Take-home message

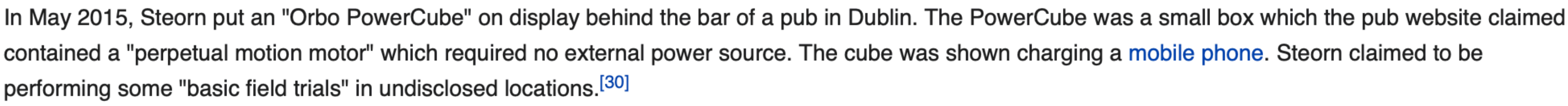
Reversibility is important to a cell as it minimises energy dissipated

All natural processes are irreversible (*but some still try to invent perpetual motion machines…Steorn 2009*)

A cell uses phase transitions to make changes with minimum energy consumption: a small change in a parameter can create a huge change in the state of a system

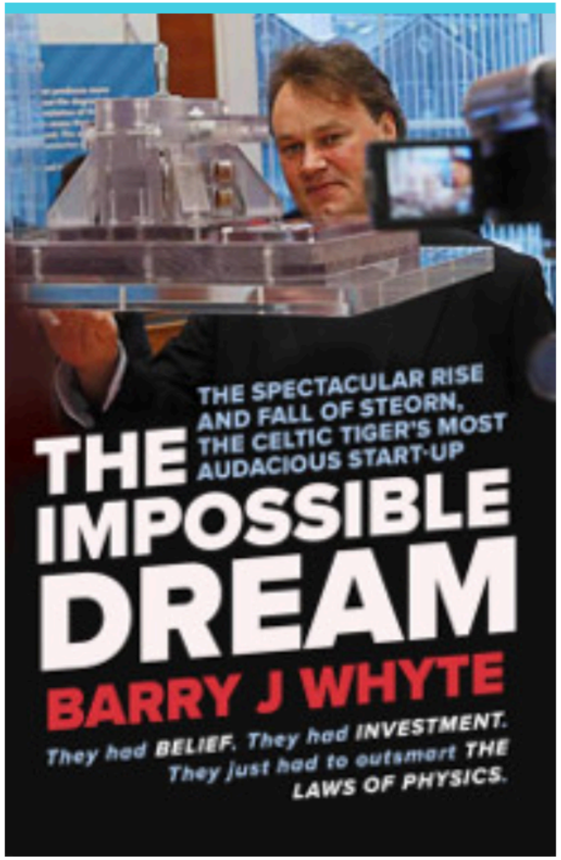
Driving force comes from the surroundings, i.e, equipartition theorem… again! I*f you kinetically poke a system you change a few d.o.f* but if you change the temperature you change all of them

Update 2015 - they're still going …



In May 2015, Steorn put an "Orbo PowerCube" on display behind the bar of a pub in Dublin. The PowerCube was a small box which the pub website claimed contained a "perpetual motion motor" which required no external power source. The cube was shown charging a mobile phone. Steorn claimed to be performing some "basic field trials" in undisclosed locations.[30]

1 Claim something impossible
2 Fail
3 Write book about it…. get rich



The Impossible Dream: book brings new account of Steorn's journey

May 18, 2020 Michael Ferrier

Irish Business Post journalist Barry J Whyte, who wrote several articles covering Steorn over the years, has published a book detailing the whole saga. The Impossible Dream: The spectacular rise and fall of Steorn, one of the Celtic Tiger's most audacious start-ups is available now through Amazon for Kindle, and will be released on paperback in September. I […]

Quote of the day:

They had belief. They had investment. They just had to outsmart the Laws of Physics

http://dispatchesfromthefuture.com

Why does this scam still work?

Surely everyone has heard about conservation of energy?

Thermodynamics is unlike other theories in physics - it’s an extrapolation from experience not a fundamental theory.

There might always an exception … cue magnetic fields, gyroscopes, …

Compare: Quantum electrodynamics

Spectral lines in atoms… Rutherford’s experiment of firing He ions at Au foil and some bounced back… nucleus and electrons

electrons … TVs …

Schrodinger equation describes behaviour of atoms and molecules and (so far) all its predictions have been demonstrated

Thermodynamics claims to describe every physical process
but we cannot check every process, maybe it doesn’t apply *here?*

But journals get fooled too …

Here’s an MD simulation that seemed to have found perpetual motion in ion flow through a channel….

