

Final Exam

09:15 - 12:15

Wednesday, 29th January

Room: PO 01

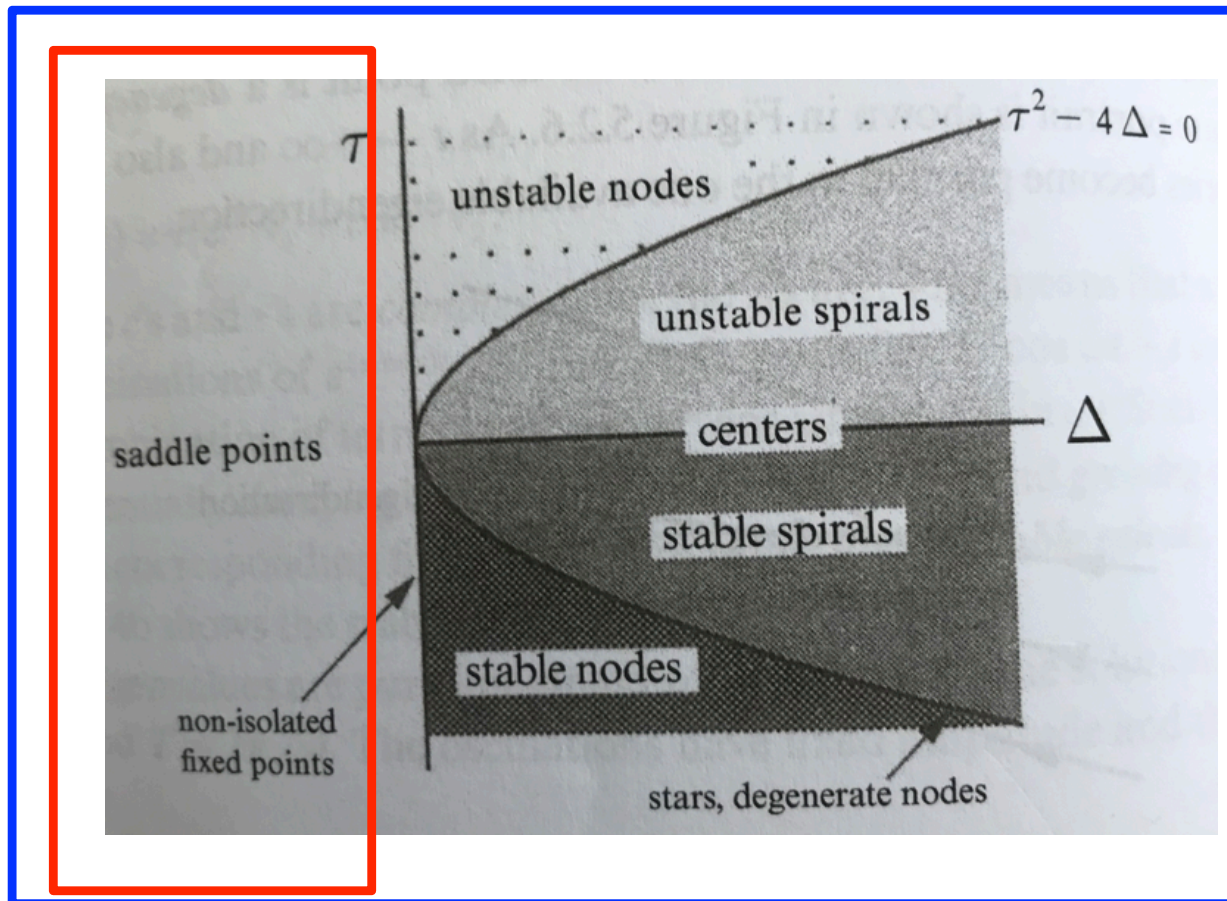
Instructions for the procedure, seating plan, etc will be distributed nearer the time.

Comments on first Graded Exercise

If a questions asks for all calculations to be shown, don't just state the answer

Saddlepoints are not stable or unstable, they're just "a saddlepoint"

Saddlepoints are not saddle nodes.



