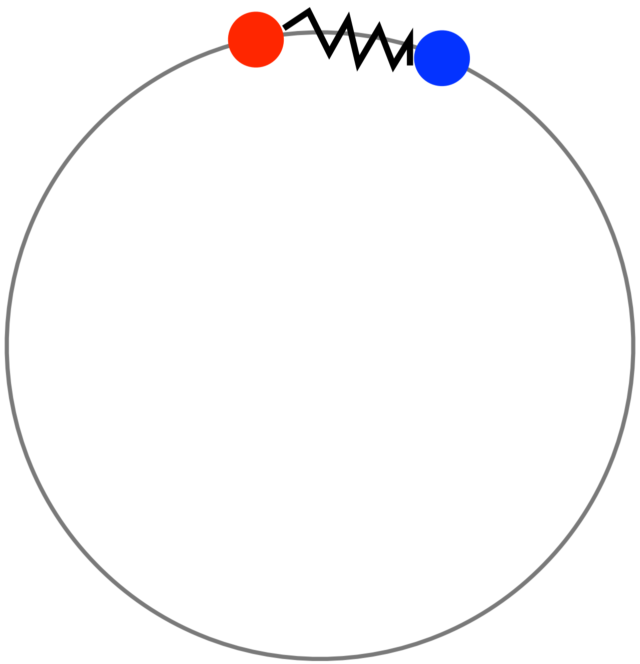
Phase oscillateurs & entrainement

Two coupling models

Model I (6.1)

$$\dot{\alpha} = \Omega$$

$$\dot{\theta} = \omega + K \sin(\alpha - \theta)$$



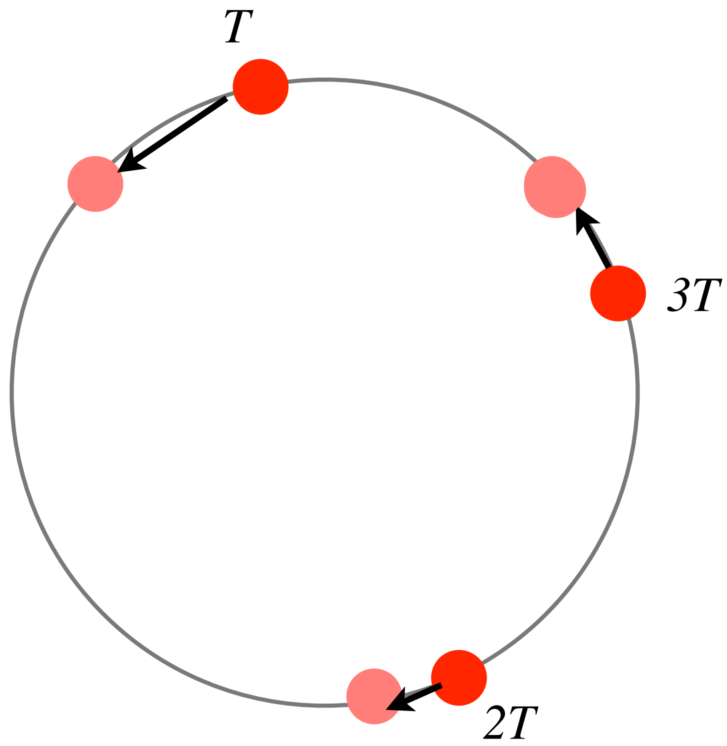
spring

Model I (6.2)