A charge-driven molecular water pump

XIAOJING GONG[1,2], JINGYUAN LI[3], HANGJUN LU[1,2,4], RONGZHENG WAN[1], JICHEN LI[5], JUN HU[1,6]*
AND HAIPING FANG[1]*

It was a mistake, of course, but you can’t tell that by looking at Nature’s website

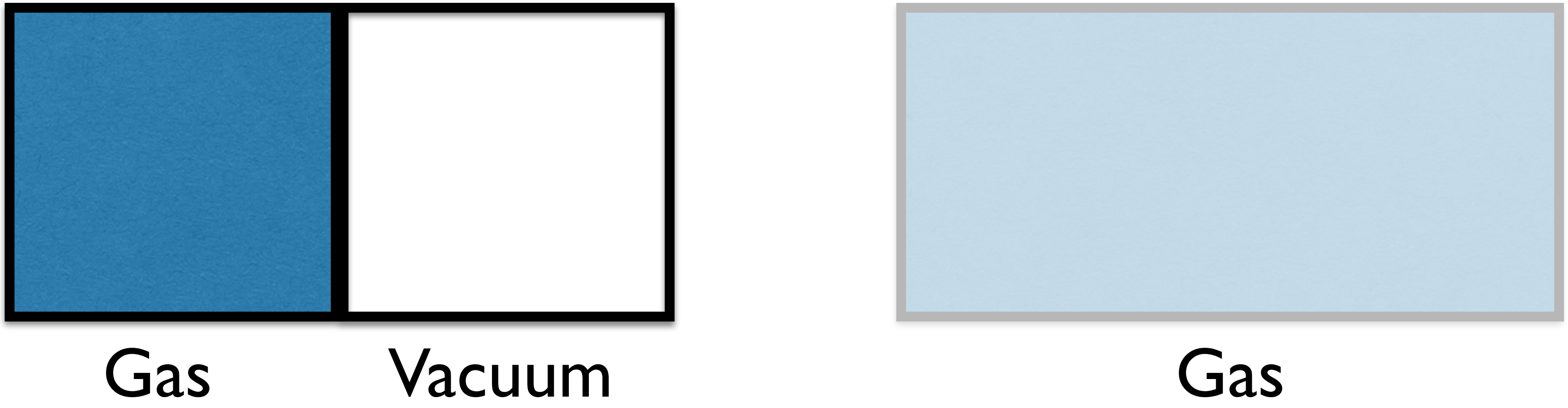
https://www.nature.com/articles/nnano.2007.320

Static charges cannot drive a continuous flow of water molecules through a carbon nanotube

NATURE NANOTECHNOLOGY | VOL 5 | AUGUST 2010 | www.nature.com/naturenanotechnology

Wong-Ekkabut et al.

Entropy



Experimentally, an isolated system (constant N,V, E) has a constant internal energy and, in equilibrium, no further spontaneous changes occur.

When a gas in an isolated box (adiabatic walls) is allowed to expand, it fills the available space.

Why does it expand?

It is overwhelmingly likely that the random thermal motion of the molecules drives them to occupy the newly-available volume. Energy is transferred between all the dof of the system until no further spontaneous changes occur, this state has the maximum ***entropy***.

2nd Law of Thermodynamics

2nd Law says that entropy always increases: it restricts the direction of spontaneous physical changes so that heat always flows from higher to lower temperatures.

Whatever can happen given the system's energy/constraints **will** happen

Whatever can happen in the most ways is what we **observe** to happen

Kelvin-Planck: "It is impossible to devise an engine which, working in a cycle, shall produce no effect other than the extraction of heat from a reservoir and the performance of an equal amount of mechanical work."

Clausius: "It is impossible to devise an engine which, working in a cycle, shall produce no effect other than the transfer of heat from a colder to a hotter body."

Free Energy

An **isolated** system has a constant internal energy (that's what *isolated* means!)

What happens if a system is not isolated?

It is found experimentally that there is still a quantity related to the internal energy that is minimised in equilibrium - **Free Energy** - whose definition depends on the boundary. There are as many types of free energy as there are boundary conditions on a system.

Free energies

Consider a system of N particles in various kinds of "box":

Isolated system: U = Internal energy: N, V, U are constant

Isothermal system: $F = U - TS$ = Helmholtz free energy: N, V, T are constant

Isothermal, isobaric system: $G = U + PV - TS$ = Gibbs free energy: N, P, T are constant

Open system with a fixed, diathermal wall: $\Omega = U - TS + \mu N$ = Grand potential:

Chemical potential μ, V, T are constant

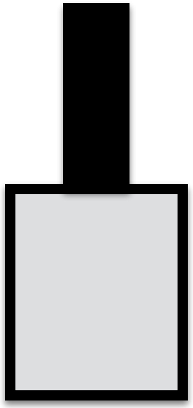
In any problem, the recipe is to identify the boundary conditions, construct an appropriate free energy, minimise it, and that gives the equilibrium state. If, as a result of changing a parameter like T, the system takes up a new phase, we have a phase transition.

