Engineering of the Musculoskeletal System and Rehabilitation

1.2 Numerical methods

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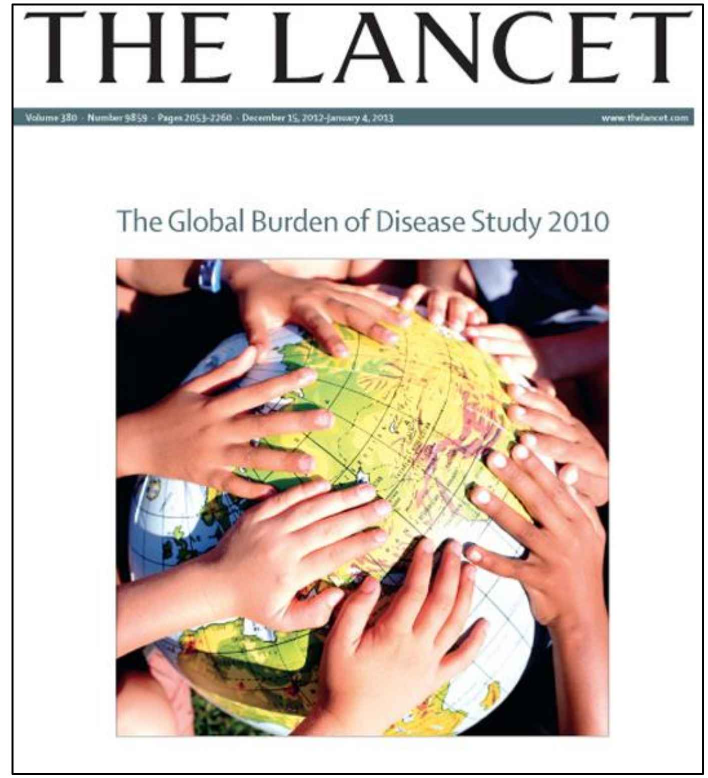
What for?

“Since 1970, men and women worldwide have gained slightly more than **ten years of life expectancy** overall, but they spend more years living **with injury and illness**.”



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Trends in the disability-free life expectancy in Switzerland over a 10-year period: an analysis of survey-based data

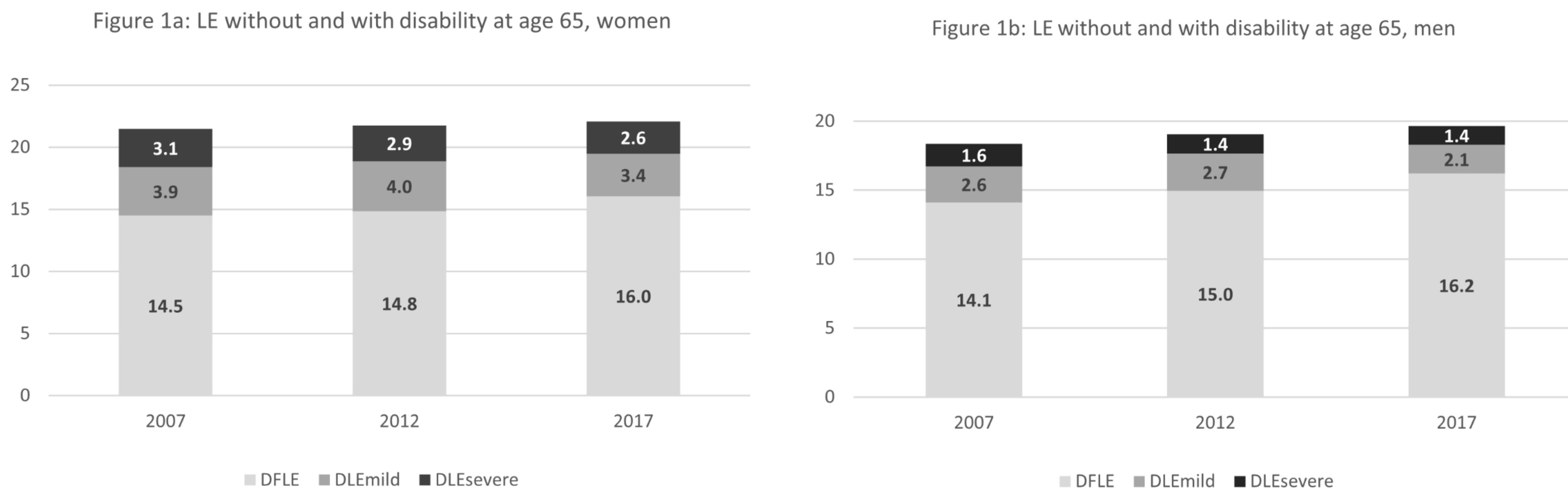
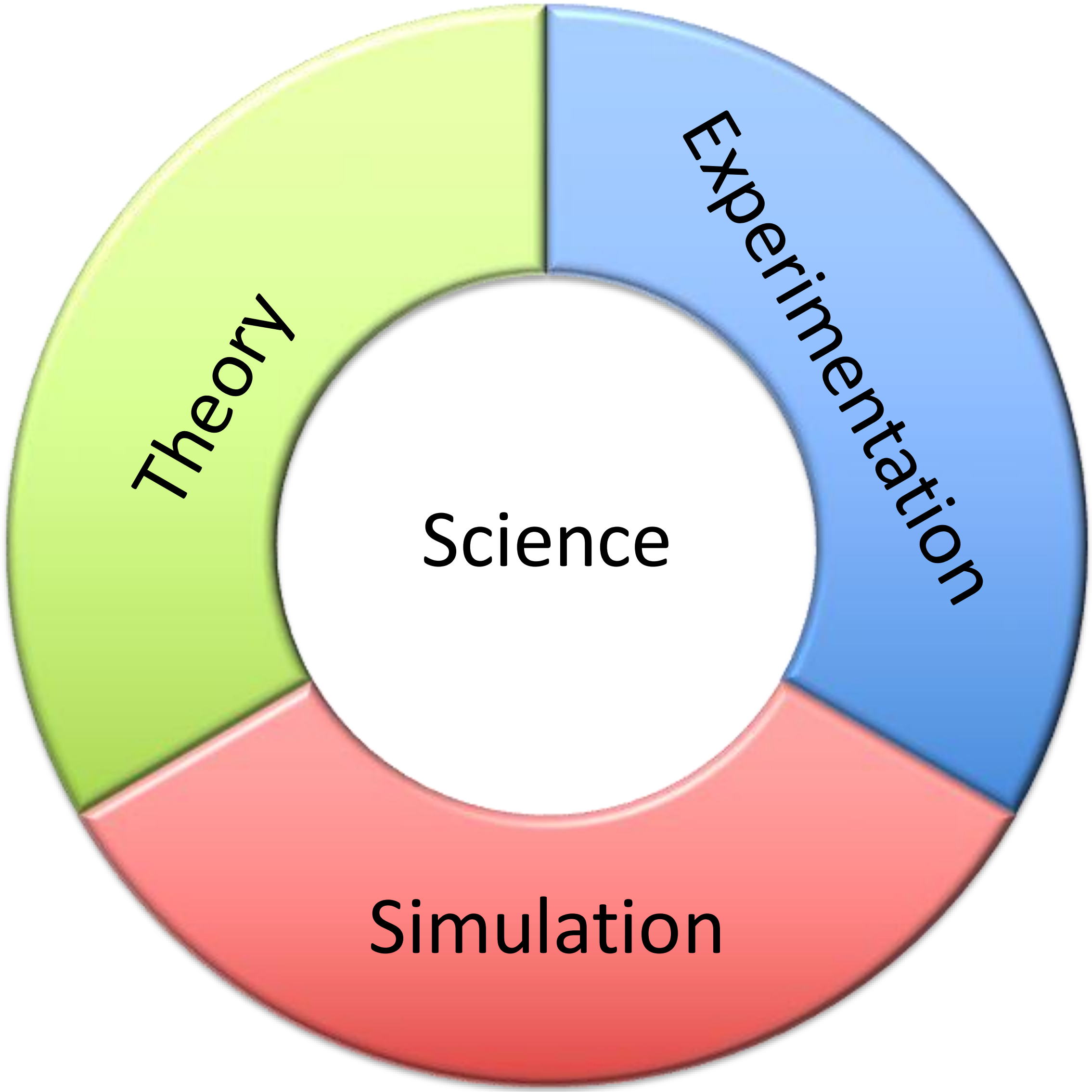


Figure 1: Trend in life expectancy without (DFLE), with mild (DLE_{mild}), and with severe (DLE_{severe}) disability at 65 and 80 years of age, by sex. Mean values for DFLE and LED_{mild} and LED_{severe}.

Musculoskeletal system pathologies

- Arthritis
  - Joint replacement (Limited lifespan)
  - Cement damage accumulation
  - Wear (of sliding surfaces)
  - Peri-prosthetic fracture
  - Dislocation, subluxation, instability
  - Osteo-integration (aseptic loosening, primary stability)
  - Bone resorption (remodeling)
  - Tissue engineering (cartilage, intervertebral disk, tendon)
- Bone fractures
  - Fixation & design of plates, rods, screws, pins
  - Stability, bone adaptation
  - Tissue engineering (bone)

3 Pillars of science



Objectives of numerical modeling

- Analyze (understand) observations
- Test hypotheses
- Design experimental setup
- Design (pre-tests) of medical devices
- Improve treatments, surgical techniques
- Improve preoperative planning

Advantages of Numerical Methods

- Efficiency to solve problem (no analytical solution)
- Large scope of problems (physics, biology, chemistry)
- Cheap (material)
- Easy (conceptually)
- Access to all system quantities (not measurable)
- Not dangerous (chemicals)
- No ethical issue (animal/human experiment)

Drawback of Numerical Methods

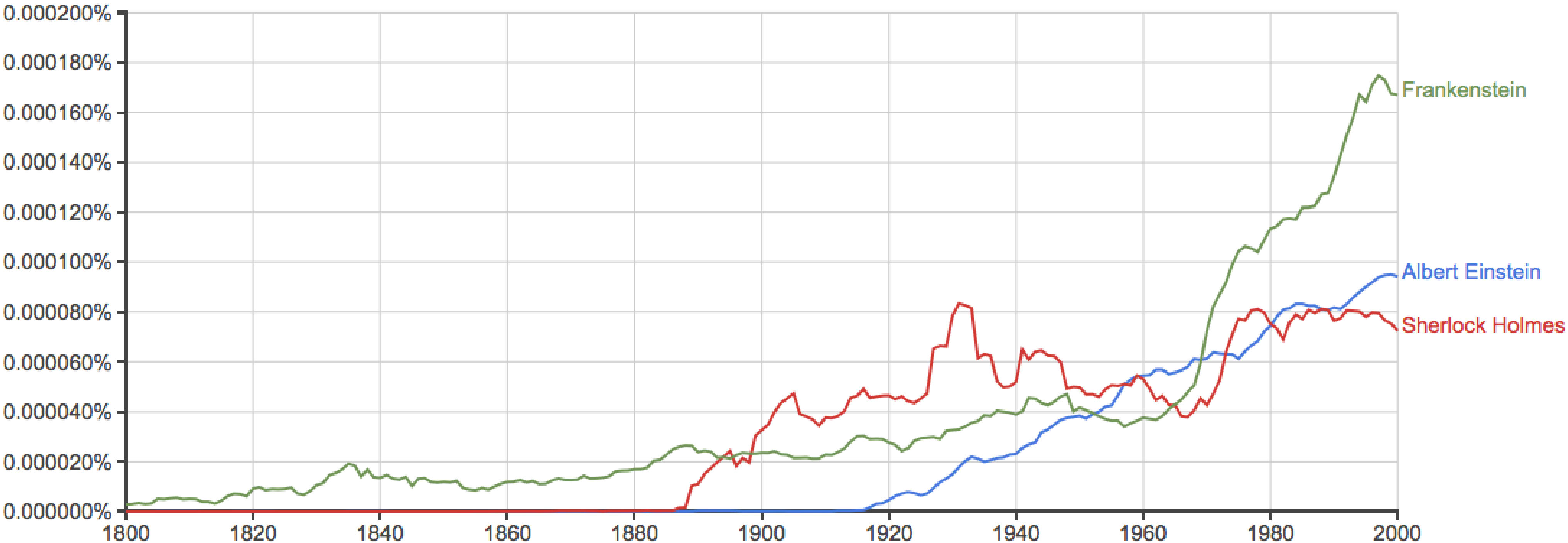
- Complexity (variability) of living tissues
- Correlation (validation) with experiments difficult
- More method-oriented than problem-oriented
- Qualitative rather then quantitative

New trend

From Specialty towards Integration

- Multi-scale (body, systems, organ, cell, molecule)
- Multi-physics (solid, fluid, reactions)
- Multi-disciplinary (engineer, biology, medicine)

Word frequency



Google Books Ngram Viewer

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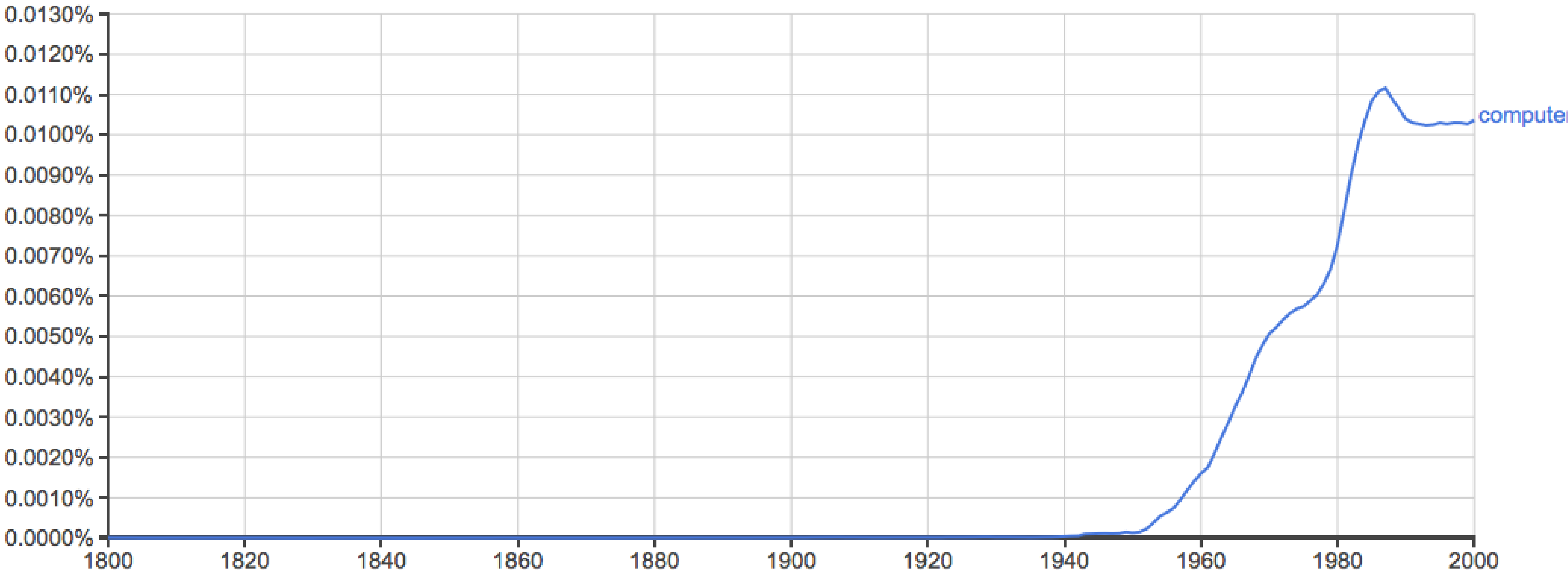
Word frequency



