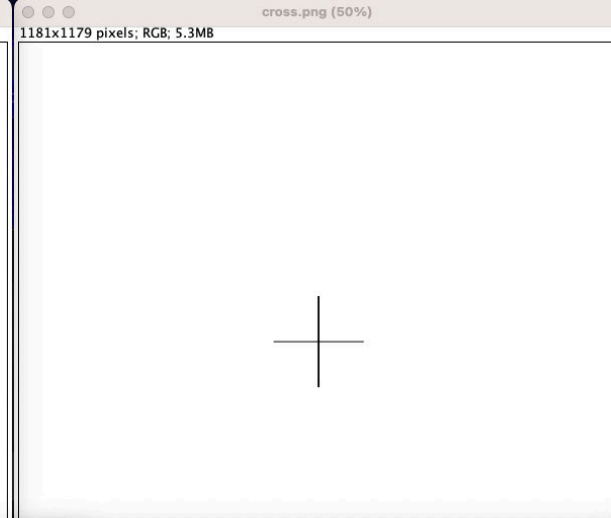
Ladder: $FT = \underbrace{\frac{A}{k_y} \sin y_0 k_y}_{\text{single slit diffraction}} + \underbrace{\frac{A}{k_x} \sin x_0 k_x (e^{-i2ky y_0} + e^{-iky y_0} + 1 + e^{iky y_0} + e^{i2ky y_0})}_{\text{5 slits interference}}$

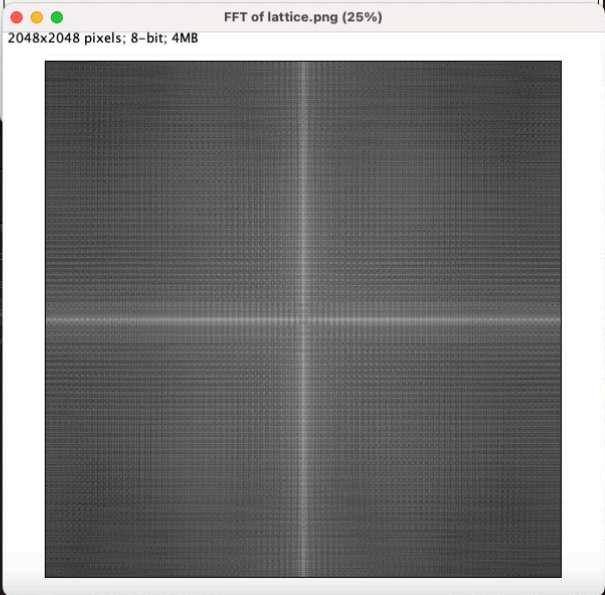
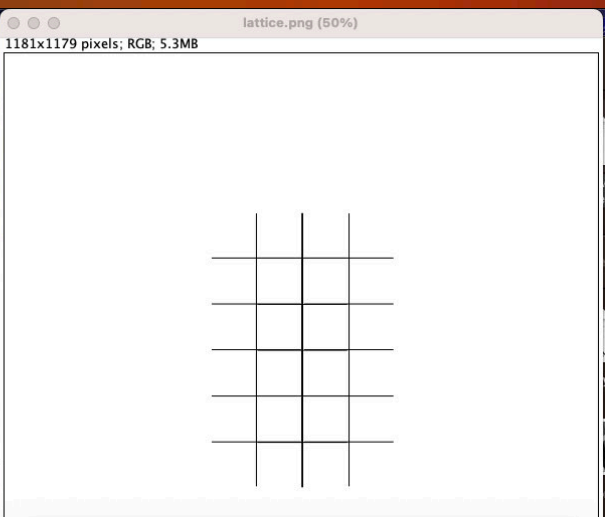
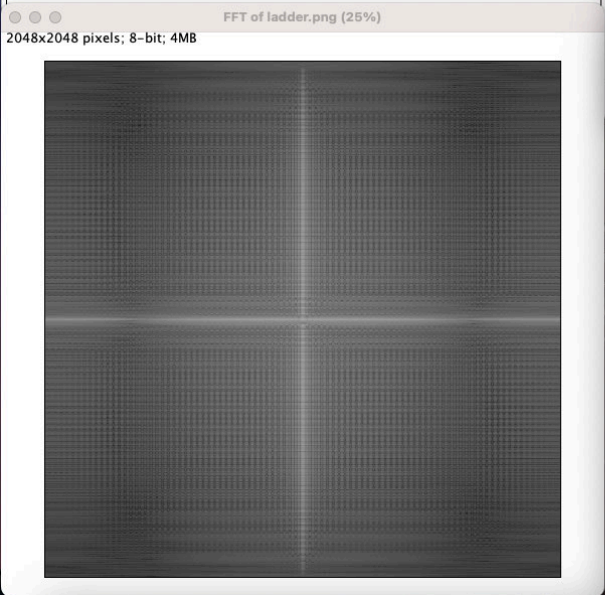
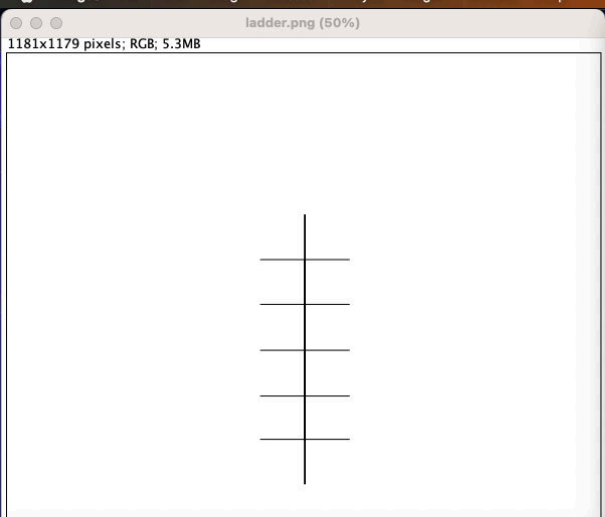
$$= \frac{A}{k_y} \sin y_0 k_y + \frac{A}{k_x} \sin x_0 k_x (1 + 2\cos ky y_0 + 2\cos 2ky y_0)$$