Phases and phase transitions

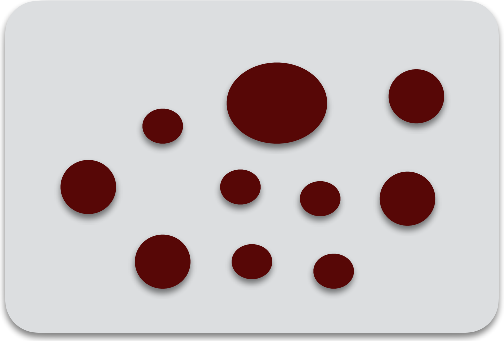
" A ***phase*** is an homogeneous region of matter bounded by surfaces across which its properties change discontinuously (e.g., solid, liquid or gas) "

" An ***equilibrium phase*** has no internal gradients nor between it and its surroundings "

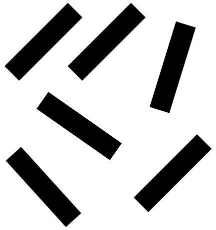
Water



Gas in isolated box with a piston



Solution of micelles



Rodlike liq. crys. / TMV in solution

"Homogeneous" contains an **implicit** average over a region of space, e.g., micelles in water.
NB. Molecular shape may or may not be relevant depending on concentration, cp. ideal gas.

There are distinct types of bounding surface (or wall):

Adiabatic = neither heat nor matter transport across the surface
Diathermal = heat transport but not matter
Open = heat and matter transport

Entropic spring

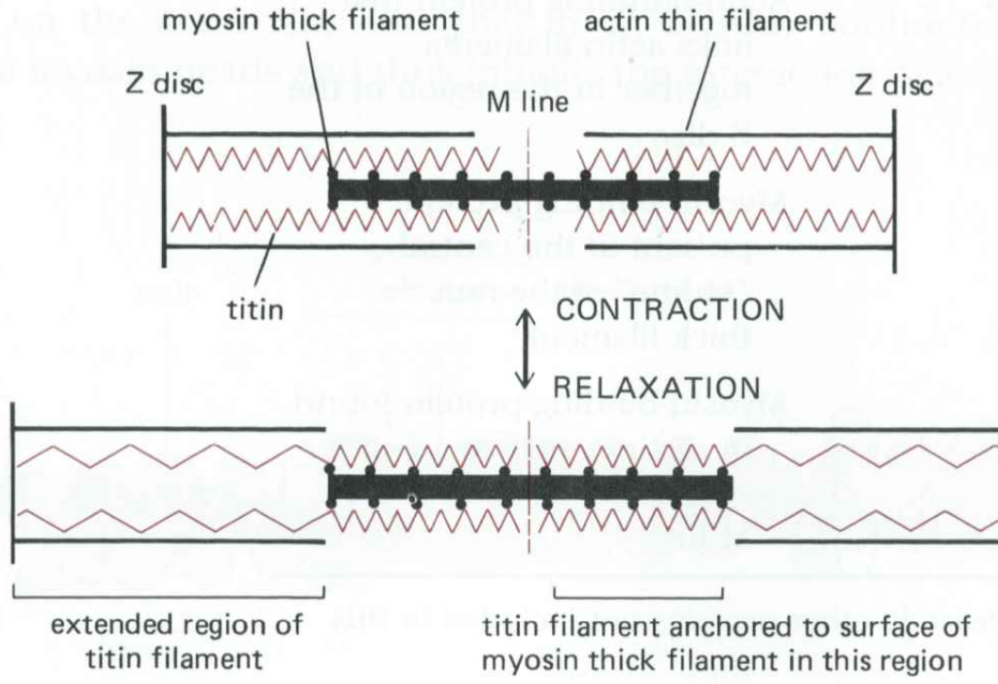


Fig.. 11-21 Mol. Biol. Cell, B Alberts et al.