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Word frequency

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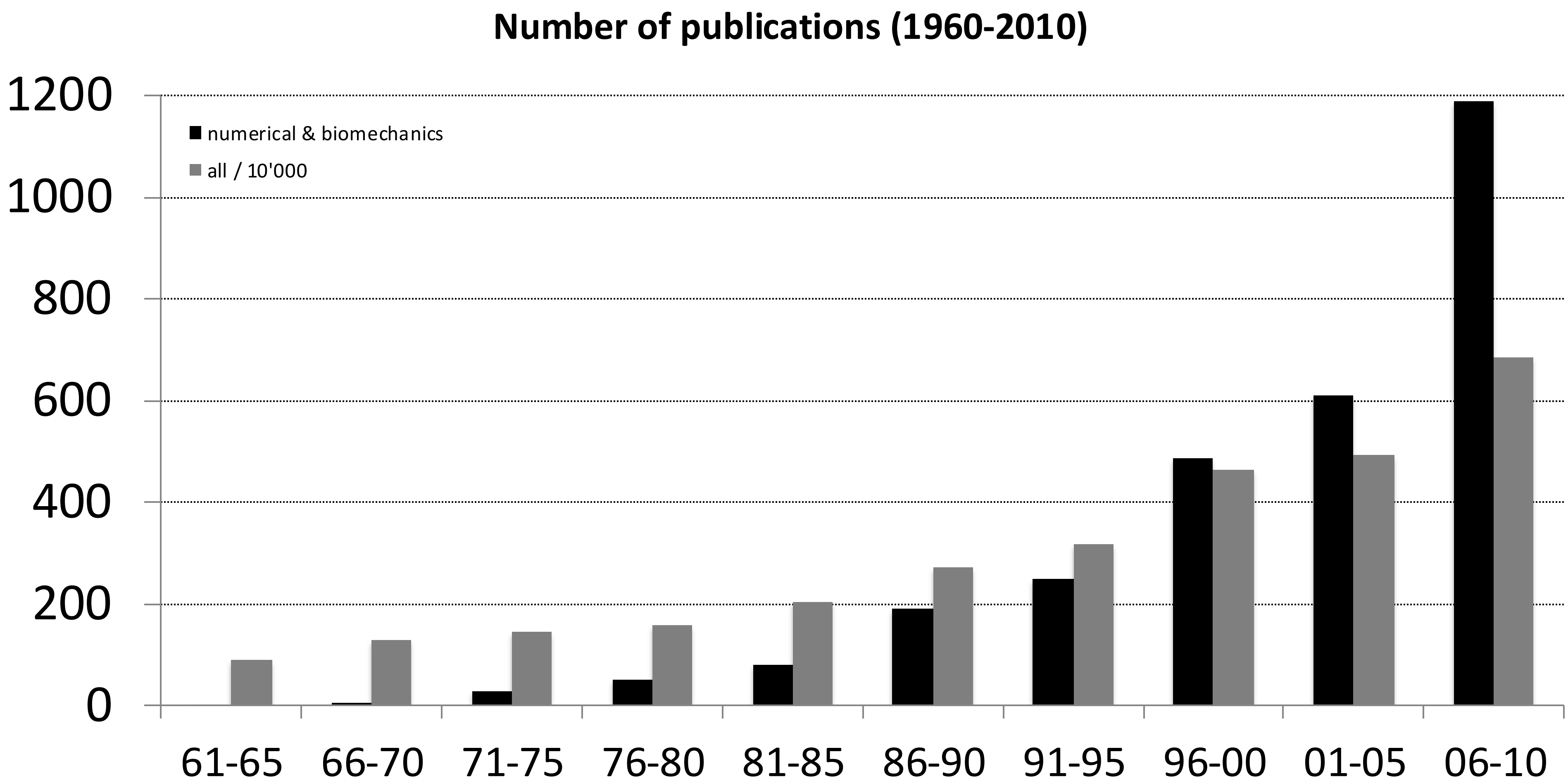
Relative Importance in Science



“numerical” + “biomechanics” from www.scopus.com

History

The development of numerical methods has followed the growth of computer power, starting slowly 50 years ago and extending very rapidly today.

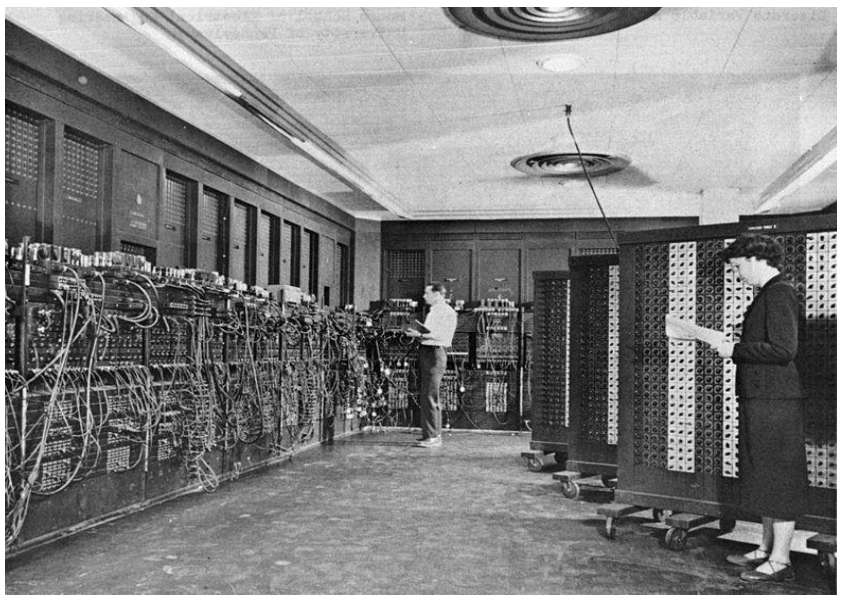
History

- Bio-engineering, as numerical modeling, can be both originated to World War II.

- Numerical modeling in bioengineering followed the recent rapid evolution of numerical techniques and computers performances.

Where it started

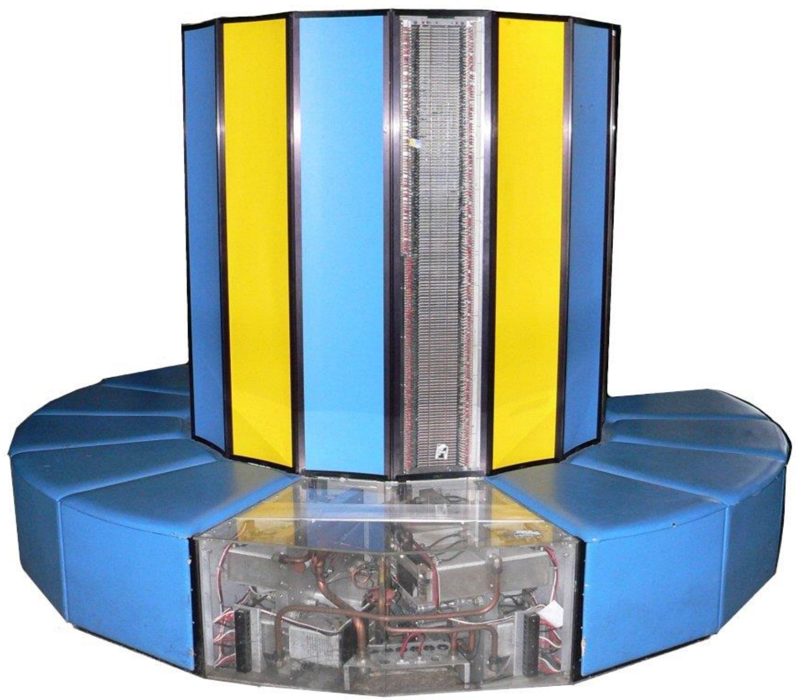
ENIAC (Electronic Numerical Integrator And Computer), 1946-55, 30 tons, 350 flops



1946, U.S. Army photo

30 years later…

CRAY-1, Los Alamos (1976), EPFL (1986-88)
100 megaFlops



Today's supercomputers

IBM Blue Gene/Q, EPFL (2013)

173 teraFLOPS (10^{12} FLOPS)

Today's supercomputers

HPE SGI 8600 system (EPFL, 2018)

> 1 petaFLOPS (10^{15} FLOPS)

https://www.hpcwire.com/2018/07/10/hpe-epfl-launch-blue-brain-5-supercomputer/

Today’s supercomputers

Summit (IBM)

200 petaFLOPS (10^{15} FLOPS)

https://www.ornl.gov/directorate/ccsd

Today's supercomputers

Swiss National Supercomputing Centre, Lugano

HPE Cray EX, 4'719 TFlops, 2024

https://www.cscs.ch/computers/alps

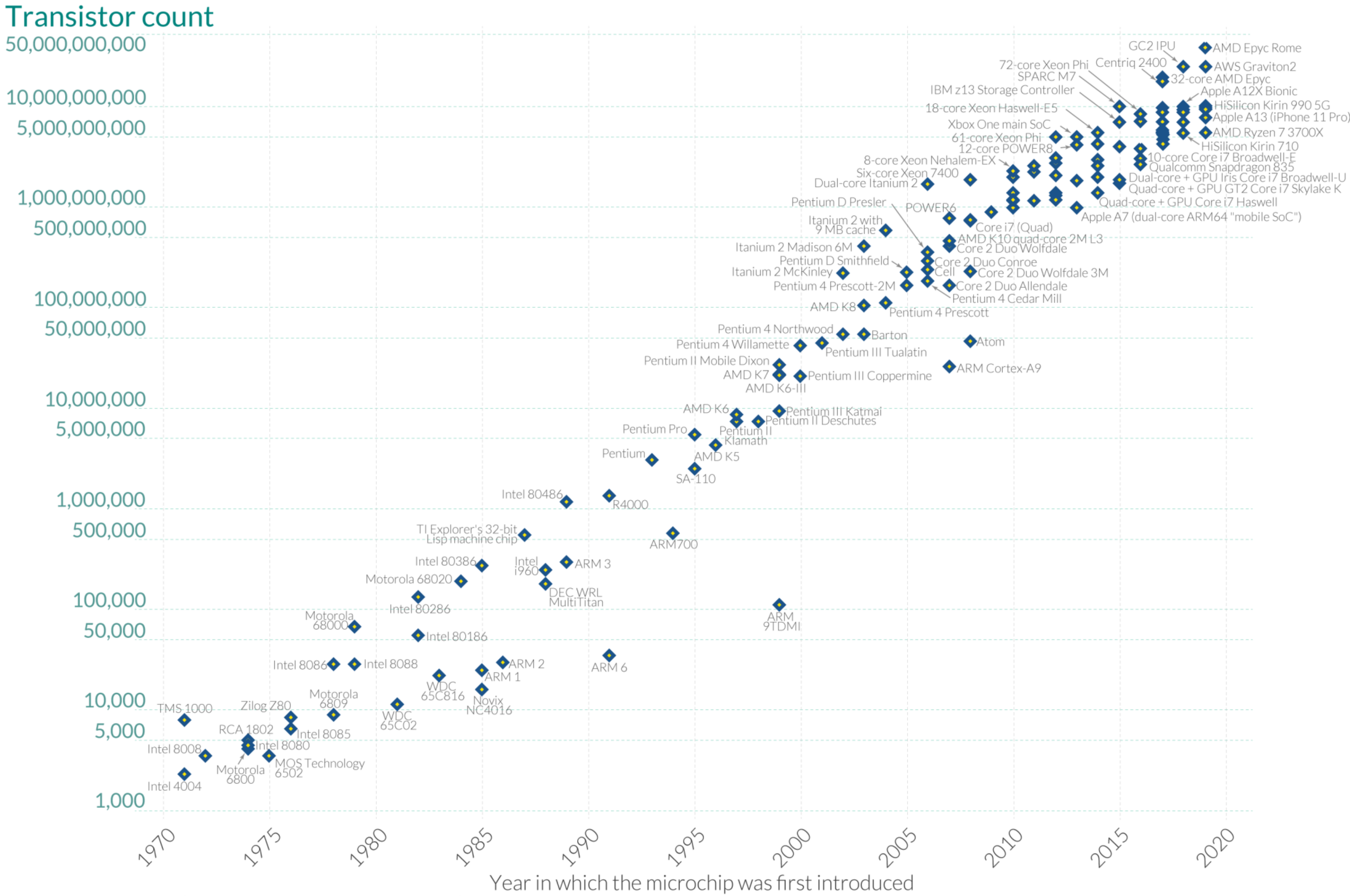
Today's laptop

≈ 1 teraFlops

Moore's Law: The number of transistors on microchips has doubled every two years

Our World in Data

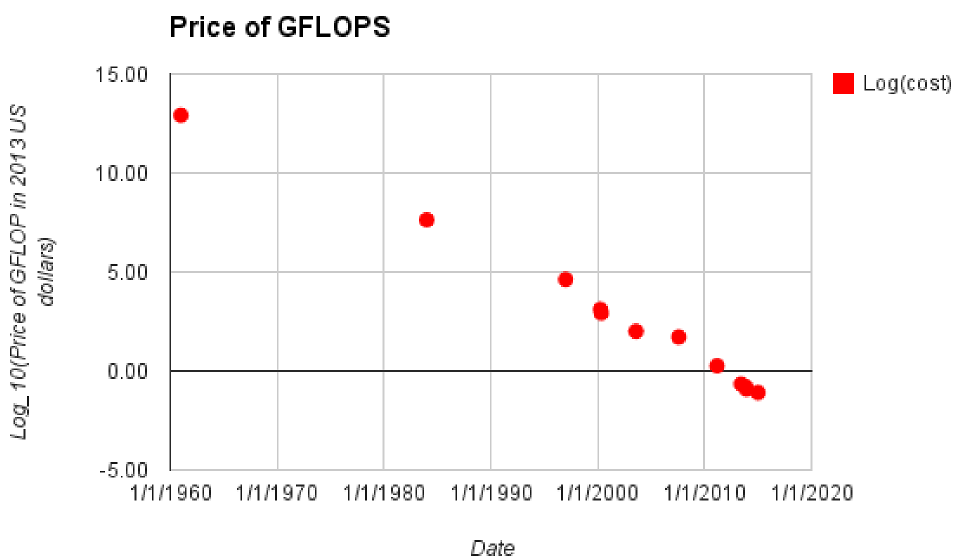
Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

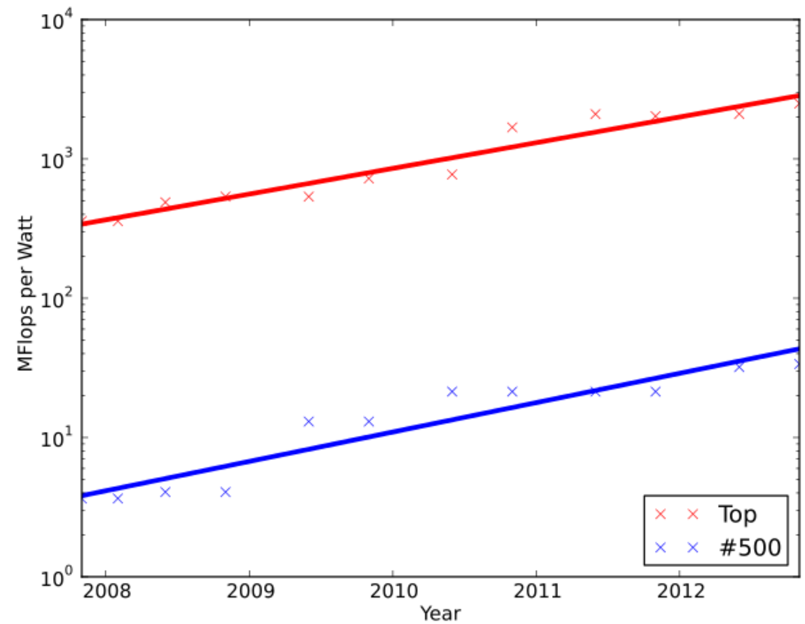


Data source: Wikipedia (wikipedia.org/wiki/Transistor_count)

OurWorldinData.org – Research and data to make progress against the world's largest problems.