Fixed points

Saddlepoints

Nodes, star nodes, spirals, centres, etc.

Lecture 6 Recapitulation

There are only a limited number of interesting elements that can be combined in a dynamical system (in any dimension):

- constant production or decay rate $\sim s$ and $-s$
- linear growth / decay $\sim r x$ / $-r x$
- logistic growth $= x (1 - x)$
- competition $\sim -xy$
- anti-competition $\sim xy$
- saturating competition $\sim -xy / (1 + x^{**2})$

The goal is to understand what these terms mean, and be able to combine them to construct new models. These can be used in many fields, from engineering, neuroscience, psychology, ecology, etc.

Lecture 7 Limit Cycles

- Where are we on the search for robustness in living organisms?
 - Fixed points were the first attempt (but they're fixed!)
 - Hysteresis was next (stable state resists small perturbations)
- We have seen all types of fixed point in *linear* 2D systems (tau delta plot), and these also occur in non-linear systems
- There is one more type of FP that only occurs in *non-linear* 2D (and higher) systems: *a limit cycle*
- Limit cycles represent systems that can *oscillate* without an external driving force, e.g., heart contraction, chemical reactions, neurons
- The amplitude, frequency, and shape of a limit cycle are set by the **equations** not by the initial conditions; if perturbed, they return to the limit cycle

Background quiz: go.epfl.ch/turningpoint

Session Id: [julian23](#)



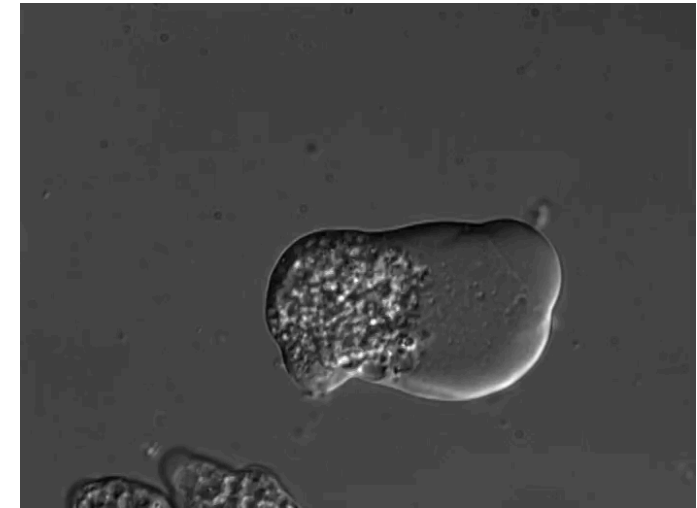
All input is anonymous; data are stored outside CH