> “ Many biological molecules have a defined mechanical function. For these molecules, their resistance to unfolding in response to an applied mechanical force is of critical physiological importance. Titin is the protein responsible for passive elasticity in skeletal muscle, where it functions as a molecular spring… ”

C. Bustamente et al., Ann. Rev. Biochem. 73:705-748 (2004)

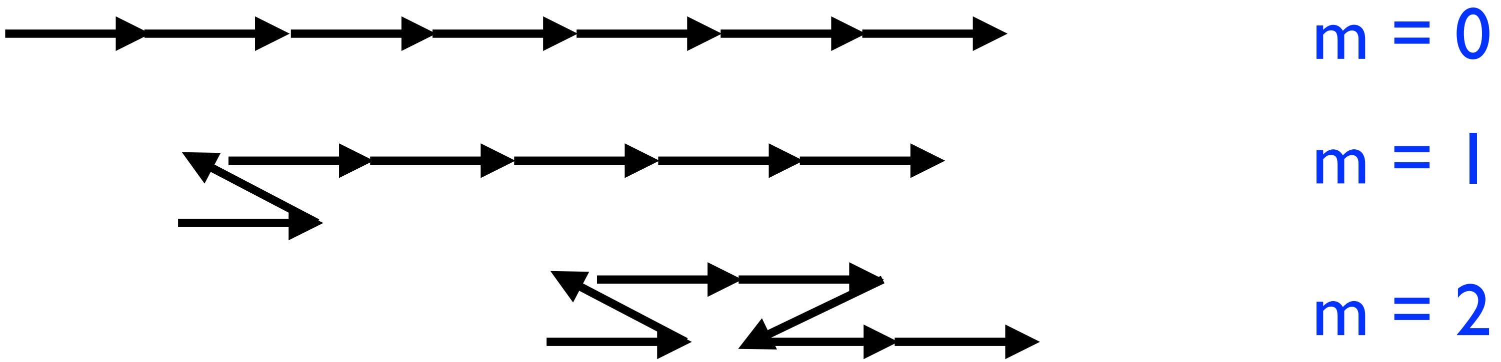
What is an entropic spring?

A 1d polymeric molecule whose resistance to an applied force depends on its length and whose conformations are accessible to thermal excitation.

Phase transition of entropic spring

A polymeric molecule will stretch and shrink as its monomers fluctuate in space due to thermal motion. In equilibrium, deviations from the mean end-to-end length will cost free energy.

Consider a 1d polymer of N identical monomers (= bonds) in which each bond can only point to the right (+X) or left (-X) independently of all others with no energy cost.



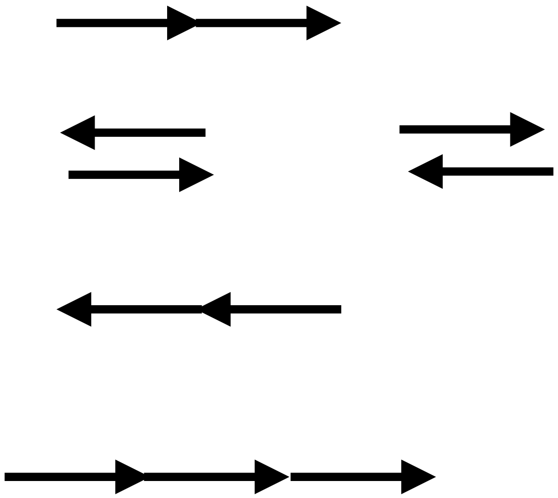
If there are m left-pointing bonds out of N bonds, there are $\Omega(m, N) = N! / m! (N-m)!$ conformations. The free energy of the chain is $F = E - TS$, but $E = 0$, and entropy is:

$$S(m, N) = k_B \ln(N! / m! (N-m)!)$$

NB Stirling's approximation:
$\ln N! \sim N \ln N - N$

What is the free energy F(L), where L = end-end length? (Required derivation)

Example conformations



N = 2, m = 0

N = 2, m = 1

N = 2, m = 2

N = 3, m = 0

N = 3, m = 1

N = 3, m = 2

N = 3, m = 3

Draw all possible configurations for

N = 3, m = 1, 2, 3 and verify the formula for $\Omega(m, N)$.

If you want - draw the configurations for N = 4

Collapse of an entropic spring

The free energy as a function of its end-to-end length is:

$$F(L/L_0) = k_BT.(\ L_0/a\ ).f(L/L_0)$$

where $f(x) = 1/2(\ (1 - x).\ln(1 - x) + (1 + x).\ln(1 + x) - 2.\ln 2\ )$

and L = end-to-end length of the polymer, L_0 = maximum length of polymer (all bonds parallel).
It exerts a force on its two ends that increases as $x = L/L_0$ moves away from 0.
(Graph f(x) what curve does it remind you of?)

Note.

1) Free energy increases with temperature, i.e., it is an *entropic spring*, with spring constant ~ k_BT/a
2) Free energy increases with polymer contour length L_0
3) Free energy is **maximised** for fully stretched chain ($L = +/- L_0$)

The most likely conformation **minimises** the free energy wrt length, and gives the force exerted by the polymer (the force-extension curve):

$$\text{Force} = -\ dF(L/L_0)/dL = -dF(x)/dx\ .\ dx/dL = (k_BT/2a)\ .\ \ln(\ (1 - x)/(1 + x)\ )$$

which has a minimum at x = 0 or L = 0. Hence, the chain **collapses** to zero length in the absence of an applied tension.