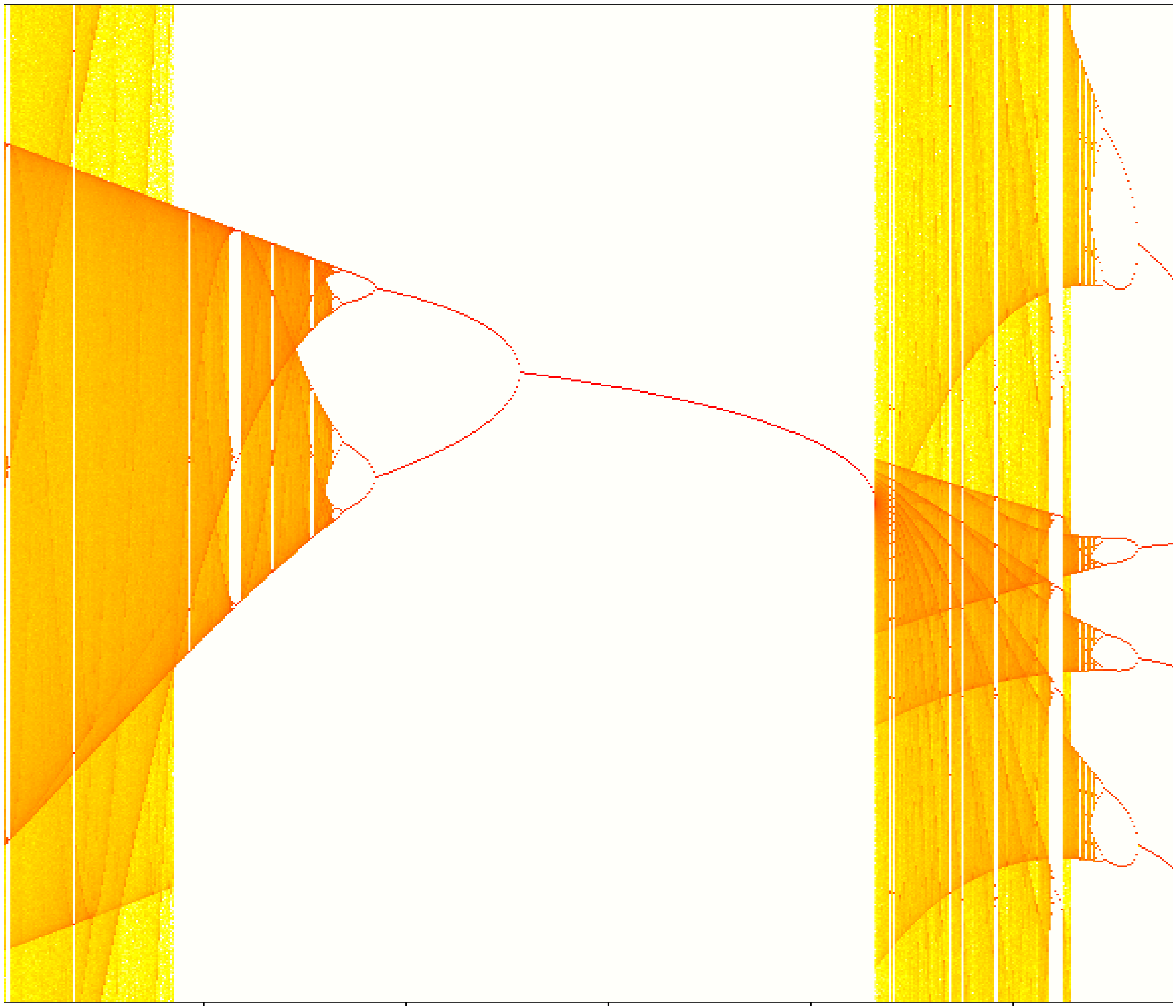
$$\dot{\theta} = \omega + \sum_n \delta(t - n\,T) g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega\,T + g(\theta_{n-1})$$