FLOPS/$ and FLOPS/W





https://aiimpacts.org/trends-in-the-cost-of-computing/

https://en.wikipedia.org/wiki/Performance_per_watt

Today’s computer

- Efficient (personal/workstation) computers
- Efficient simulation software
- User-friendly simulation software
- Numerical technics are commonly used as a tool by engineers and scientists

History of Numerical Methods (PDE)

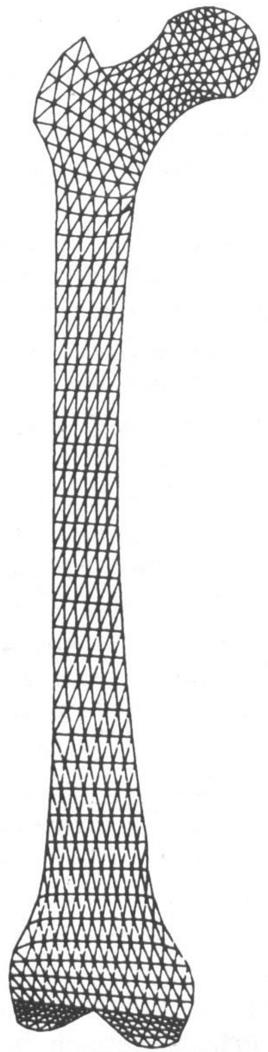
- Variational principale (1900)
- Finite Difference Methods (1930)
- Implicit methods (1950)
- Finite Element Method (1960)

Thomee, V. (2001). From finite differences to finite elements - A short history of numerical analysis of partial differential equations. Journal of Computational and Applied Mathematics 128(1-2): 1-54.

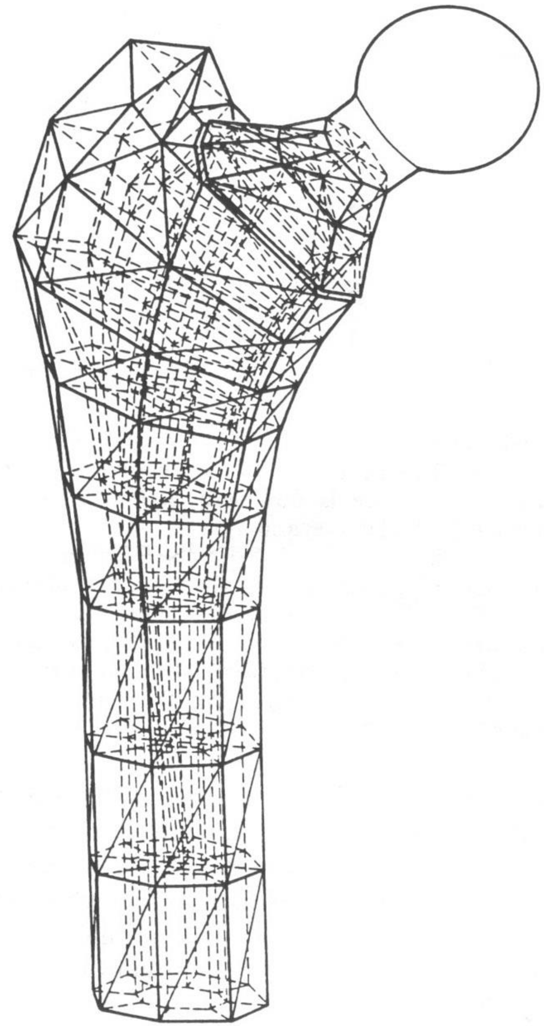
Typical problems

- Joint prostheses
  - Mechanical failure
  - Wear
  - Osteo-integration
- Tissue engineering
- Surgical technique

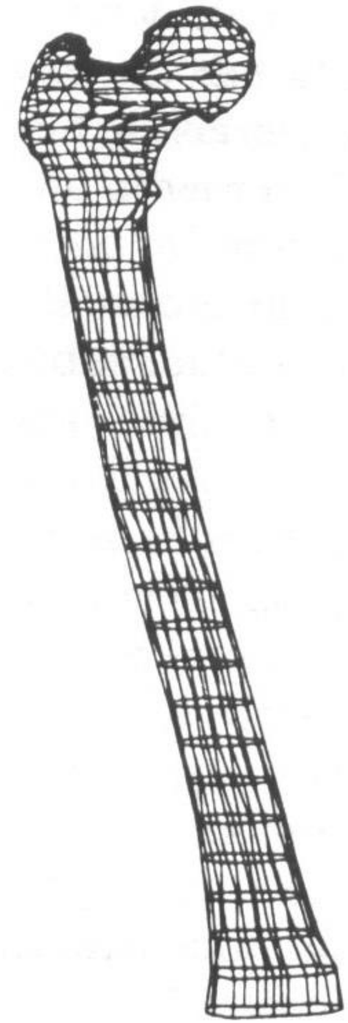
First numerical models



Brekelmans 72

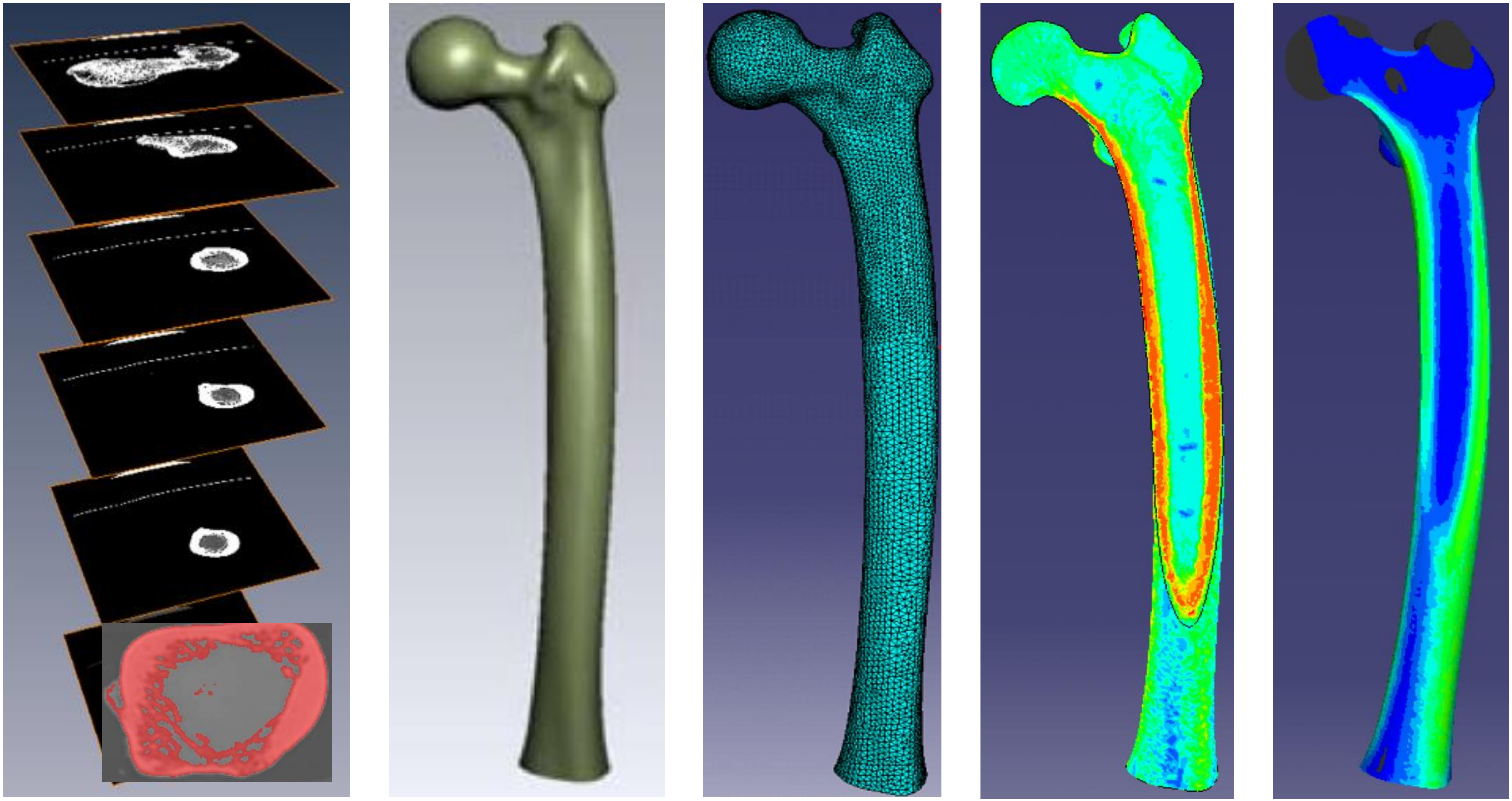


Roehrle 77



Rohlmann 82

Hip

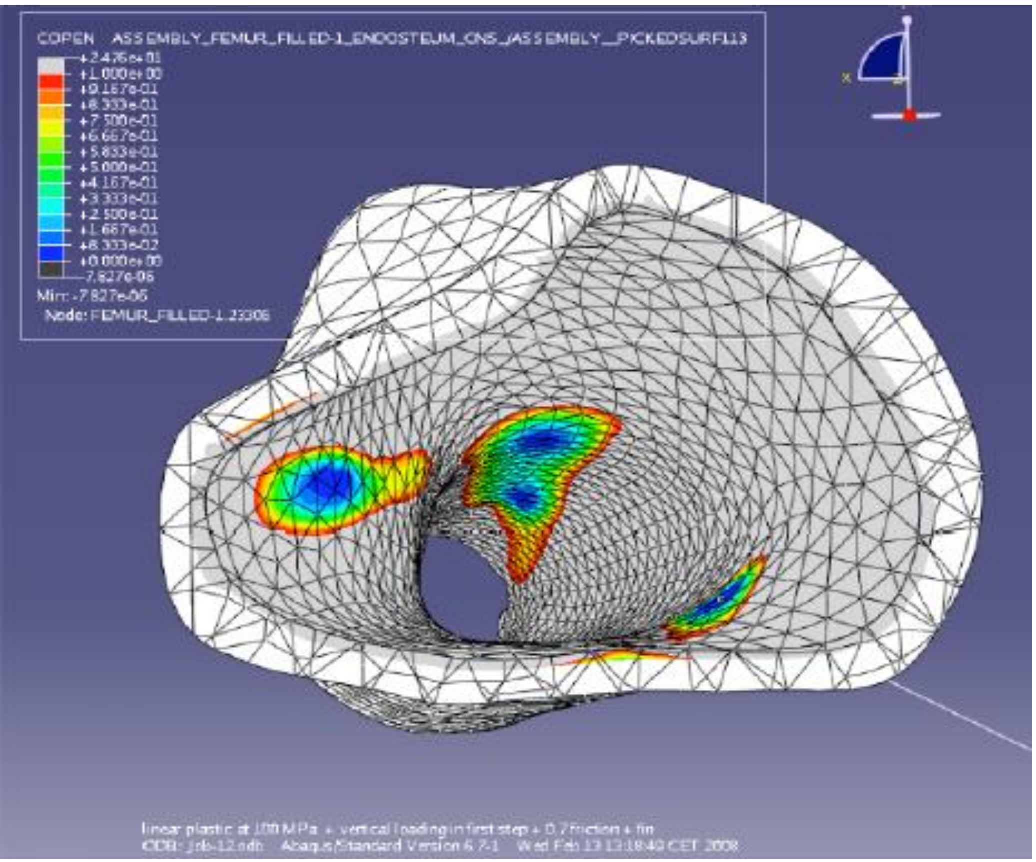


EPFL-LBO (Gortchacow, 2011)