Relation of entropic spring to phantom chain

What is the mean-square end-to-end length of the entropic spring in equilibrium?

Starting from: $F(x) = k_BT.(L_0/a)((1 - x).\ln(1 - x) + (1 + x).\ln(1 + x) - 2.\ln2)$

where $x = L/L0$, and using: $\ln(1 + x) \sim x - x^2/2 + x^3/3 + \ldots$

We get

$$F(x) \sim k_BT.(L_0 / 2a)(x^2 - 2 \ln 2)$$

Ignoring the constant term, we use the Equipartition theorem to get

$< F / k_BT > = (L_0 / 2a) < x^2 > = 1/2$

or $< x^2 > = a / L_0$, but $x = L/L_0$ and $L_0 = Na$, so

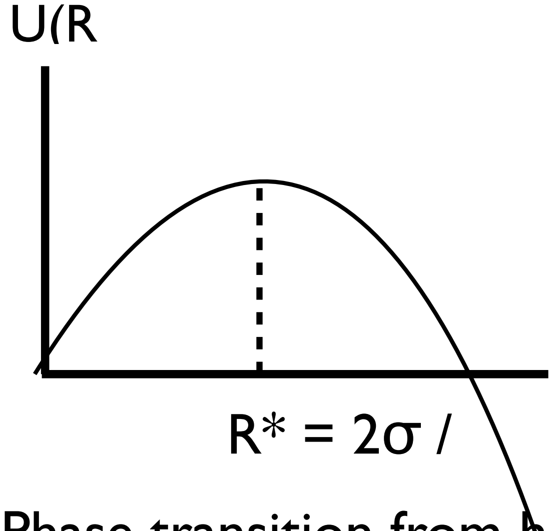
$$< L^2 > = Na^2$$

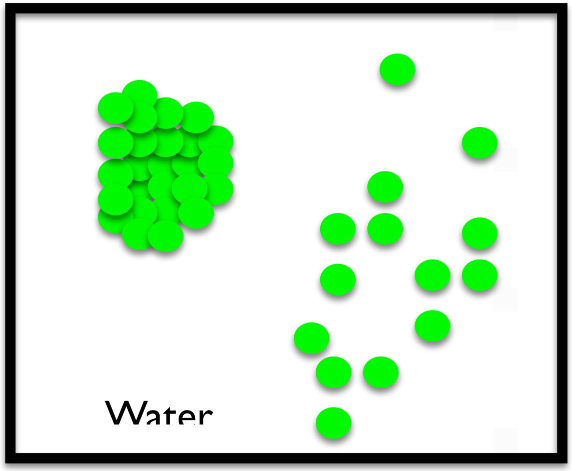
and the entropic spring has the same end-to-end length dependence on its contour length (N) as a phantom chain.

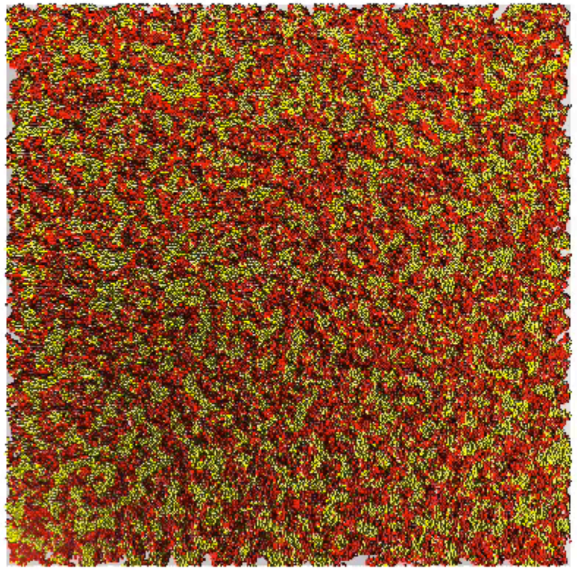
Oil/water phase separation

Recall from Lecture 3, when oil molecules aggregate into a sphere, energy is lowered by a volume term (γ), but repulsive surface energy is proportional to the surface area (σ):