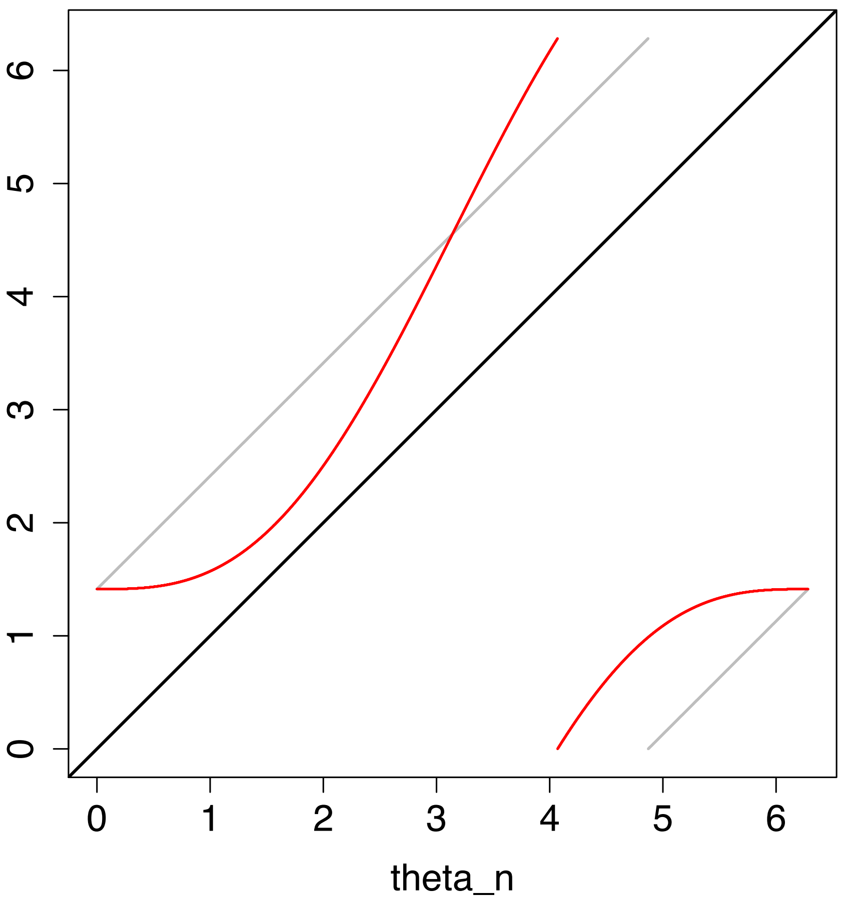
$$g(\theta) = e\,\sin(\theta)$$

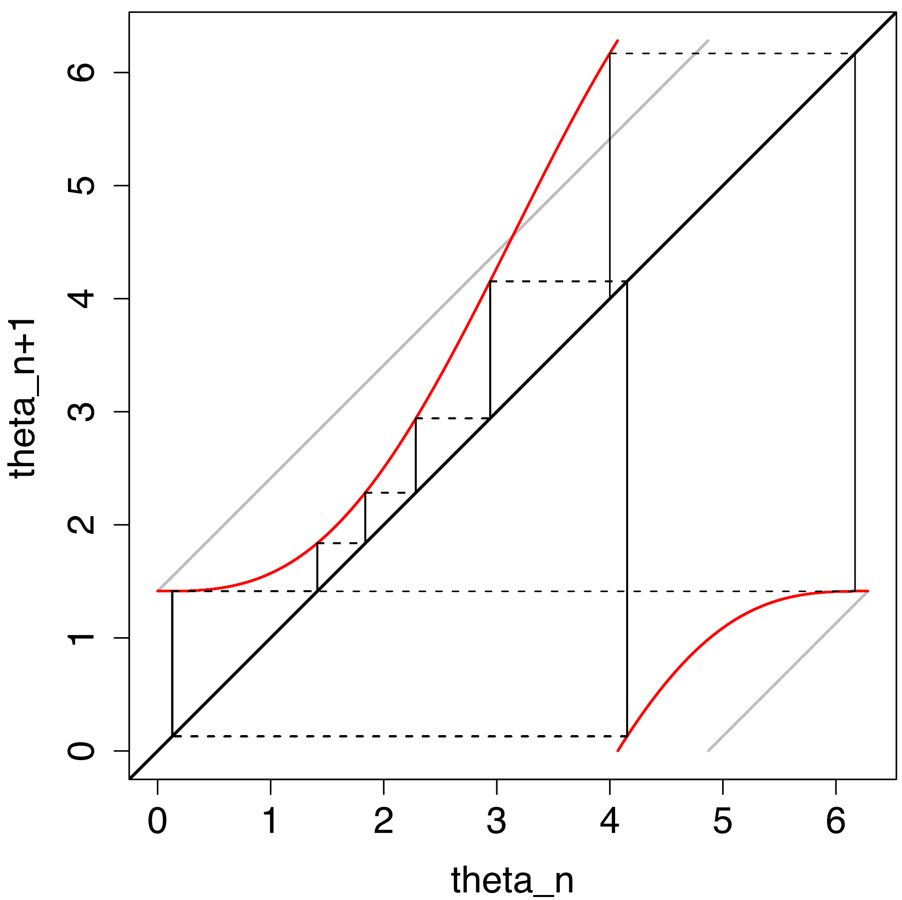