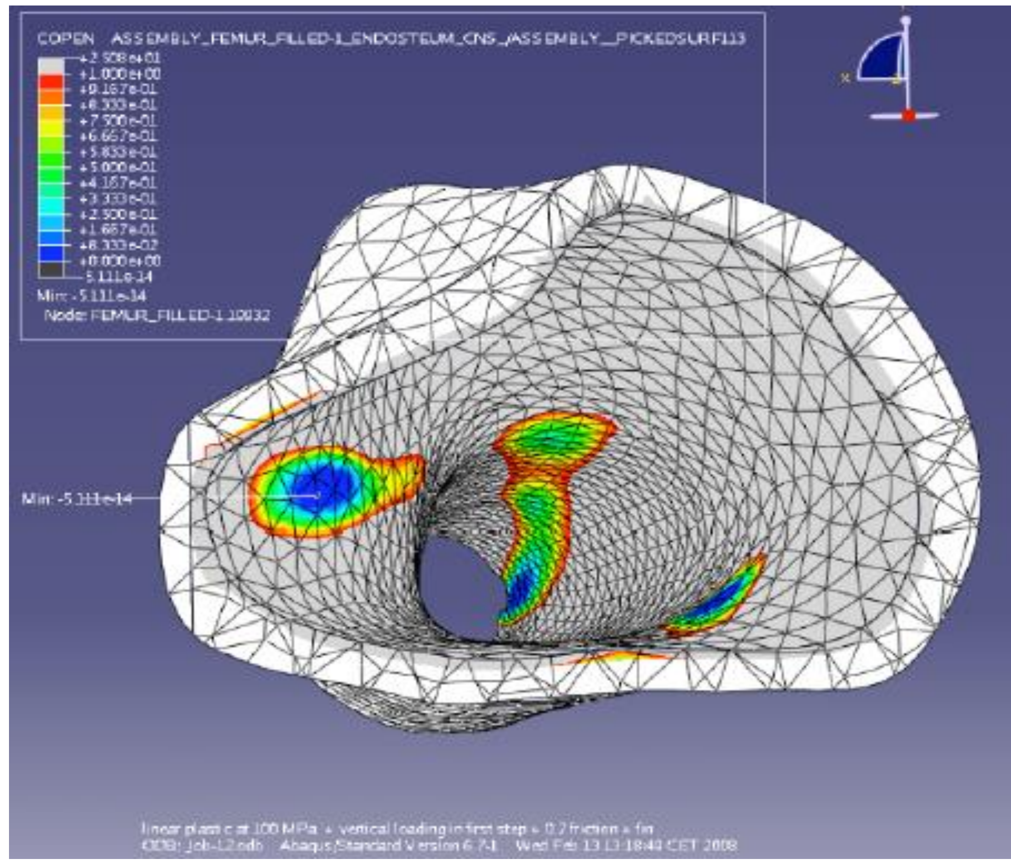
Hip

Bone-stem gap

No load



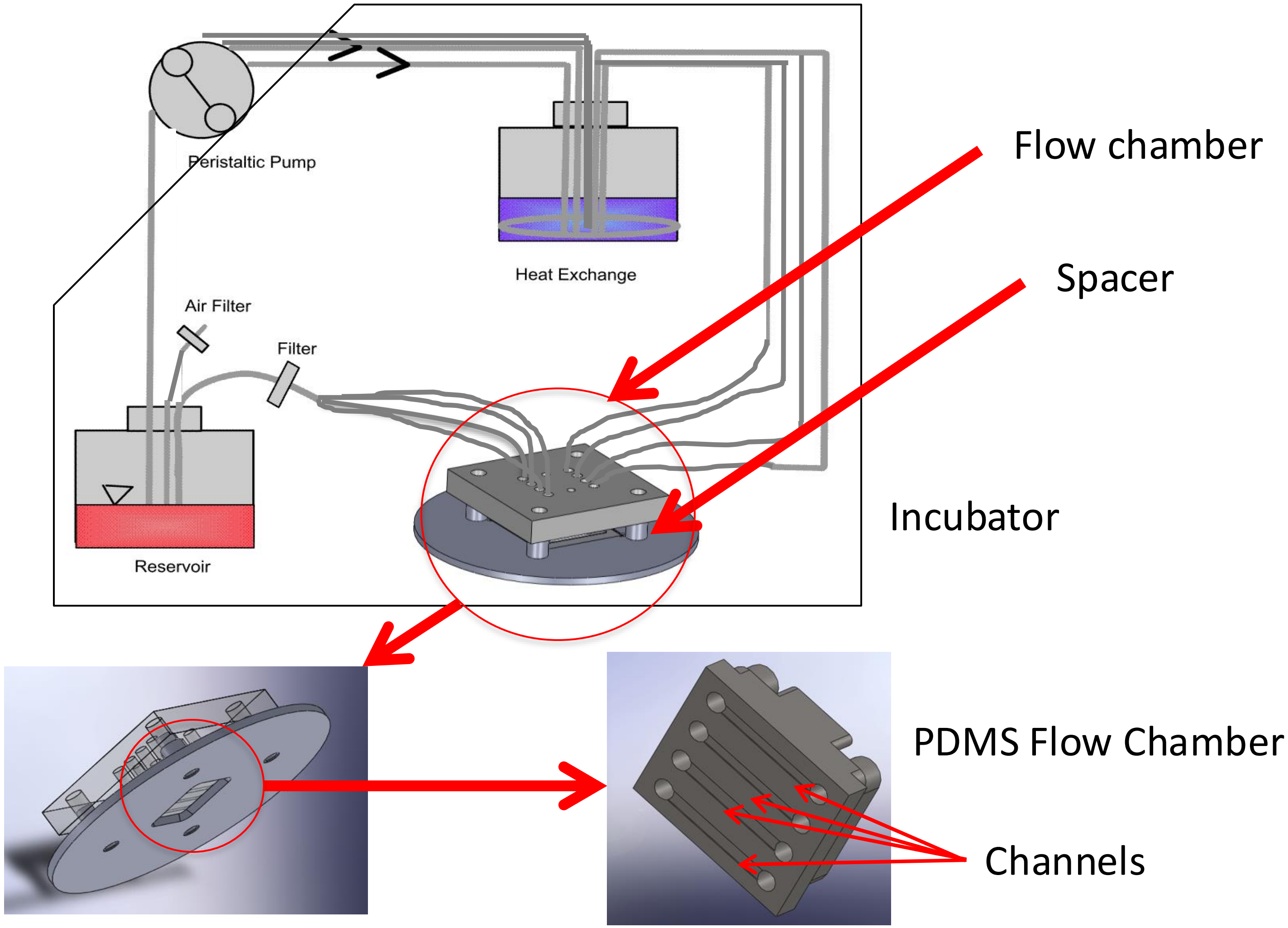
Walking load



EPFL-LBO (Gortchacow, 2011)

Design of a flow chamber

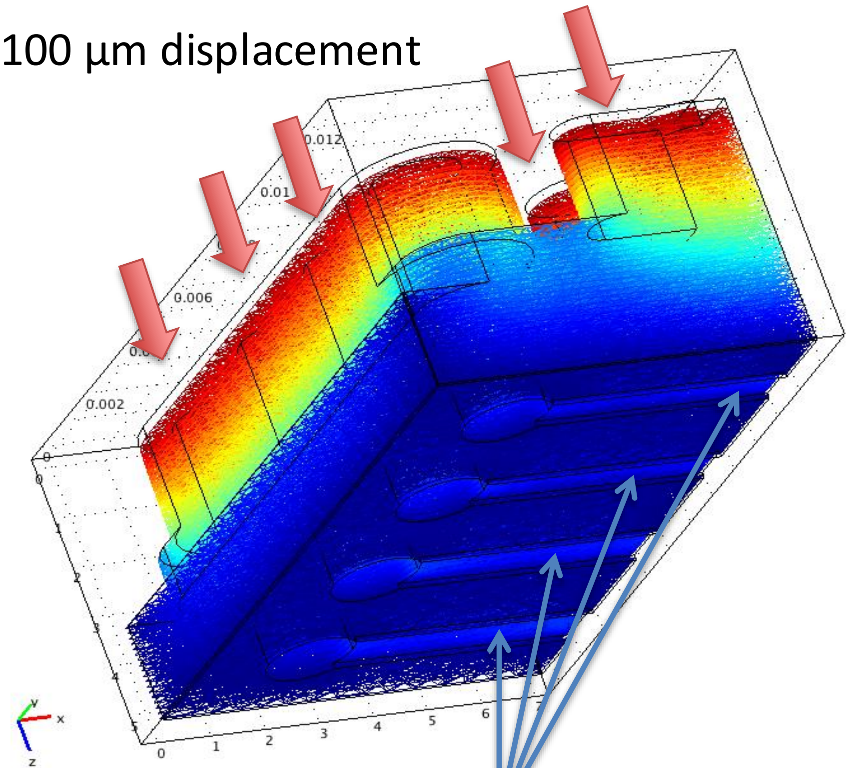
Effect of spacer height on channel deformation



EPFL-LBO (Gortchacow, 2011)

Flow channel deformation

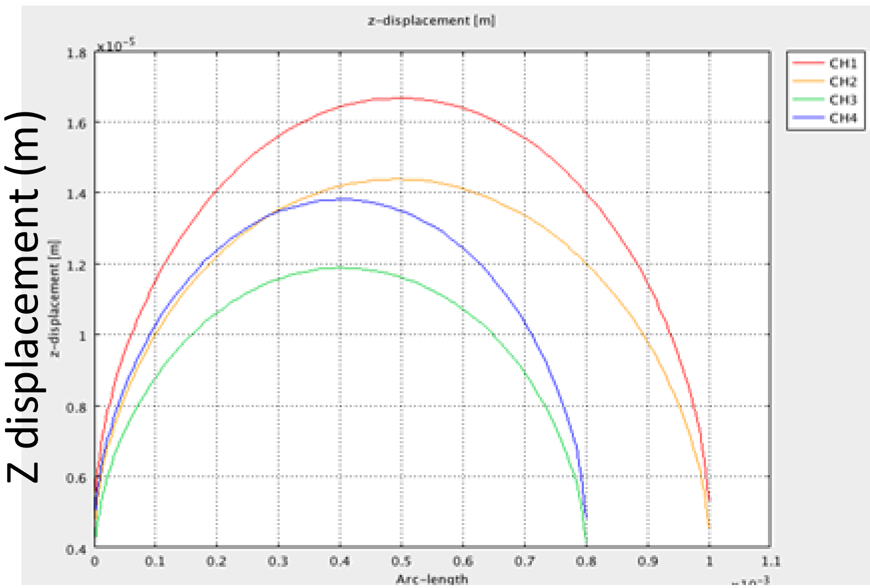
100 µm displacement



Channels

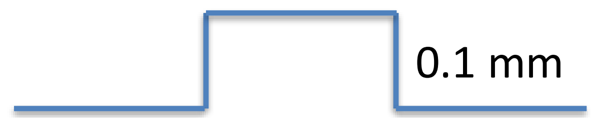
Vertical Displacement at channel center

Z displacement (m)



Channel width

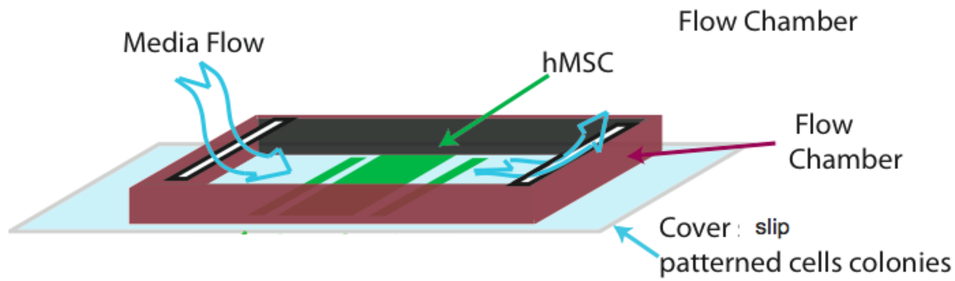
0.8 or 1 mm

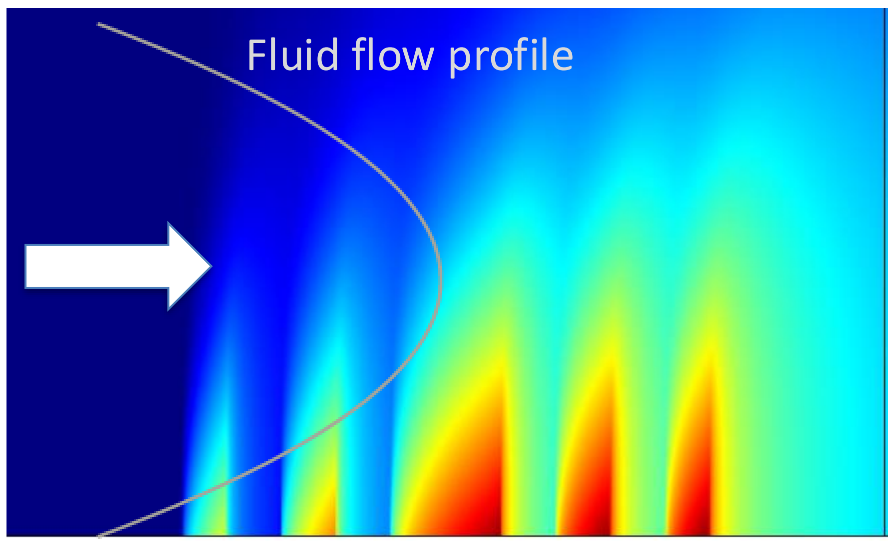


0.1 mm

EPFL-LBO (Gortchacow, 2011)

Design of the experiment

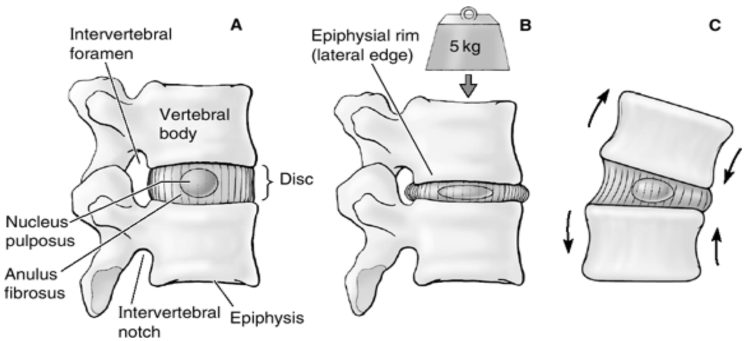


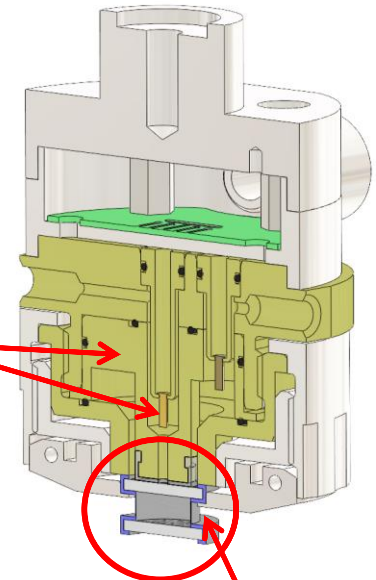


Morphogen concentration

EPFL-LBO (Gortchacow, 2011)

Design of a micro-calorimeter





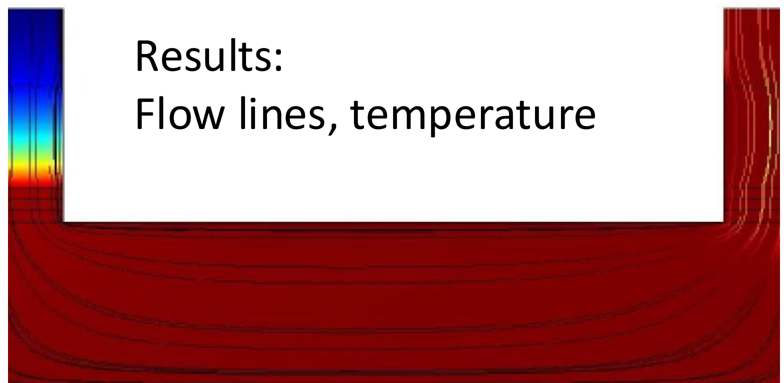
Thermistors

Hydrogel sample

Inlet

Outlet

Perfusion tubes
Fluid flow (Navier-Stokes equ.)