$$U(R) \sim 4\pi\sigma R^2 - 4/3\pi\gamma R^3$$







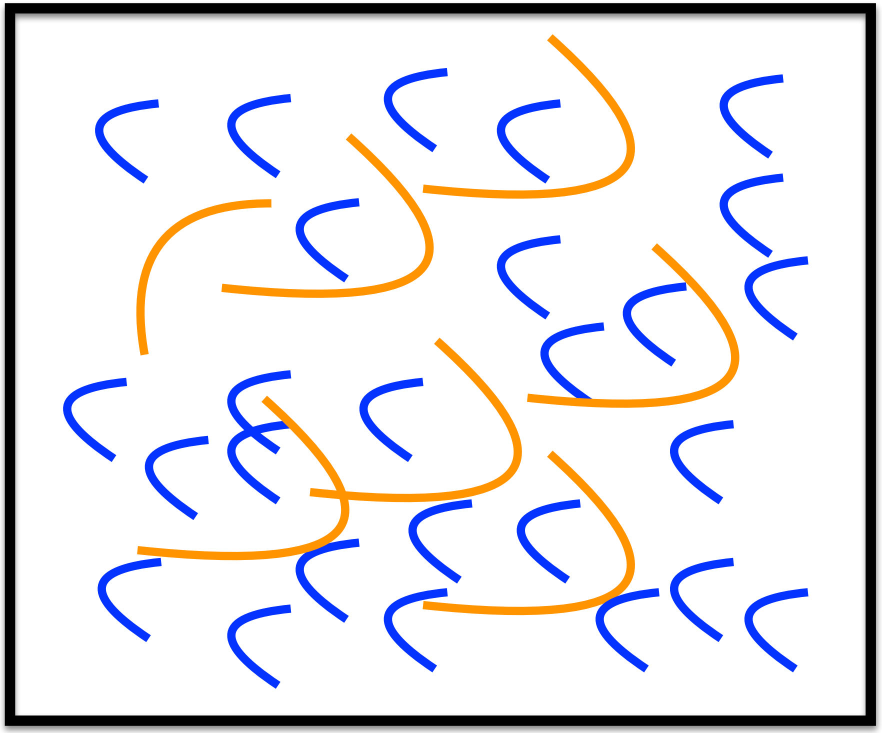
Phase transition from homogeneous phase to macroscopically separated oil and water phases. No entropy term? Surface tension keeps drops spherical so no fluctuations in shape.

What changes if we consider polymers instead of small oil/water particles?

Polymer phase separation

Consider a mixture of a polymer in a solvent (which may be another polymer):

Do they mix? Do they phase separate?



We can construct a thermodynamic theory of the mixture that predicts a phase separation as as function of the polymer/solvent interactions.

Assume: composition, V, T are constant.

Helmholtz free energy is:

$$F = U - TS$$

U ~ energetic interaction between polymers
S ~ number of configurations of polymers/solvent

How do we find U and S? Just as the entropic spring's behaviour was dominated by the largest number of microstates (bond flips), the polymer mixture's behaviour is dominated by the most likely "number" of interactions

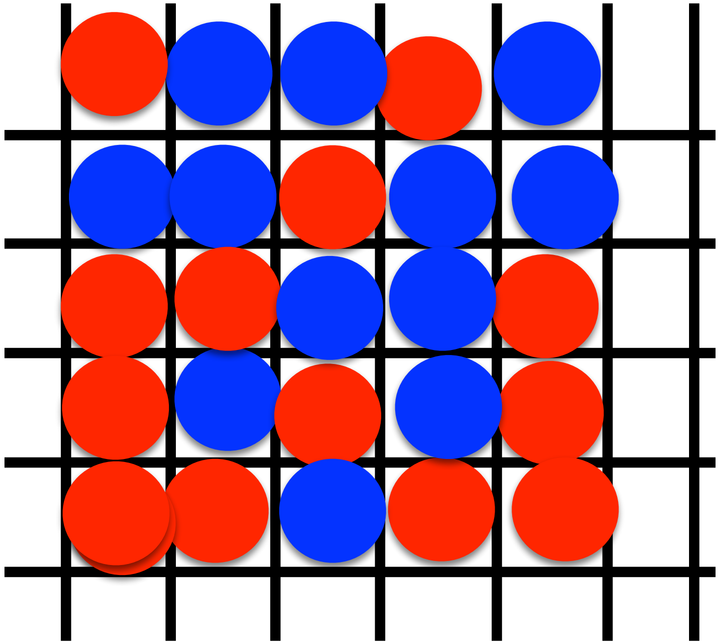
pre-Flory Huggins theory

Consider a lattice with N sites that is filled with **monomers** such that

n_1 monomers of type 1 🔵

n_2 monomers of type 2 🔴

and $N = n_1 + n_2$



How many ways **$\Omega(n_1, n_2, N)$** are there of placing $n1$ (blue) monomers and $n2$ (red) monomers on the lattice?