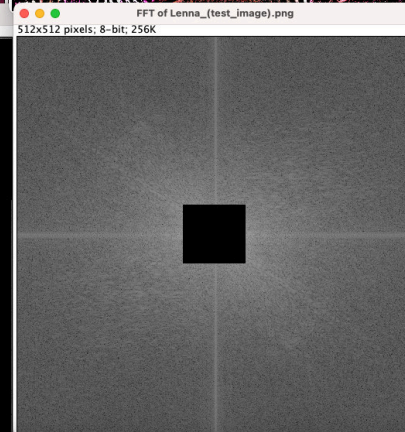
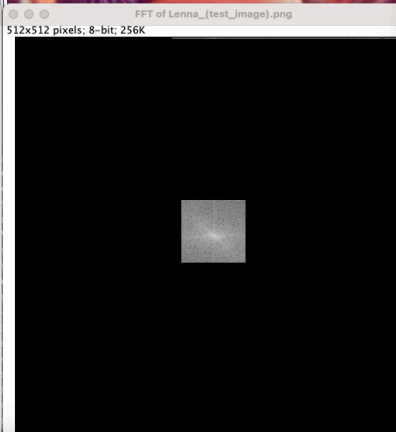
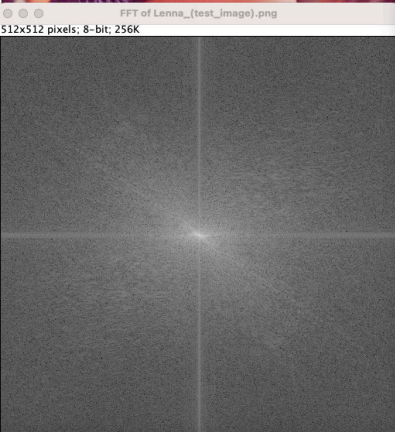
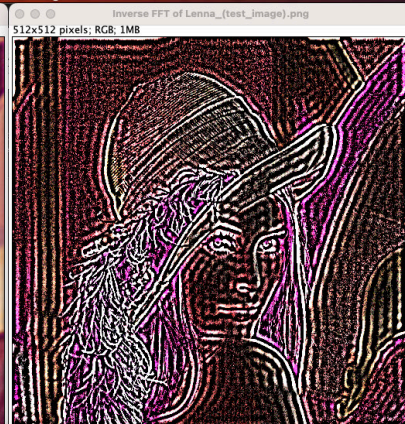
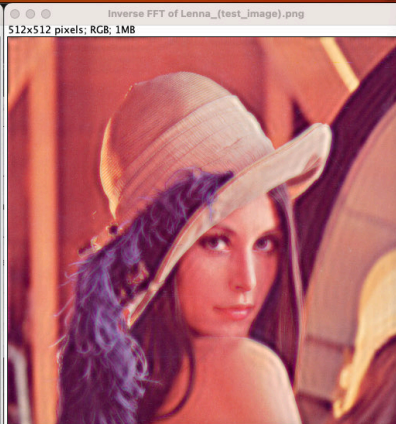
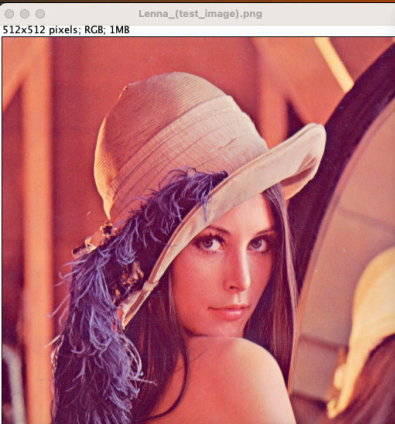
Wiggle along k_x direction with period $\frac{2\pi}{x_0}$. Along k_y direction: Beat.

(Still get peak at $k_x = 0$ & $k_y = 0$ so still a clear cross of FFT)

Grid: Beat in both k_x and k_y directions. (Still get peak at $k_x = 0$ & $k_y = 0$, so still a cross in F space)







3. Choose a spatial component in your original image and mask the respective frequency components in the Fourier transform image. Explain your choice of mask.

I choose Lena standard test picture to show what will happen if we filter out the low frequency information (right) or we filter out the high frequency information (middle)

4. Perform inverse Fourier transform on your masked Fourier transform image. Compare with your expectations.

If we filter out high frequency information (Middle):

① Loss of sharp edge. ② loss of details (blurred) ③ Reduced noise (good aspect).

But actually in this picture and the FFT, I don't introduce noise, so we cannot conclude this based on the example I showed. But it's true, high frequency components are generally weak and will be more easily affected by noise.

④ Gentle gradients or smoothness get reserved (continuous color)

If we filter out low frequency information (Right):

① Emphasize of edges & details ② Higher contrast ③ Loss of gradual changes (color change ^{is} very fast) ④ Hollow or ghost appearance

⑤ Possible higher noise.