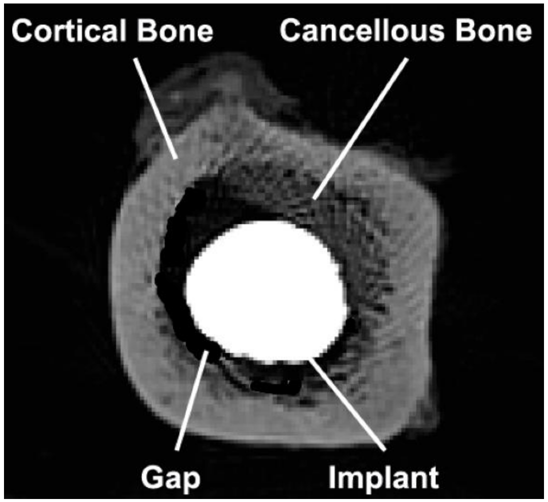
Results:
Flow lines, temperature

Porous filter
Fluid flow (Brinkman equ.)
Temperature (Heat equ.

Hydrogel sample

EPFL-LBO (Vogel, 2011)

Fluid Flow Within Bone-Implant Gap



$$-\nabla \sigma = 0$$ Navier

$$S\frac{\partial p_f}{\partial t} + \nabla \cdot \left[-\frac{\kappa}{\mu}\nabla p_f\right] = -\alpha_B \frac{\partial \varepsilon_{vol}}{\partial t}$$ Darcy's & continuity

$$\sigma = \mathbf{C}(E,\nu)\varepsilon(\mathbf{u}) - \alpha_B p_f$$

$$p_f = \frac{1}{S}(\zeta - \alpha_B \varepsilon_{vol})$$ Biot

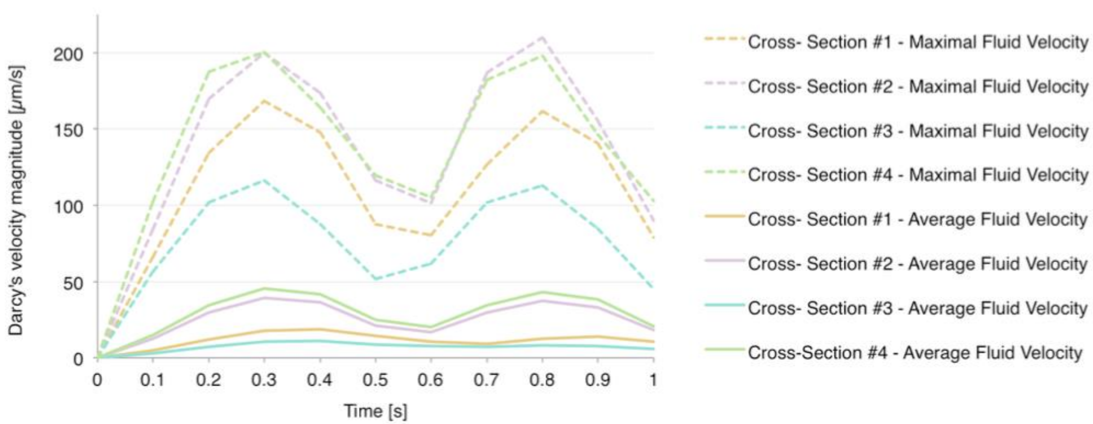


Figure 1 - Average and maximal interstitial fluid velocity in granulation tissue during one load cycle

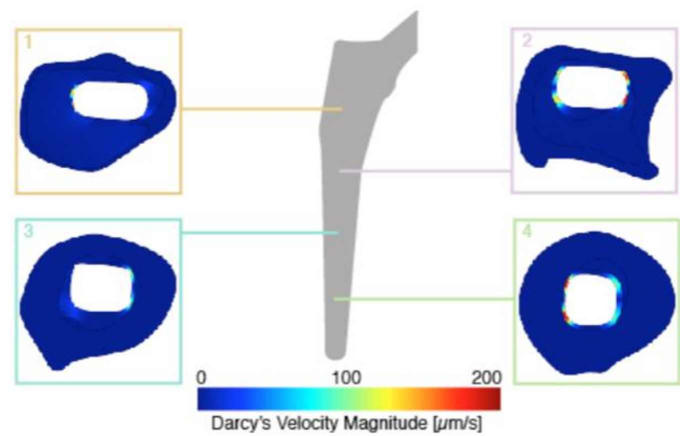


Figure 2 - Interstitial fluid velocity at the bone-implant interface around a cementless femoral stem

EPFL-LBO (Malfroy Camine 2015)

Orthopedic companies

- Conception / Improvement
- Development
- Optimization
- Evaluate uncertainties
- Risk analysis
- Prepare for mechanical testing (ISO, ASTM)
- Communication, marketing

Musculoskeletal system

- Musculoskeletal system
  - Muscles (tendon, ligaments, cartilage, soft tissues)
  - Bones
  - Joints
- Engineering (mechanics)
  - Forces
    - Muscles
    - Joints (natural, artificial)
  - Stress/strain
    - Bone, Cartilage, Tendons, Ligaments, soft tissues
    - Implants (joint prostheses, cement, screws, etc)

Modeling Techniques

- Multi-scale (level) modeling
  - Organ, Tissue
  - Limb, joint, tissue, interfaces, micro-structures, cells
  - Sequential: pre-computed micro-scale for macro-scale
  - Concurrent: on-the-fly micro-scale for macro-scale
- Multi-physics
  - Mechanics (rigid multi-body, deformable solid)
  - Fluid (transport)
  - Heat (cement polymerization)
  - Chemicals (biological reactions)
  - Electromagnetism

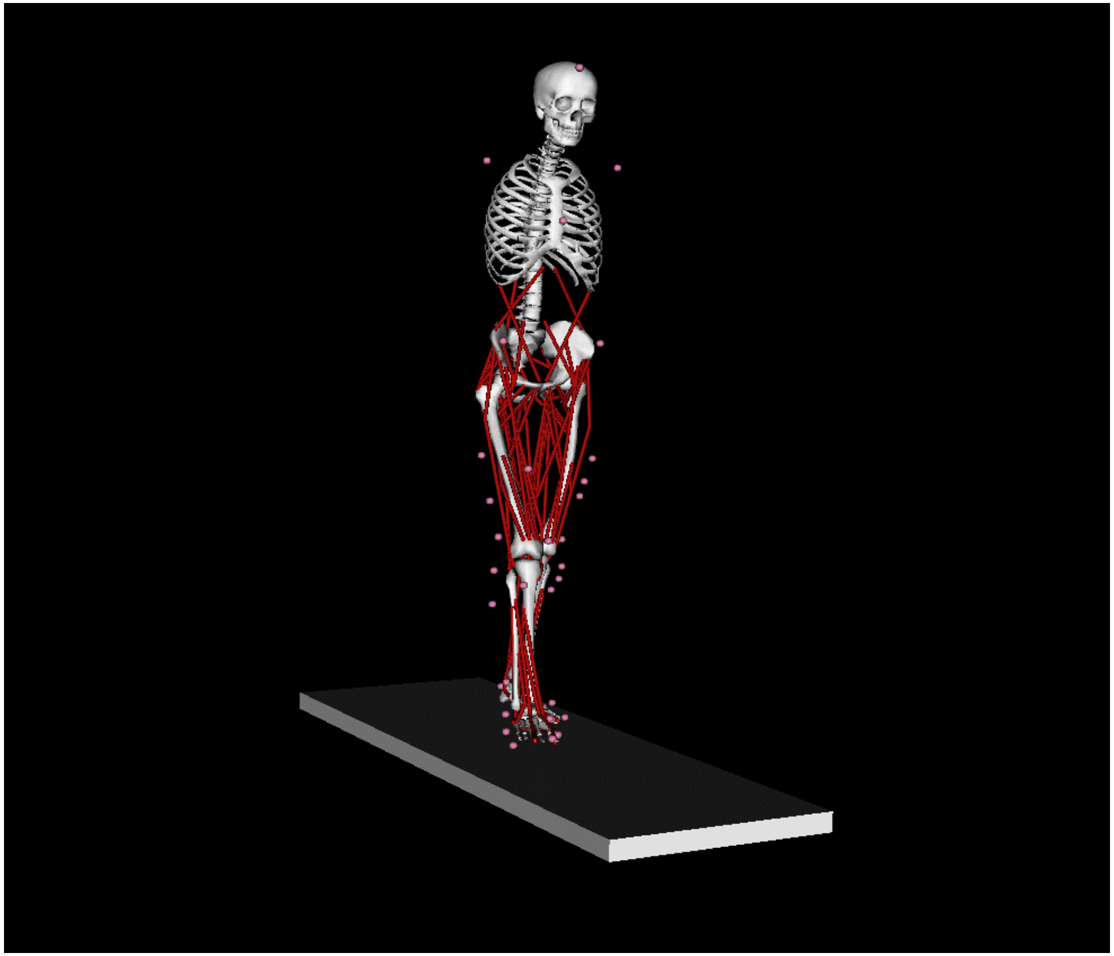
Multi-level: Joint – Tissue Decoupling

- Joint: Musculoskeletal modeling
  - Joint and muscles forces
  - Indeterminate system
    (more degrees of freedom than equilibrium equations)
    - Inverse dynamics (Motion -> muscle forces)
    - Forward dynamics (Muscle forces -> motion)
    - Simplified system (determinate)
- Tissue: Finite Element Modeling
  - Partial differential equations
  - Boundary conditions
  - Constitutive Laws

Joint: Musculoskeletal Modeling

- Multi-body (rigid) system
- Newton equation of motion
- Lagrangian mechanics
- Inverse dynamics: forces from movement
  - Inverse kinematics: joint angles from motion captors
  - Joint torques from angles kinematics & inertia & ground reaction
  - Muscle forces (optimization of physiological criteria)
- Available software for inverse dynamics
  - OpenSim
  - SIMM
  - Anybody
  - Lifemodeler

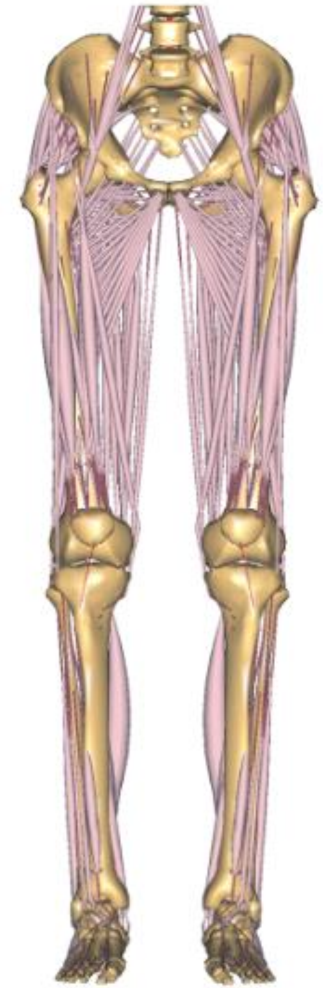
Musculoskeletal Model



http://simtk-confluence.stanford.edu:8080/display/OpenSim/Gait+2392+and+2354+Models

Musculoskeletal modeling

Automated 3-D image-analyzing tools to parameterize musculoskeletal system



Tissue: Finite Element Modeling

Most biomechanical systems can be described by

- A set of Partial Differential Equations (PDEs)
- Completed with constitutive equations
- Boundary conditions (← Joint model)
- Initial conditions

PDEs

- Solid (deformation) mechanics (stress, strain)
- Fluid mechanics (fluid velocity, pressure)
- Heat (temperature)
- Transport (diffusion, advection, concentration)
- Electromagnetism (electric & magnetic potential)
- Wave propagations (EM, acoustic)
- Coupling in multi-physics

Ordinary Differential Equations (ODE)

- Differential equation with 1 independent variable
- Linear/nonlinear
- Order (highest derivative)
- Homogeneous/nonhomogeneous
- Existence, uniqueness (Cauchy–Lipschitz theorem)

$$\frac{dy}{dt} = F(t, y), y(t_0) = y_o$$

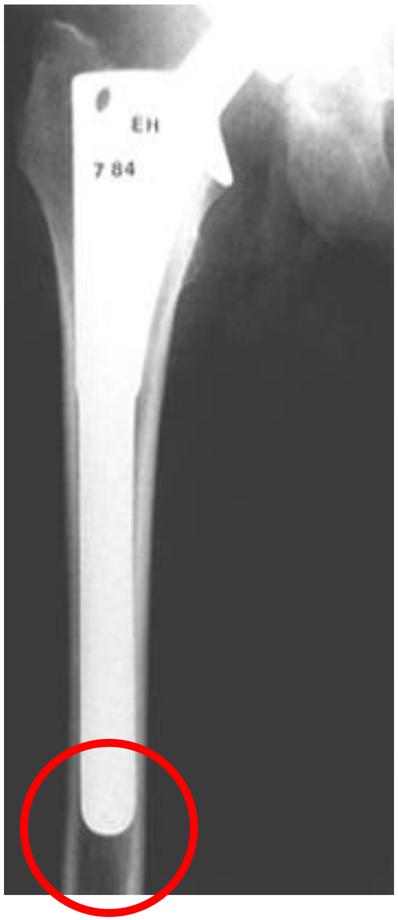
Ordinary Differential Equations (ODE)

- System of linear ODEs
- ODE of order n can reduce to n 1st order ODEs
- Non constant coefficients : $A = A(x,\ t)$
- Nonhomogeneous ODE ($b \neq 0$)
- Boundary problem (x), or initial value problem (t)

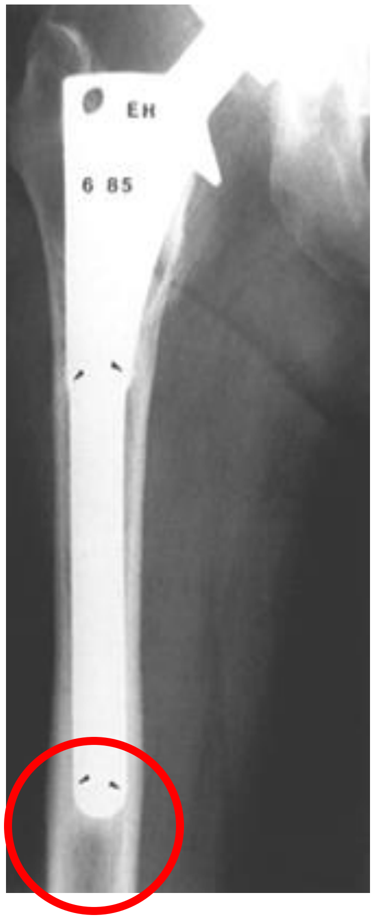
$$\frac{dy}{dt} = A(y) + b(t), y(t_0) = y_o$$