Express the result in terms of the volume fractions $\varphi_1 = n_1 / N$ and $\varphi_2 = n_2 / N$

$$\ln \boldsymbol{\Omega}(\varphi_1, \varphi_2) = ?$$

Flory-Huggins theory of polymer mixtures

$\ln \boldsymbol{\Omega}(\varphi_1, \varphi_2)/N = -\varphi_1 \ln \varphi_1 - \varphi_2 \ln \varphi_2.$ (for monomers)

If species 1 is a polymer with N monomers, and volume fraction φ_1
and species 2 is a monomeric solvent with volume fraction φ_2

such that $\varphi_2 = 1 - \varphi_1$, the result is altered somewhat.

But the essence of the argument is that the connectivity of the polymers is *ignored* when placing their monomers on the lattice, and the entropy of the polymers is just reduced by a factor 1/N

U = energetic interactions among monomers and solvent proportional to their volume fractions $\sim \varphi (1 - \varphi)$

S = translational entropy of the polymers and solvent

Flory-Huggins theory of polymer in solvent

Why this form?

$$\beta F = (\varphi/N)\ln(\varphi) + (1-\varphi)\ln(1-\varphi) + \chi\,\varphi\,(1-\varphi)$$

polymer entropy

solvent entropy

energy favours separating if $\chi > 0$

entropy favours mixing

Plot F for several χ

Notes

1) The first term is usually negligible because polymers have N >> 1 (PEG has a molecular weight ~ 10,000 Da or more)
2) Polymers have very low translational entropy compared to solvent
3) Energetic term: every monomer in the polymers interacts with the solvent, a small, repulsive χ increases the energetic term very rapidly with polymer length
4) Temperature dependence of $\chi(T) \sim A + B/T$ and values of A, B are tabulated for different polymer mixtures.

Assumptions of Flory Huggins Theory

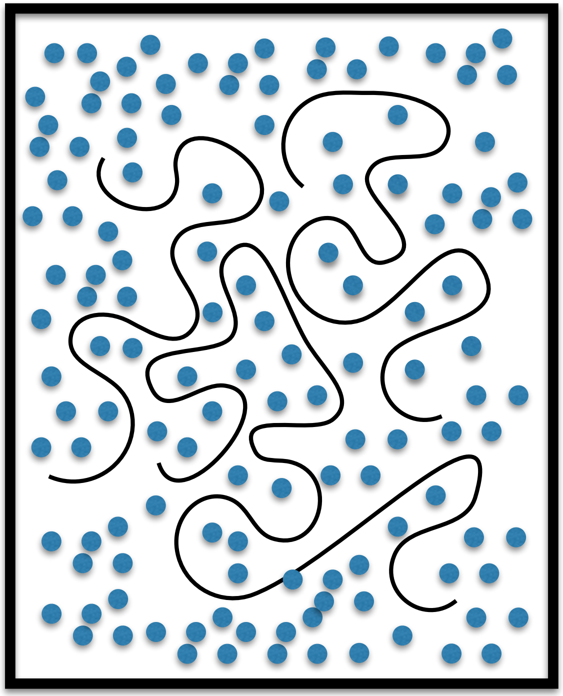
A) Polymers move and explore all available conformations
⇒ Mixing increases their entropy

(cp. lots of entropic springs pressing on each other)

B) Only translational entropy is considered and this is greatly reduced for long polymers because of $(\varphi/N)\ln(\varphi)$ term

C) All monomers interact equally on average (Mean Field Theory)
⇒ a small difference in χ parameter is magnified by polymer

lengths



$$\beta F = (\varphi_1/N_1)\ln(\varphi_1) + (\varphi_2/N_2)\ln(1 - \varphi_2) + \chi\,\varphi_1\varphi_2$$

e.g., $N_1 = N$
$N_2 = 1$ (water)

where $\beta = 1/k_BT$ and F is the free energy.