instantaneous kicks



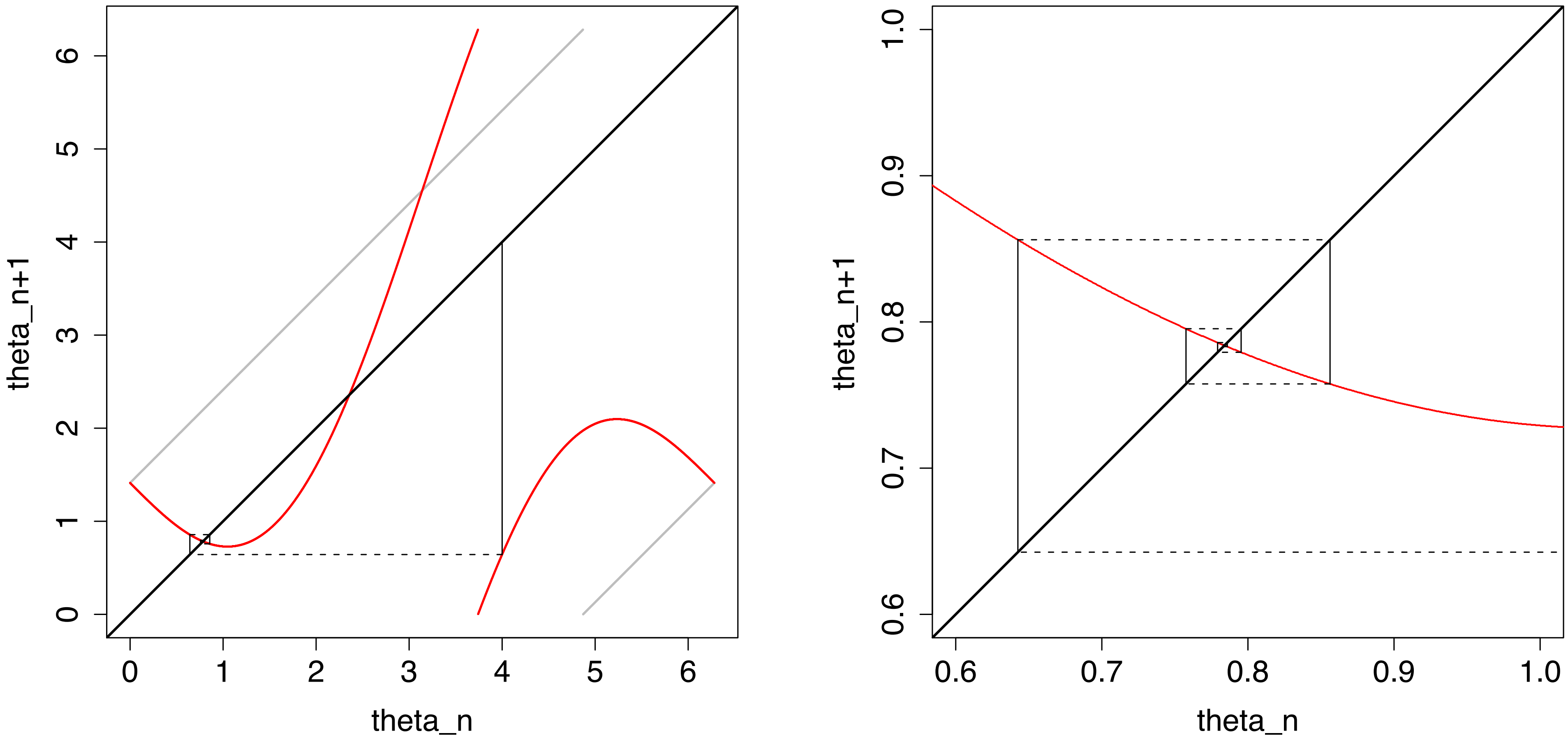
Stability (no fixed point) $|e| < T\omega \sim 1.4$