ODE example: bone adaptation

post-surgery



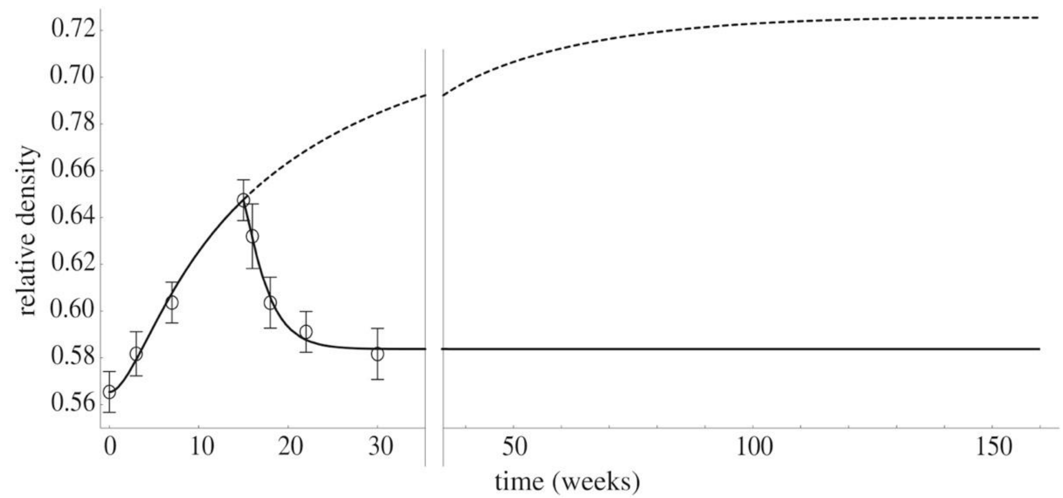
after 1 year



J. D. Bobyn and C. A. Engh, Non-Cemented THA, Raven Press (1988)

$$\frac{\partial \rho(x,t)}{\partial t} = v(\psi(x,t) - \psi_e)$$

$$\frac{d\rho}{dt} = v\left(\frac{{\rho_c}^2}{2E_c}\frac{\sigma^2}{\rho^3} - \psi_e\right)$$



Terrier et al, Clin Biomech, 2005

Partial Differential Equation (PDE)

$$F(x_1, \cdots x_n, u, \frac{\partial}{\partial x_1}u, \cdots \frac{\partial}{\partial x_n}u, \frac{\partial^2}{\partial x_1 \partial x_1}u, \frac{\partial^2}{\partial x_1 \partial x_2}u, \cdots) = 0$$

- x_i independent variables, u_i dependent variables
- $x_1 = t$ (time)
- $x_2 = x$, $x_3 = y$, $x_4 = z$ (space)
- No time dependency (steady state or equilibrium)
- Existence and uniqueness not guaranteed in general, but usually locally if well-posed

Well-posed PDE

- Regular PDE and domain (Cauchy problem)
- Constitutive equations
- Boundary conditions
- Initial conditions

$\Rightarrow$ A unique & stable solution exists (locally)

Boundary conditions

- Value of the dependent variable u (and/or its derivative) on the boundary for all time t
- **Dirichlet**: u imposed on the boundary
- **Neumann**: du/dn imposed on the boundary
- Mixed: Dirichlet and Neumann on boundary parts
- Cauchy: Dirichlet and Neumann on boundary
- Robin : $u + du/dn$ imposed on the boundary

Initial conditions

- Value of the dependent variable, and/or its derivatives, at time $t = 0$, for the entire domain
- Initial value problem

$$u(x, t = 0) = u_0(x)$$

PDE: scalar coefficient form

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \boldsymbol{\alpha} u + \gamma) + \beta \cdot \nabla u + au = f \qquad \text{in } \Omega$$

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - h^T \mu \quad \text{(Generalized Neumann) on } \partial\Omega$$

$$u = r \qquad \text{(Generalized Dirichlet) on } \partial\Omega$$

- Coefficients $(c, \alpha, \gamma, \beta, a, h)$ and f, g, and r can depend on x, y, z, t
- PDE is linear when coefficients depend only on (x, y, z), or constant
- A PDE is nonlinear if coefficients depend on u (or its derivatives)
- **Initial condition is required**

Well-posed problem

- Domain (geometry)
- PDE (physics)
- Constitutive laws (material)
- Initial conditions
- Boundary conditions

Summary

- Most biomechanics (bioengineering) systems (problems) can be represented by Partial Differential Equations (PDEs)
- Well-posed PDEs have a unique solution
- Most PDEs can’t be solved analytically
- Many PDEs can be solved by numerical methods
- Some PDEs can’t be solved by numerical methods

Quotes

"Among all of the mathematical disciplines the theory of differential equations is the most important…
It furnishes the explanation of all those elementary manifestations of nature which involve time."

Sophus Lie (1842-1899)

References (library.epfl.ch/en/ebooks)

- Partial Differential Equations I, Basic Theory, Michael E. Taylor
- Partial Differential Equations II, Qualitative Studies of Linear Equations, Michael E. Taylor
- Partial Differential Equations III, Nonlinear Equations, Michael E. Taylor
- Partial Differential Equations, Emmanuele DiBenedetto
- Partial Differential Equations, Modeling and Numerical Simulation, Roland Glowinski and Pekka Neittaanmäki
- Linear Partial Differential Equations for Scientists and Engineers, Tyn Myint-U and Lokenath Debnath
- Partial Differential Equations, Laurence C Evans

Numerical methods for PDE

Most PDEs can’t be solved algebraically

Numerical method should be robust and reliable:

- Stable (convergence, increasing/oscillating error)
- Accurate “enough” (constant error)

Numerical methods for PDE

- Approximate (discretized) solution of the PDE
- Subdivide the domain in small sub-domains
- 3 major discretization methods:
  - Finite difference method (FDM)
  - Finite volume method (FVM)
  - Finite element method (FEM)

Finite Element Method (FEM)

- PDE with dependent (solution) variable u(x)
- Strong form (PDE) → weak form (integral)
- Discretization: $u(x) = \sum u_i\, v_i(x)$ v_i: basis functions
- Weak form (integral) → Matrix form: $Ku = L$
- Matrix solving
- Non-linearity (convergence criteria)
- Time integration

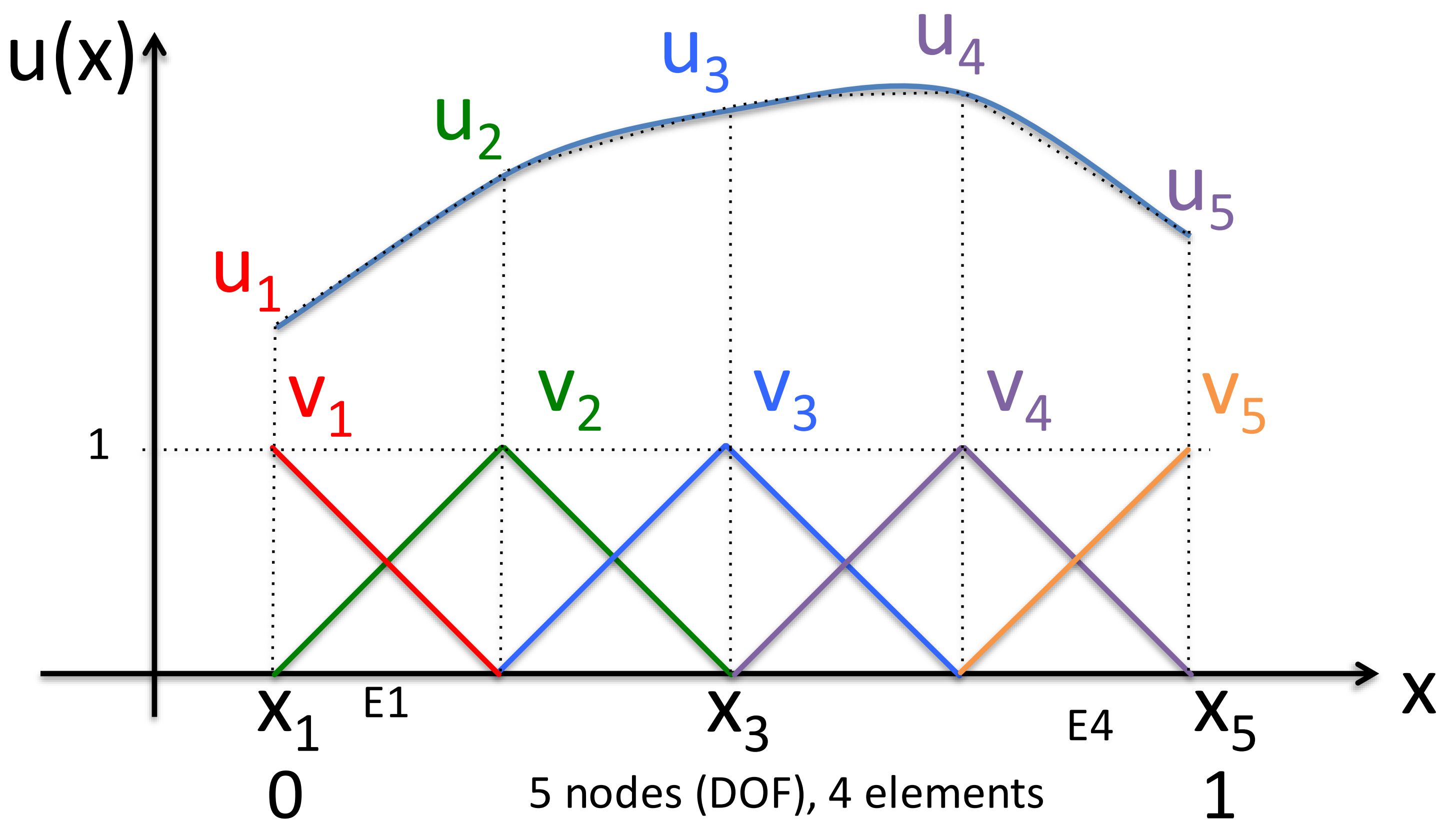
Example

- Strong form: $-u_{xx} = 1,\ u(0) = u(1) = 0$
- Exact solution: $(x - x^2)/2$
- Weak form: $$-\int u_{xx} \underset{\substack{\uparrow \\ \text{Test function } v}}{v} \overset{\substack{\text{Integration by parts} \\ \downarrow}}{=} \underbrace{-u_x v\big|}_{\substack{= \\ 0 \\ v(0)=v(1)=0}} + \int u_x v_x = \int u_x v_x = \int \underset{\substack{\uparrow \\ \text{Test function } v}}{v}$$

Strong form → week form: Multiply by a test function and integrate over the domain

Discretization: basis function u_i

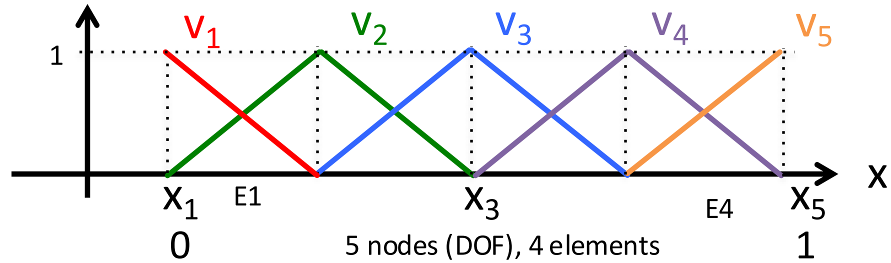
$$u(x) \approx \sum u_i\, v_i(x)$$



Matrix form

- $\int u_x \, v_x = \int v$ (weak form)
- For each element : $\sum u_i \int v_{xi} \, v_{xj} = \sum \int v_j$
- $\mathbf{K} = (K_{ij}) = (\int v_{xi} \, v_{xj})$, $v_{xi} = dv_i/dx$
- $\mathbf{u} = (u_i)$
- $\mathbf{L} = (L_j) = (\int v_j)$
- $\mathbf{K} \, \mathbf{u} = \mathbf{L}$
- Find **u** with **K** and **L** known

Stiffness matrix



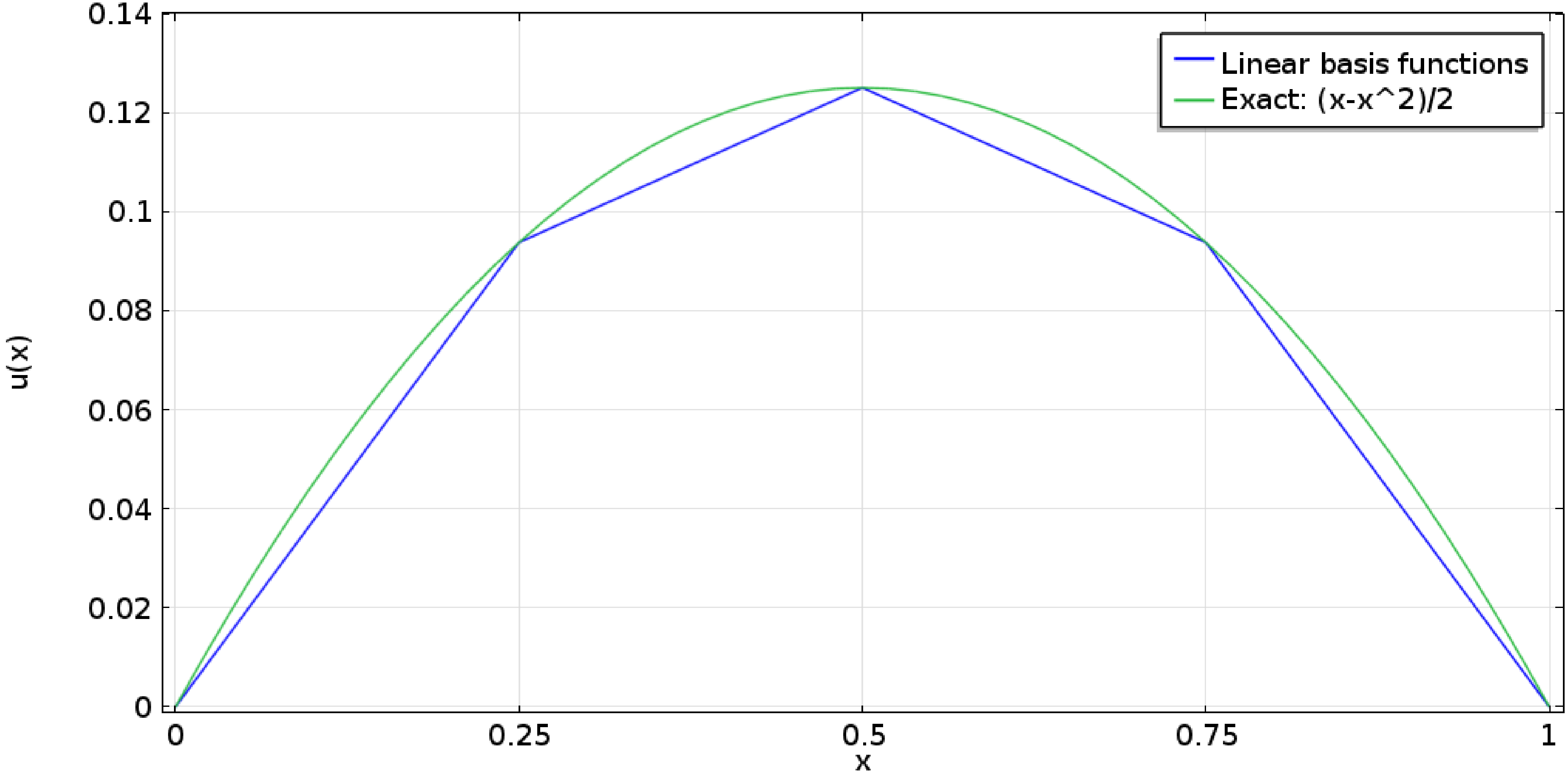
- $\mathbf{K} = (K_{ij}) = (\int v_{xi}\, v_{xj})$