N_i = No of monomers in polymers of type i = 1, 2

φ_i = Volume fraction of type i and $\varphi_1 + \varphi_2 = 1$

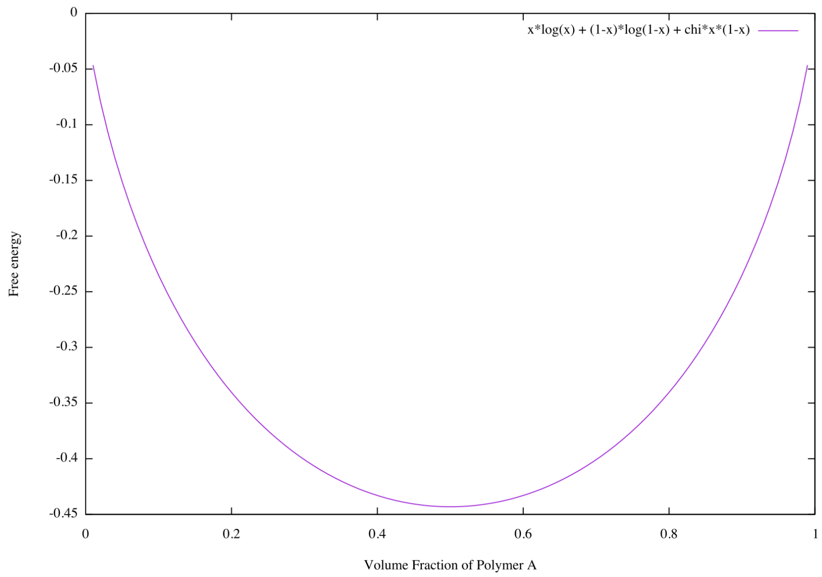
χ = "Mixing" parameter ~ how much the polymers dislike each other

$$\beta F = (\varphi_1/N_1)\ \ln(\varphi_1) + (\varphi_2/N_2)\ \ln(\varphi_2\ ) + \chi\ \varphi_1\ \varphi_2$$

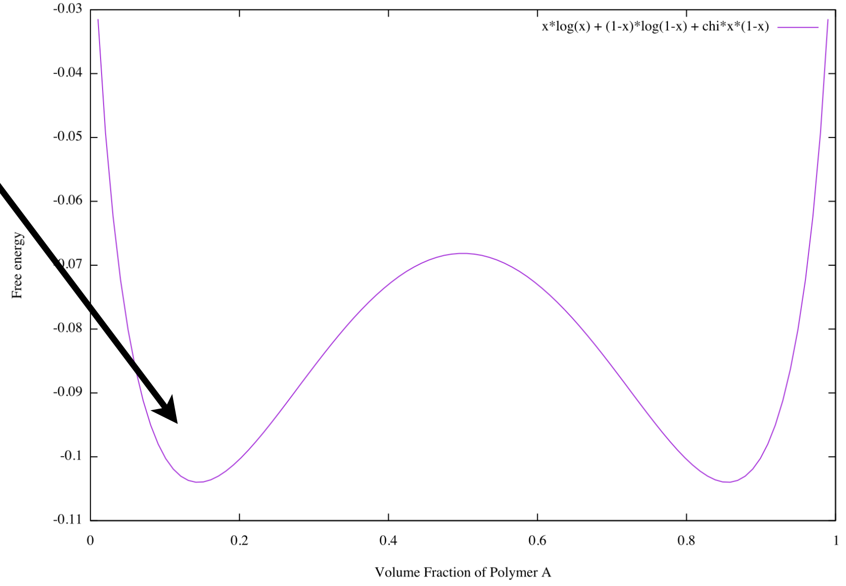
Minimising βF with respect to φ_A, φ_B predicts phase separation for parameters satisfying

$$\chi > \chi_C = 0.5\ (1/\mathrm{sqrt}(N_A) + 1/\mathrm{sqrt}(N_B)\ )^2$$

The condition $\partial F/\partial \varphi_A = 0$ (with $N_1 = N_2$) leads to $\chi N_1 = \log((1-\varphi_1)/\varphi_1)/(1-2\varphi_1)$



$\chi = 1$



$\chi = 2.5$

$N_1 = N_2$ here

If $N_1 = N_2 = 1$ then $\chi_c = 2$. For general $N_1 = N_2 = N$, we have $\chi_c = 2/N$,

so that for long polymers, χ_c is small, which is why different polymers don't mix well.

Questions for the lecture discussion

Question 1. Why are the free energies of the entropic spring and monomers in the pre-Flory Huggins model the same even though one is 1dimensional and the other is 2 dimensional?

Question 2. Suppose you pull the ends of the spring as in the DPD entropic spring simulation: how could you modify the free energy to include this pulling force.

What would the force do the bond flips?

Summary

Phase transitions are reversible *macroscopic* changes driven by *microscopic* interactions (equipartition thm. provides energy for the change to all molecules simultaneously)

Entropic spring and Flory Huggins theory are mean field theories of phase transitions - ignore connectivity - but give reasonable predictions

Small changes in a control parameter (temp., conc., etc) can have huge effects (water-ice)

Cells use phase transitions to organise their biochemistry, e.g., organelle membranes, intrinsically-disordered proteins