Exercise 5: Models for competitive exclusion: microbial ecology

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1 Week 5: Graded Series

1.0.1 Course: BIO-341 *Dynamical systems in biology*

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SSV, BA5, 2024 (Graded exercise series)

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interact
from scipy.integrate import odeint
```

All working by hand and calculations need to be shown

2 Linear phase portraits (10 pts)

Consider the following linear system:

$$\dot{x} = x + 2y$$
$$\dot{y} = 4x + 3y$$

1. Find the fixed point(s) and specify its/their stability. (by hand)

2. Find the eigenvalues and eigenvectors of the matrix associated with this system. (by hand)

3. Use 2. to write the general solution of the system. (by hand)

4. Plot the phase portrait in the (x, y)-plane.
Plot **by hand**:

- the fixed point(s)
- the nullclines
- the eigenvectors

Plot **using python**:

- the vector field
- Representative trajectories for different initial conditions listed in the array defined below (X_0s).

Hint: To plot the vector field you can use: - np.meshgrid to define the positions at which you want to represent the arrows. - Use plt.quiver to plot the arrow field - You can also look at previous corrections to understand how to plot a phase portrait using Python.
Hint: To plot trajectories, use the explicit solution that you have found in 3.

Plot for values of x and y in the interval [-5,5]. For the trajectories, use the tspan vector defined below.

[2]:
```
# Time domain
tspan = np.linspace(0, 10, 1000)

# Initial conditions
X0s = [(0,-2), (-3,1),(3,-2),(3.1,-3.5),(-5,5.01)]
```

3 Competitive exclusion in microbial ecology (20 pts)

In this model, two populations of bacteria N, M (e.g., *Escherichia coli* and *Lactobacillus acidophilus*) compete for the same resources in the same environment (for example in the gut). A general model that describes this interaction takes the form :

$$\frac{dN}{dt} = N\left(1 - \alpha N - M/\alpha\right)$$
$$\frac{dM}{dt} = M\left(1 - N - M\right)$$

with $\alpha \geq 0$.

1. Calculate the nullclines, find the fixed points, and an expression of the Jacobin for the system depending on α. (by hand)

2. Calculate the Jacobian matrix for $\alpha = 1/4$ and $\alpha = 4$ at every fixed point. In both cases, plot the nullclines and state the stability of the fixed points (by hand)

3. Plot the phase portraits for the two cases $\alpha = 1/4$ and $\alpha = 4$.

Plot **using Python**:

- the nullclines
- the fixed points
- the vector field
- Some representative trajectories for the initial conditions listed in the array below (X_0s)

  Hint: Since here you don't have the exact solution,you can use the odeint function of scipy. Look at previous corrections to understand how this works

Plot the phase portait for x and y in the interval: [-0.5, 5]. For trajectories, use the tspan defined below.

[3]:

```
# Declare the initial conditions
X0s = [(5,1), (5,4), (1,5), (3,1),(2,0.1)]

#time domain
tspan = np.linspace(0, 10, 1000)
```