Break

MAKE Project Announcement: Designing Life with AI

Kickoff event on the 30th of October

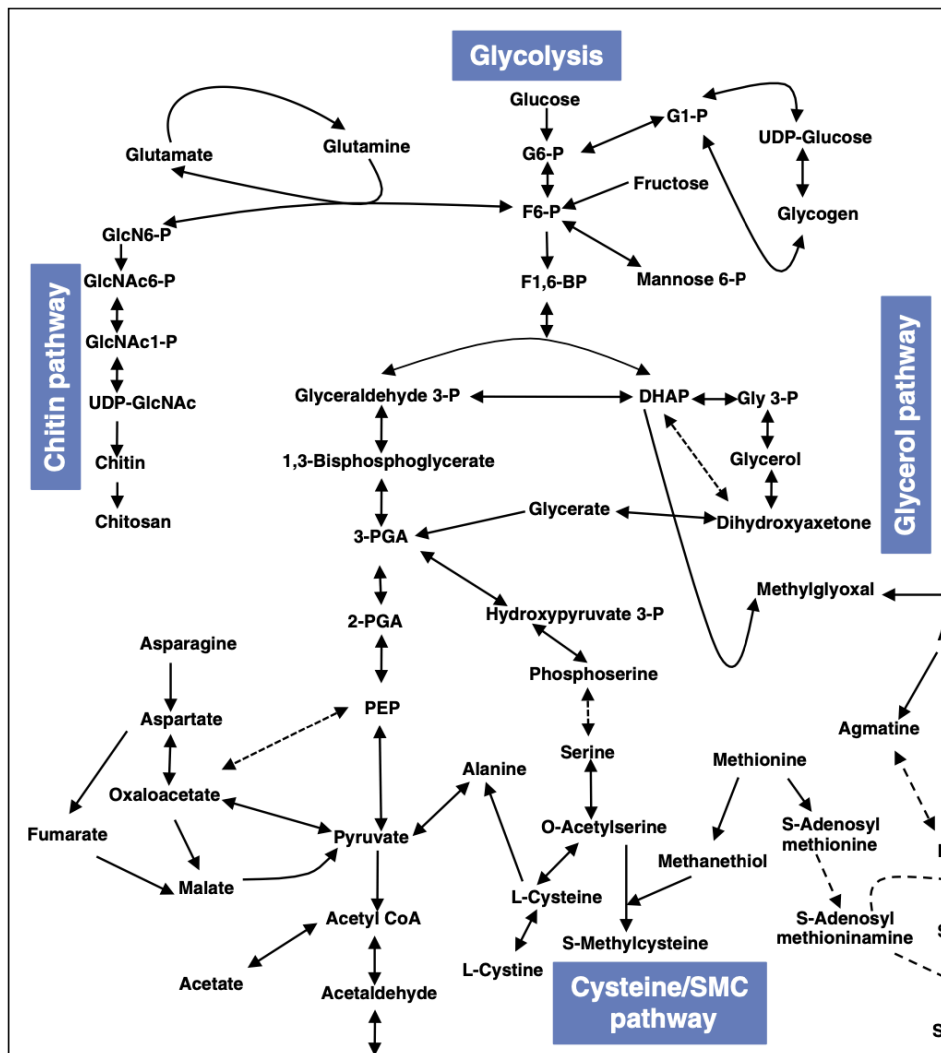
Glycolysis (course notes Ch. 5.2)



Entamoeba histolytica

Anaerobic
No oxidative phosphorylation
No mitochondria

Produces ATP in cytoplasm



Metabolomic analysis of *Entamoeba*,
Jeelani and Nozaki, Current Op. Microbiol. 20:118 (2014)

Sel'kov model of glycolysis

Strogatz, Ex. 7.3.2, page 207, course notes Ch. 5, page 42.

Sel'Kov, Self oscillations in glycolysis. A simple kinetic model.
Euro. J. Biochem. 4:79 (1968)



Let $x = [\text{ADP}] = \text{product}$
 $y = [\text{ATP}] = \text{reactant}$

The model is:

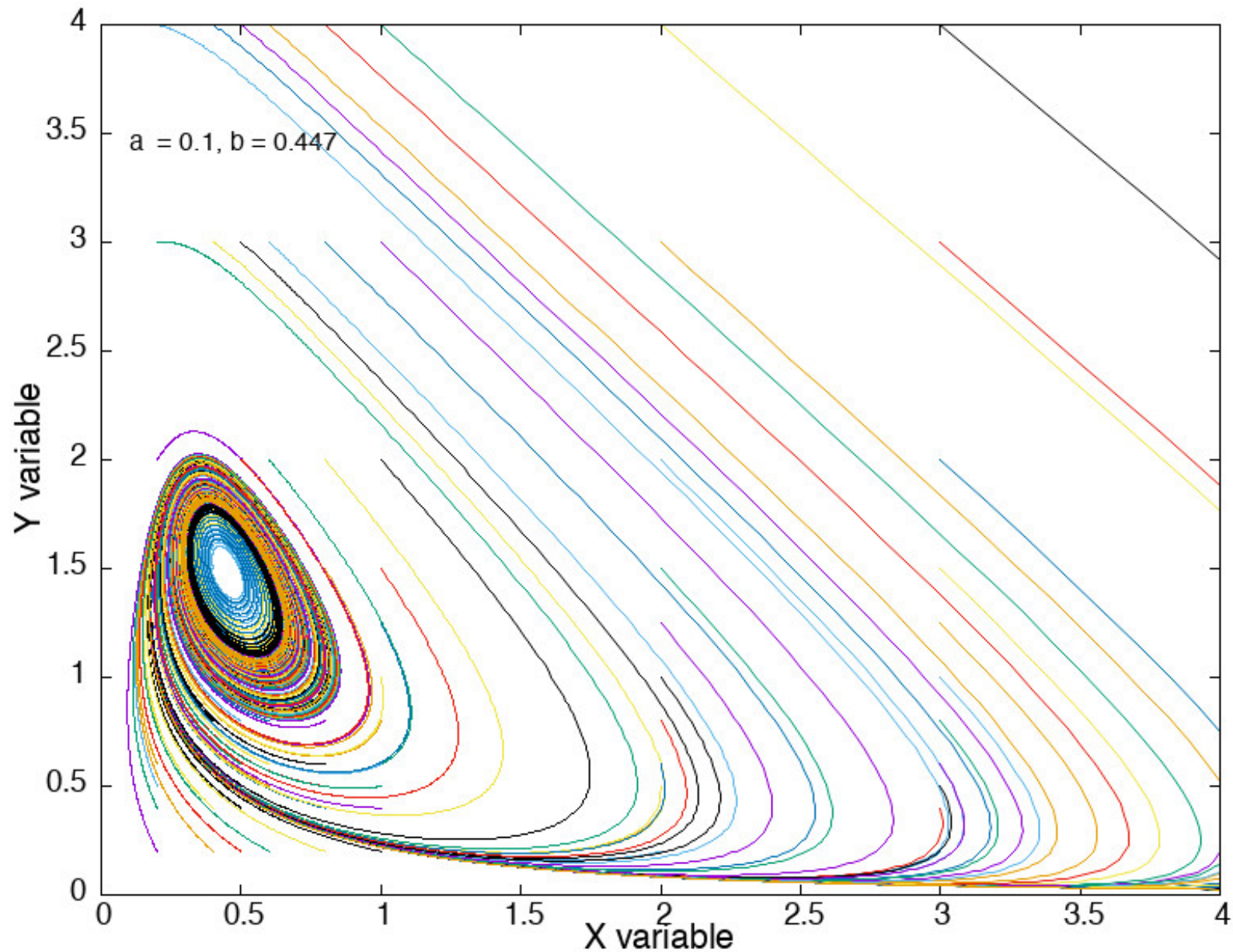
$$dx/dt = -x + a y + x^2 y$$

$$dy/dt = b - a y - x^2 y$$

with $a, b > 0$.

where do the oscillations
come from?