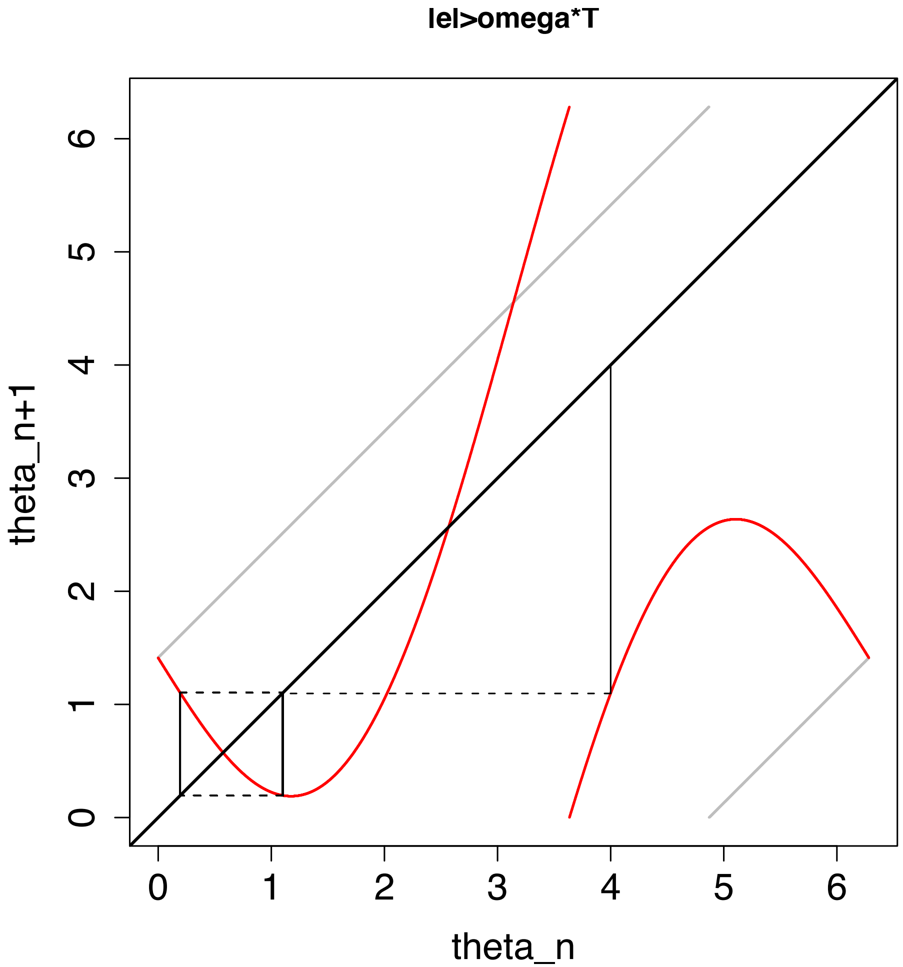
e=-1

Stability (period 1 orbits) $|e| > T\omega$



e=-2

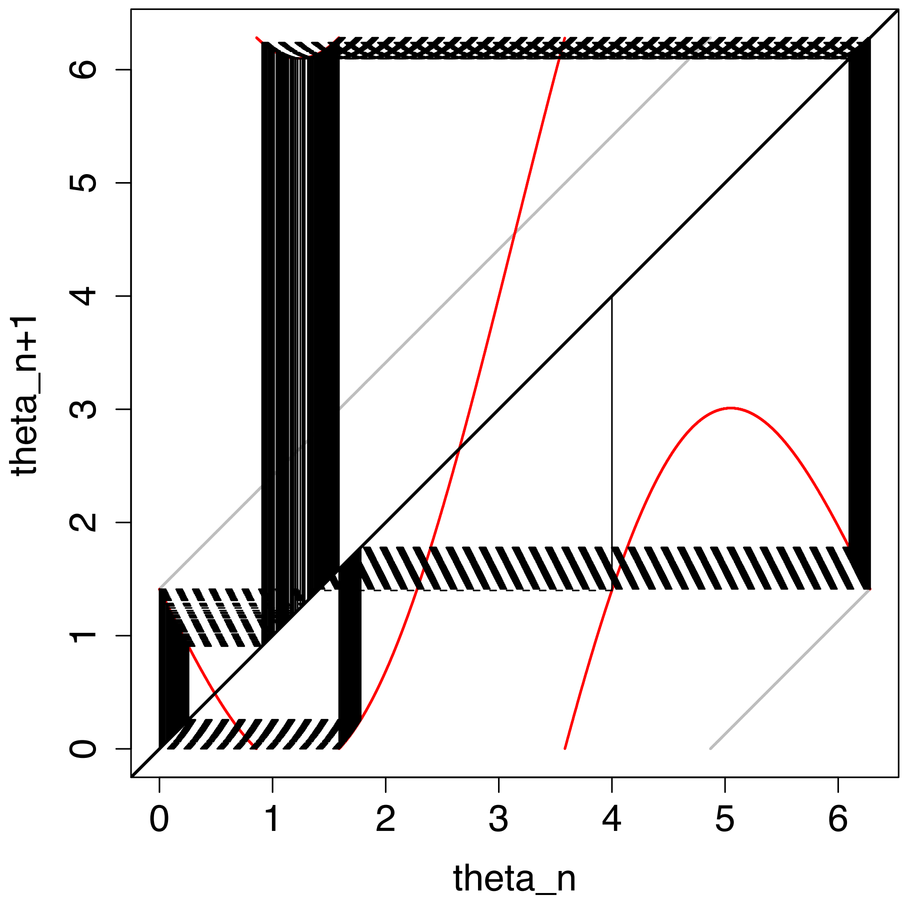
Stability (period 2 orbits) $|e| > T\omega$