- $v_{x1} = dv_1/dx = -1/(1/4) = -4 \;\rightarrow\; K_{11} = (-4)(-4)\underbrace{(1/4)}_{\text{Elements length}} = 4$
- $v_{x2} = dv_2/dx = \pm\, 4 \;\rightarrow\; K_{22} = (4)(4)\underbrace{(1/4)\ 2}_{\text{Elements length}} = 8$

- $$K = \begin{bmatrix} 4 & -4 & 0 & 0 & 0 \\ -4 & 8 & -4 & 0 & 0 \\ 0 & -4 & 8 & -4 & 0 \\ 0 & 0 & -4 & 8 & -4 \\ 0 & 0 & 0 & -4 & 4 \end{bmatrix}$$

- Stiffness matrix of each element → assembly

Solution



Linear basis functions
Exact: (x-x^2)/2
u(x)
x
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.25
0.5
0.75
1

Solving a linear system

- Ku = L, K is sparse and large
- Algorithm to avoid inverting the matrix
  - Direct
    - Exact (LU & Cholesky factorization)
    - Might require a lot of memory
    - For “small” problems
  - Iterative
    - Approximate (Preconditioning)
    - Convergence might be an issue
    - For “large” problems

Matrix form

$$\mathbf{E\,u'' + D\,u' + Ku = L}$$

- **E**: mass matrix
- **D**: damping matrix
- **K**: stiffness matrix
- **L** : load vector

$$\mathbf{u' = du/dt,\; u'' = d^2u/dt^2,}$$

FEM

- Discretization of the domain
- Evaluate the local displacement
- Evaluate the locale force
- Sum local forces
- Assembly of the matrices
- Solve the matrix system

Nonlinear Partial Differential Equations

↓ FEM

Nonlinear Ordinary Differential Equations

↓ Newton method (nonlinearity)

Linear Ordinary Differential Equations

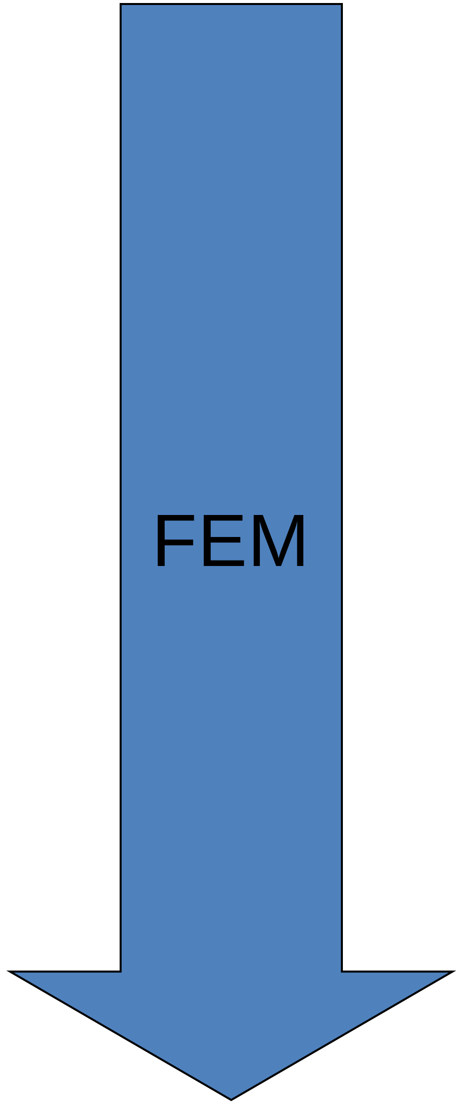
↓ Euler method (time)

Linear Algebraic Equations

↓ Ku = L (direct/iterative, preconditioning)

Solution

Nonlinear Partial Differential Equations



Solution

Nonlinearity

- PDE Navier-Stokes
- Geometrical: E = E(u) Large displacement
- Material: S = S(E) Soft tissues
- Contact: interfaces between device and tissue
- Plasticity, wear, etc.

→ Newton method

Nonlinearity

Newton method

- Solve $R(u) = Ku - L = 0$ (Residue)
- Iterative method (linearization)

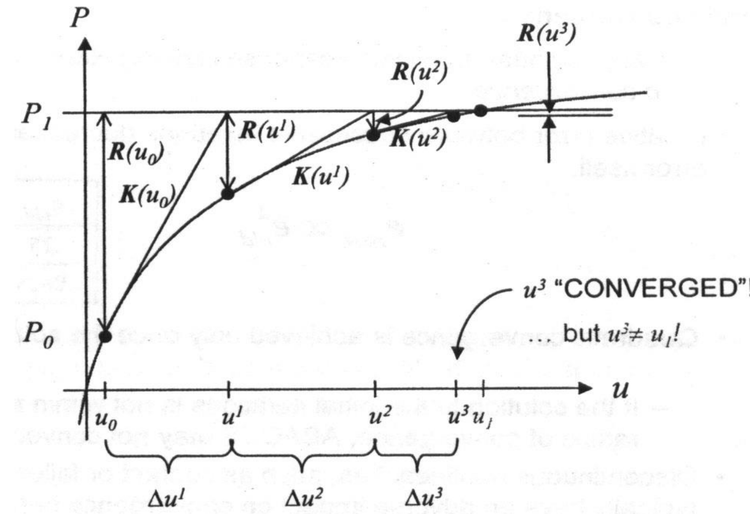
$$R(u^{i+1}) \approx R(u^i) + K^i(u^{i+1} - u^i) = 0$$

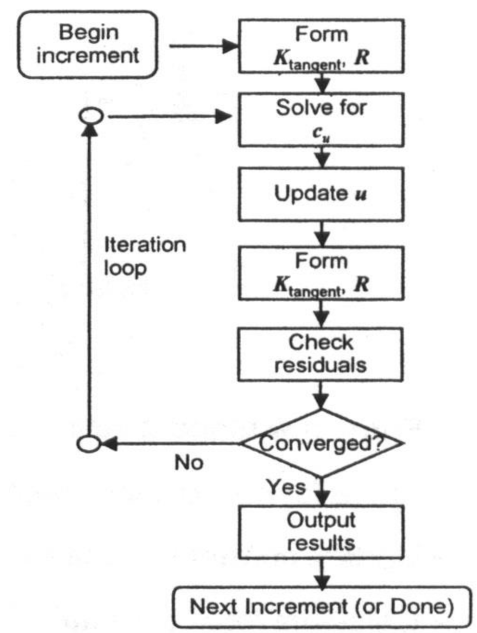
$$K^i = \text{estimate of } \boxed{dR/du(u^i)}$$

$$u^{i+1} = g(u^i) = u^i + \Delta u \qquad (\Delta u : \text{correction})$$

- Convergence tests:
  (F is an average force)

$$\boxed{\begin{array}{ll} R/F & < \varepsilon \\ \Delta u/u & < \varepsilon \end{array}}$$





Nonlinearity

- Provide (good) initial values
- Use parametric solver
- Use time-dependent instead of stationary
- Check for singularities
- Refine mesh in high gradient zones
- Solve physics sequentially, and then coupled
- Stabilization techniques (artificial diffusion, viscosity)

Time dependency

- Solve time dependency of PDE
- Linear Ordinary Differential Equations
- Time dependency in coefficient u_i
- $u(x,t) = \sum u_i(t)\, v_i(x)$
- Euler method

Time dependency

Euler method (Finite Difference Method)

- $u'(t) = f[u(t)]$, $u(0) = u_0$

- $u_{i+1} = u_i + (1\text{-}a)\ \Delta t\ v_i + a\ \Delta t\ v_{i+1}$
  $v_i\ \ \ = f(u_i)$
  $v_{i+1} = f(u_{i+1})$

  a = 0 : explicit (Euler), conditionally stable
  a = 1 : implicit (backward Euler), unconditionally stable
  a = 1/2 : Crank-Nicolson

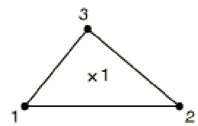
- Newmark: generalization for 2nd order diff equ.

Implicit/Explicit

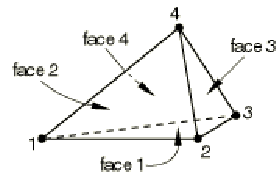
- Implicit
  - Static, quasi-static, dynamic slow
  - Unconditionally stable
- Explicit
  - Dynamic
  - Complex contacts
  - Very large models
  - Stability more difficult to achieve

Elements

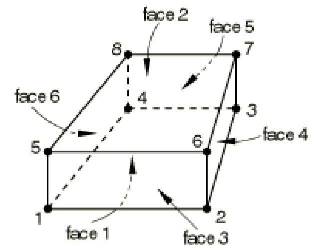
- Element type:
  - (1D) Edge,
  - (2D) Triangle, Quadrilateral,
  - (3D) Tetrahedron, Hexahedron, Prism, Pyramid
- Element (shape function) shape and order
  - 1 (linear) to 5 order polynomia
- Meshing techniques, optimal element type
  - Structured/unstructured
  - Seeding (size)
  - Avoid sharp corners
  - Tools: sweep, boundary layers (fluid)
  - Mesh quality (Jacobian: ideal (1), bad (0), inverted (<0)



3 - node element

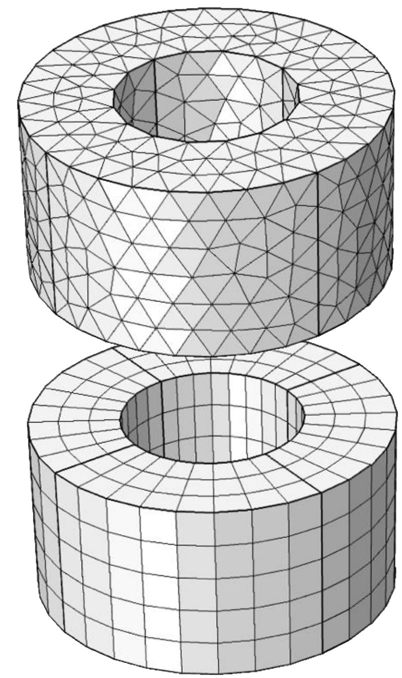


4 - node element



8 - node element

Default unstructured mesh



Structured mesh

Contact

- Source-destination (master-slave) approach
  - Destination (slave) can’t penetrate source (master)
  - Source on stiffer boundary
  - Source on concave boundary
  - Destination mesh 2 times finer than source
- Iterative algorithm
  - Find contact region (penetration)
  - Apply forces to push back penetration
- Normal direction discontinuities
  → oscillations (contact chatter)

Error control

- Error = $O(h^{p+1})$
- h: size of the mesh (element)
- p: order of the polynomial basis functions
- Adaptive meshing based on error indicator

  h-adaptive refinement (mesh size)

  p-adaptive refinement (polynomial order)
- A posteriori methods (energy)

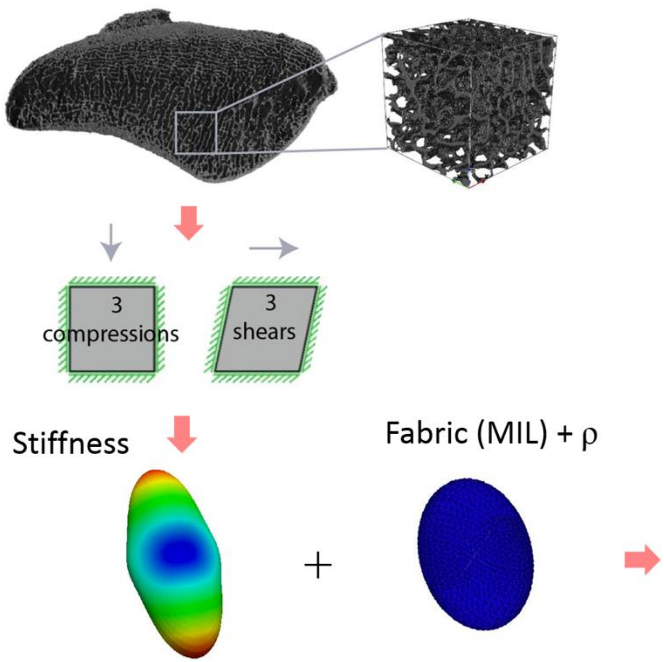
Micro-Finite Element

- Micro-FE: FE based on bone structure
- Micro-CT imaging (10 μm vs 300 μm for CT)
- Used to evaluate bone strength (fracture), in vivo
- Identify (validate) homogenized models
- No need for (non-homogeneous) constitutive law
- CPU power

Van Rietbergen, B. and K. Ito (2015). "A survey of micro-finite element analysis for clinical assessment of bone strength: the first decade." J Biomech 48(5): 832-841.

Micro-Finite Element

Identify, Validate, Extend (homogenized-) FEM



Anisotropic linear elastic material $\varepsilon = C\,\sigma$

$$
C = \begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & & & \\
-\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & & 0 & \\
-\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & & & \\
 & & & \frac{1}{2G_{23}} & & \\
 & 0 & & & \frac{1}{2G_{13}} & \\
 & & & & & \frac{1}{2G_{12}}
\end{bmatrix}
$$

Morphology – elasticity relationship
(Zysset – Curnier)[1]

$$E_i = E_0 \rho^k m_i^{2l}$$

$$\frac{E_i}{\nu_{ij}} = \frac{E_0}{\nu_0} \rho^k m_i^l m_j^l$$

$$G_{ij} = G_0 \rho^k m_i^l m_j^l \quad \forall\, i \neq j = 1,2,3$$

M – fabric tensor[2]
ρ – BV/TV

E_0, ν_0, G_0, k, l – constants to identify

EPFL-LBO (Latypova, 2015)

Physics coupling

In some situations (experiments),
physical (and chemical) phenomena
are strongly interdependent and
Physics (PDE) must be coupled to get
a reasonable solution

$\Downarrow$

multiphysics modeling

Coupling in biomechanics

- Peri-implant healing (fluid-solid)
- Prostheses analyses (solid-solid contacts)
- Tissue engineering (poroelasticity)
- Cell mechanics (advection-diffusion-reaction)
- Medical devices (EM, heat, piezo)
- Adaptive process (bone, muscles)

Where appears the coupling?

Different physics, with different dependent variables, but where appears the coupling?