Try $a = 0.1$, $b = 0.447$
Fixed point at $(0.447, 1.49)$



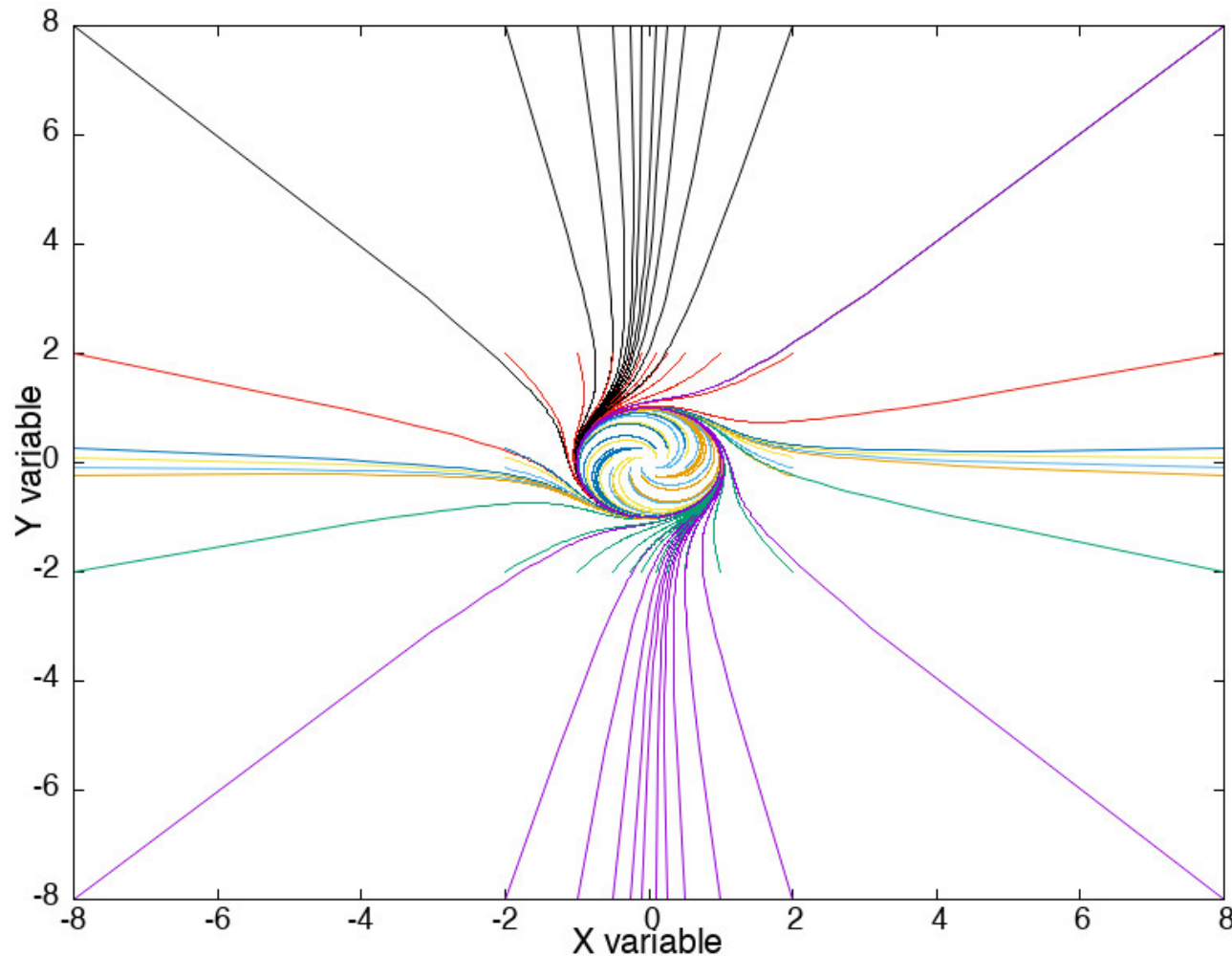
$$\begin{aligned} dx/dt &= -y + x(1 - x^2 - y^2) \\ dy/dt &= x + y(1 - x^2 - y^2) \end{aligned}$$

$$dr/dt = r(1 - r^2)$$

$$d\phi/dt = 1$$

in plane polar coordinates

Why are trajectories at large x, y straight lines? Shouldn't they be rotating?



“Tricky” points

- Distinction between a limit cycle and a centre is that the limit cycle is isolated, whereas centres are closed cycles infinitesimally close together
- Poincare Bendixson theorem needs two things: a node/spiral inside a closed bounding curve / trapping region. Finding a trapping region is usually the harder part.
- Boundaries of a trapping region don't have to be straight lines
- Check the field vectors on the corners of a trapping region: trajectories can squeeze through a tiny space (see slide 3 in the population model)
- Be able to find the direction of the trajectories for large X , Y coordinates far from any fixed point (e.g., curve 3 of the trapping region in the Selkow model)