e=-2.6

Chaotic orbits

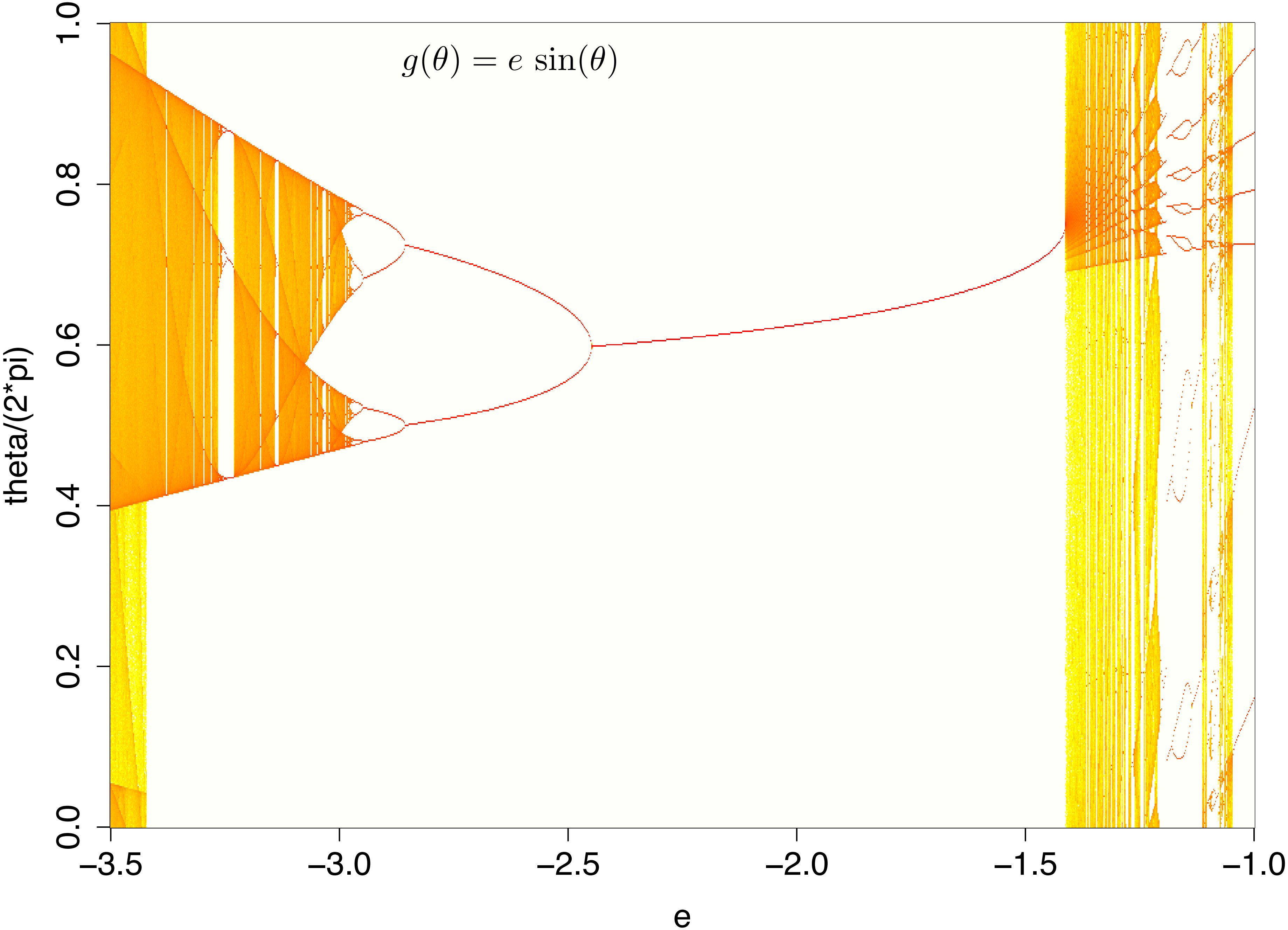
$|e| > T\omega$



e=-3

$$\dot{\theta} = \omega + \sum_n \delta(t - nT)g(\theta(t))$$

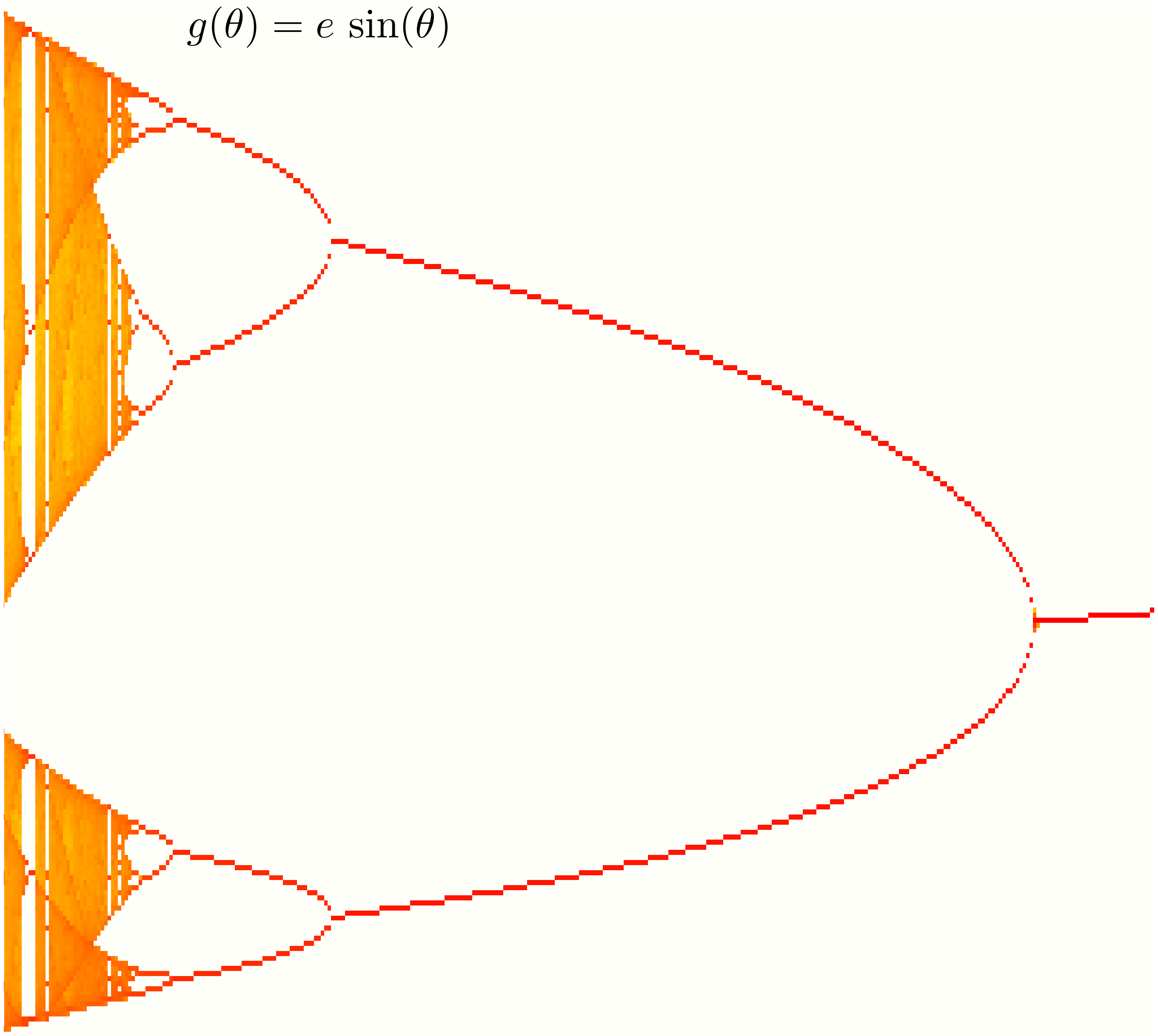
$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$



$$\dot{\theta} = \omega + \sum_n \delta(t - nT) g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

$$g(\theta) = e \sin(\theta)$$



Remarks

- <u>no chaos</u> in 2D ODE system (but chaos in 3D)
- very complex (and beautiful) behavior in the kicked phase oscillator
- chaos in 1D maps (discrete dynamical systems)