- PDE (convection, poroelasticity)
- Constitutive laws (non-isothermal flow)
- Boundary conditions (fluid-solid interaction)
- Contacts between solids
- Other (medical devices control, MEMS)

Types of coupling

- Physical coupling
  - Coupling between physics (field)
  - Coupling within materials (constitutive laws)
  - Coupling at interfaces
- Modeling coupling (operators)
  (modeling simplification between source and destination submodels)
  - Extrusion (2D → 3D)
  - Projection (3D → 2D)
  - Integration (2D/3D → 1D)

Coupling modeling

- 1-way coupling
  - One physics influence the other one, but it is not reciprocal
- 2-way coupling
  - Both physics influence each other

- Weak coupling: slight coupling effect
- Strong coupling: important coupling effect

Coupling: Joint - Tissue

- Joint → Tissue
  - Multi-level, one-way coupling
  - Not always acceptable (shoulder, patella)
- Joint ⟷ Tissue
  - Multi-level, two-way coupling

Patient-specific modeling

- Replicate patients with model
- Specific prediction for specific patient
- Identify groups of patients by prediction
- Group control studies
  - Pathology group (of patients)
  - Control group (no pathology)

Patient-specific modeling

- Advantages
  - Closer to clinical reality
- Difficulties
  - Material properties, boundary conditions
- Methods
  - Redo from scratch each new patient
  - Modify a generic parametric model

Verification and Validation

Verification: solving the equations right

Validation: solving the right equations

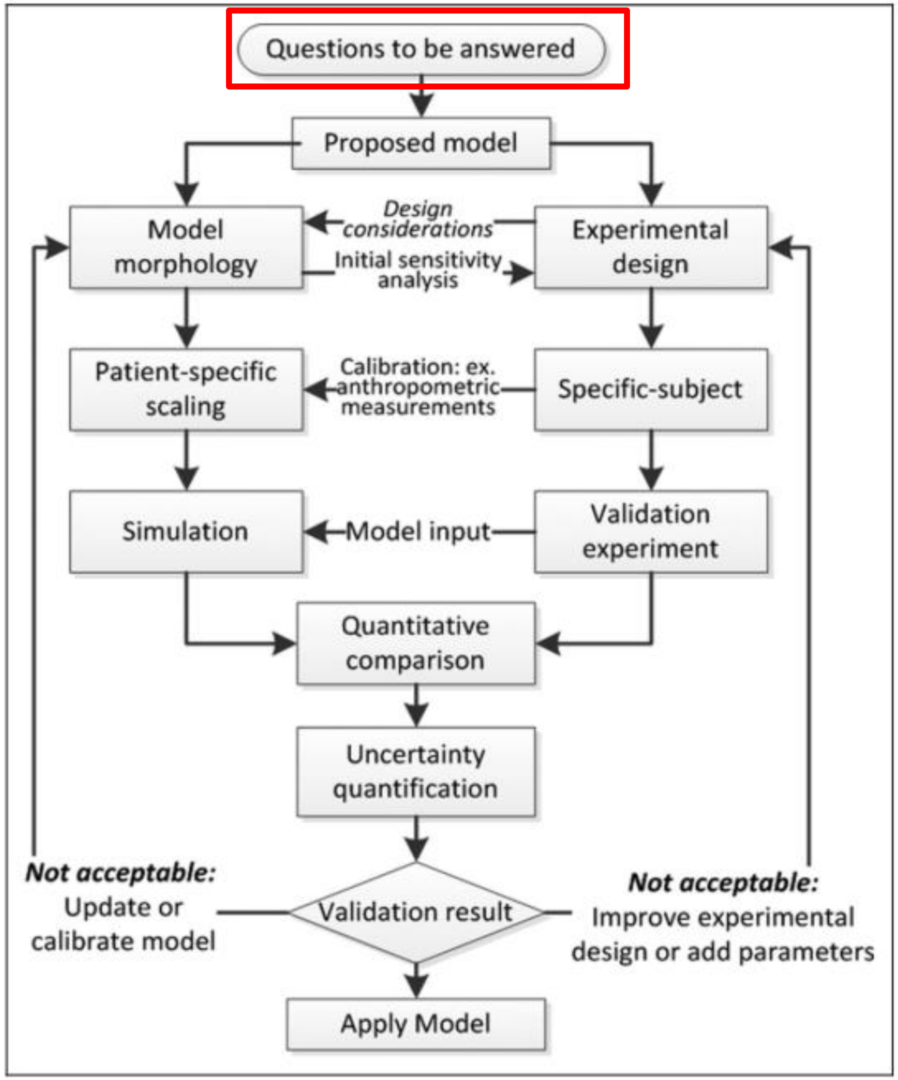
Verification

- Poor sister of validation
- Code/Calculation verification
- **Mesh convergence (discretization error)**
- Expected smoothness of solution
- Expected physical behavior
- Sensitivity analysis of (all) model parameters
- Compare with (semi-) analytical solution (simplified model)

Validation

- Accuracy assessment (error between num. & exp.)
- Experimental error: random + systematic (value ± SD)
- Numerical error: solver type, input parameters (value ± SD)
- Statistical analysis
- Reject null hypothesis: no correlation between num. & exp.
- Predictive capability of the model
- Limitations, field of application
- Validation ≠ (experimental) identification (calibration)
- Direct validation: comparison with specific experimental data
- **Indirect validation: comparison with literature**

Validation

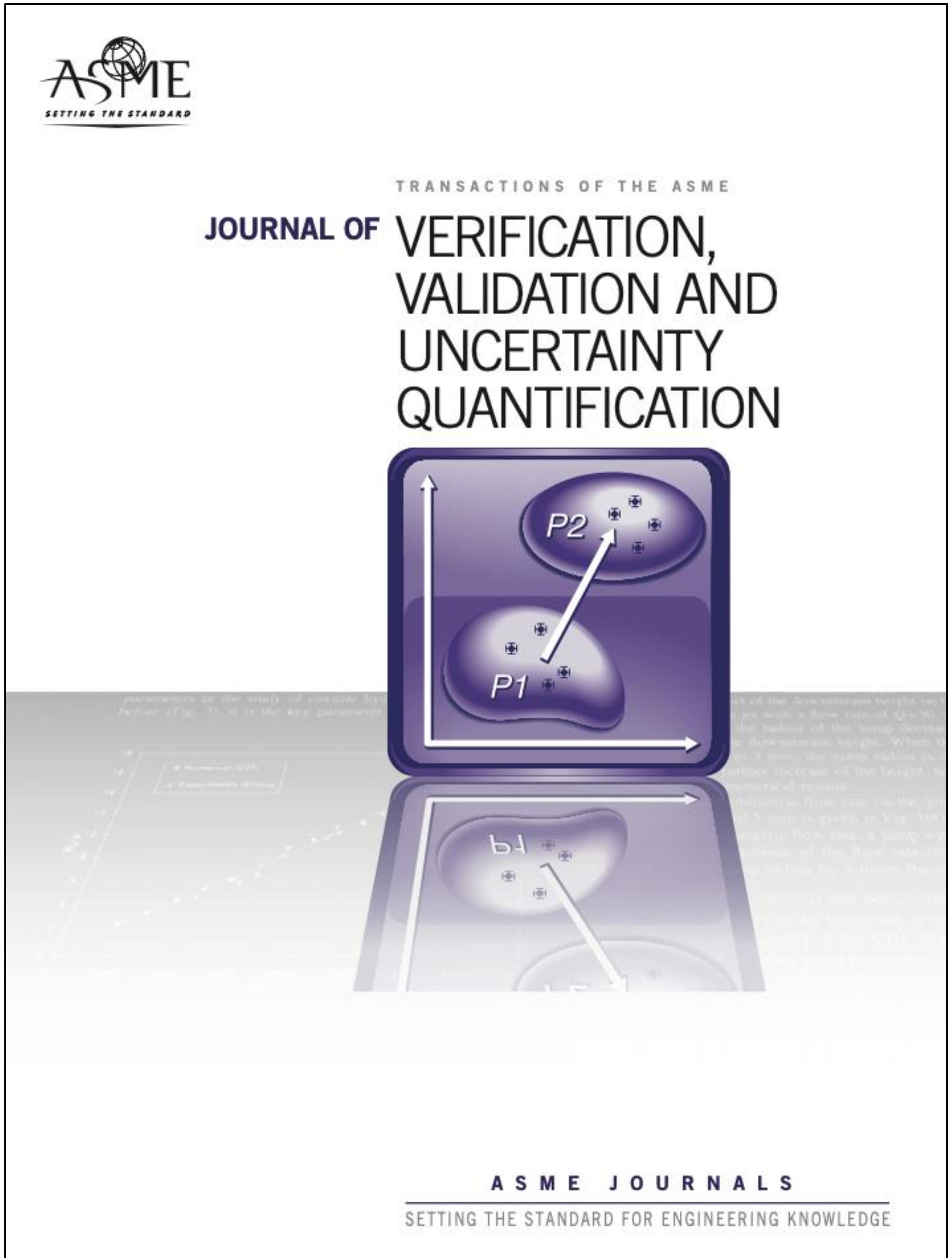


Lund et al (2012) Proc Inst Mech Eng H **226**(2): 82-94.

Uncertainties quantification

- Uncertainties (prediction, measurements)
- Sensitivity analysis
- Effect of (unknowns) “input” parameters on “output” quantities
- Evaluate most critical “input” parameters

Since 2016…


ASME
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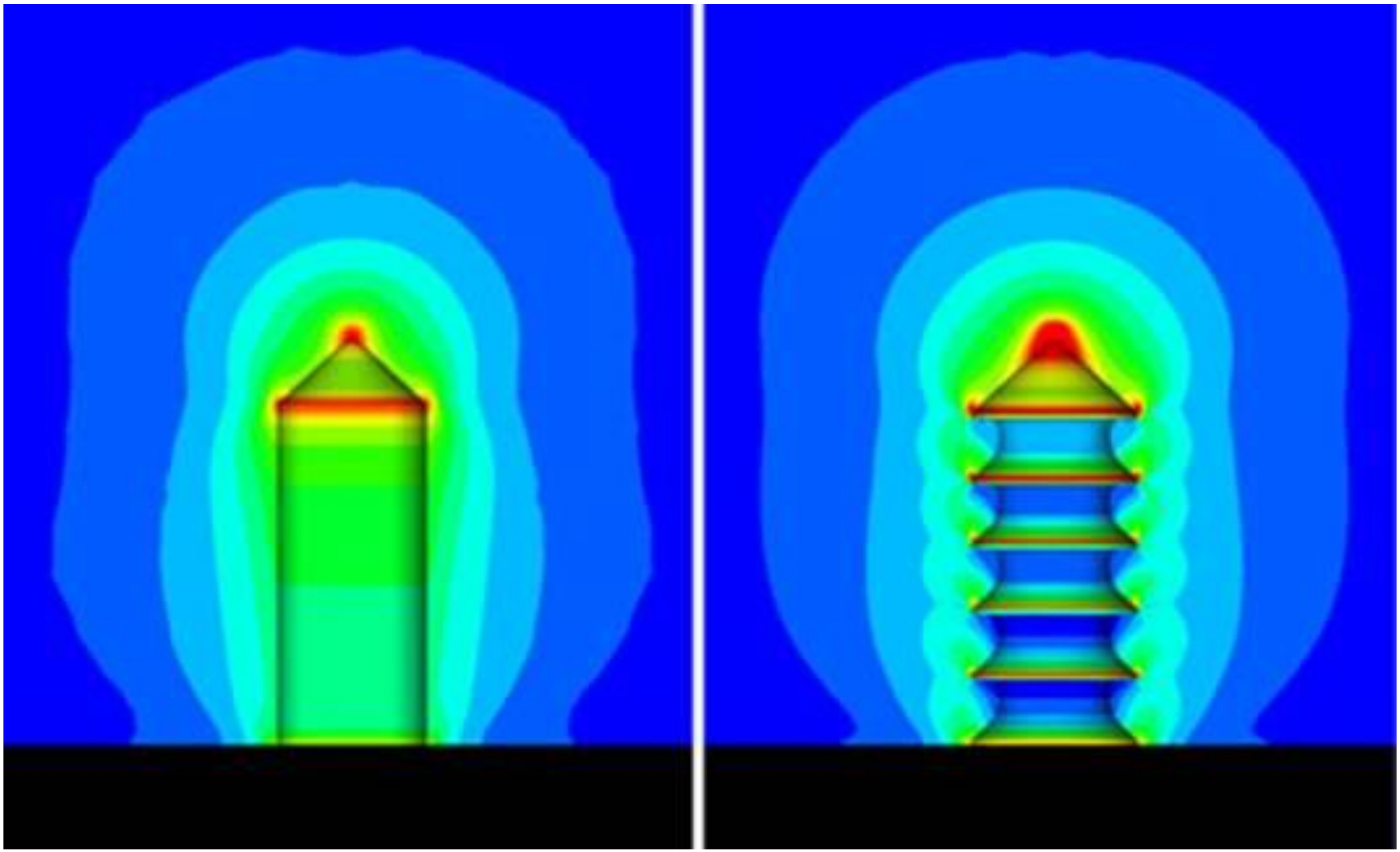
TRANSACTIONS OF THE ASME
JOURNAL OF VERIFICATION, VALIDATION AND UNCERTAINTY QUANTIFICATION
P2
P1
ASME JOURNALS
SETTING THE STANDARD FOR ENGINEERING KNOWLEDGE

General advices

- Start VERY simple
- Add complexity gradually
- Verify as much as you can while adding complexity
- Use symmetry as much as possible
- Do not over-complex your model
- Evaluate the range of validity of your model

Screw pullout test



Terrier, 2005

Summary

- Advantages of numerical methods
  - Between theory and experiment
  - Reduce time of development
  - Reduce costs
  - Reduce risks
  - Increase understanding
  - Increase creativity
- Applications
  - Test hypotheses
  - Parameters analysis
  - Phenomena understanding
  - Rapid prototyping (feasibility analysis) of complex systems or devices
  - Patient-specific modeling

Progress, Difficulties & Future

- Over-complexity of models
- Material properties (in vivo)
- Loading conditions (activities of daily living)
- Patient-specific models (statistical analyses)
- Better corroborate (validate) models with reality
- Retrospective study to explain failure mechanisms
- Preoperative tool
- Adaptation processes

Taylor & Prendergast (2015). "Four decades of finite element analysis of orthopaedic devices: Where are we now and what are the opportunities?" J Biomech 48(5): 767-778.

FEM software

- Comsol (www.comsol.com)
  - Multiphysics
- Ansys (www.ansys.com)
  - Solid, fluid (Fluent), heat, electromagnetics
- Abaqus (www.simulia.com)
  - Solid, heat, (fluid, electromagnetics)
- BETA CAE (www.beta-cae.com)
  - Solid, multiphysics
- Elmer (www.csc.fi/elmer)
  - Multiphyisics, free
- FreeFEM (www.freefem.org)
  - Multiphysics, free
- FEBiO (www.febio.org/)
  - Solid, free
- ArtiSynth (artisynth.magic.ubc.ca/artisynth/)
  - Coupling Rigid body dynamics & Solid deformation, free

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