Recap of Lecture 2 - Population models

Let $N(t)$ = size of a population, e.g., a single species like rabbits or E Coli, tumour cells, etc. that grow by reproduction/division only (i.e., no other influx of individuals).

We can write: $dN/dt = N\,R(N)$

and focus on dynamics of $R(N)$ = *relative rate of growth*. $N = 0$ is always a fixed point of such models, and $N >= 0$ by definition.

If $R(N) = r\,(1 - N / K)$ = logistic model.

But the two parameters r, K can be removed by non-dimensionalising:

$dM/d\tau = M\,(1 - M)$, where $M = N/K$, and $\tau = rt$.

Plotting dynamical systems is an art … (see Alle effect)_

Given $dN/dt = F(N) = rN (1 - N/K)$,

or $dN/dt = F(N) = rN - N/a (N - b)^2$,

it's important to be able to plot F(N) versus N (Graph 1) *by hand*.

Some general tips are:

- What is F(N) at N = 0?
- What does F(N) do as N tends to infinity?
- Where does F(N) cross the X axis? i.e., where are fixed points.
- Where does F(N) have turning points?

When F(N) has several parameters (as in Allee effect — r, a, b), you have to make an assumption about the magnitude of the parameters, plot the graph, then change the relative magnitude of parameters, replot, etc, until you have identified all the possible cases.

Lecture 3 Introduction

- Cells/organisms live in unpredictable environments
- Key properties: robustness, adaptability, control
- Fixed Points regulate the *shape* of trajectories in a system's dynamics, they control what possible behaviours can occur (*nothing interesting happens between fixed points in 1D, trajectories are monotonic*)
- If Fixed Points were unchangeable, they would be of limited use, e.g., cell needs to produce more of a protein under certain conditions and less under other conditions
- How can a Fixed Point be moved in response to an cell/ organism's need?

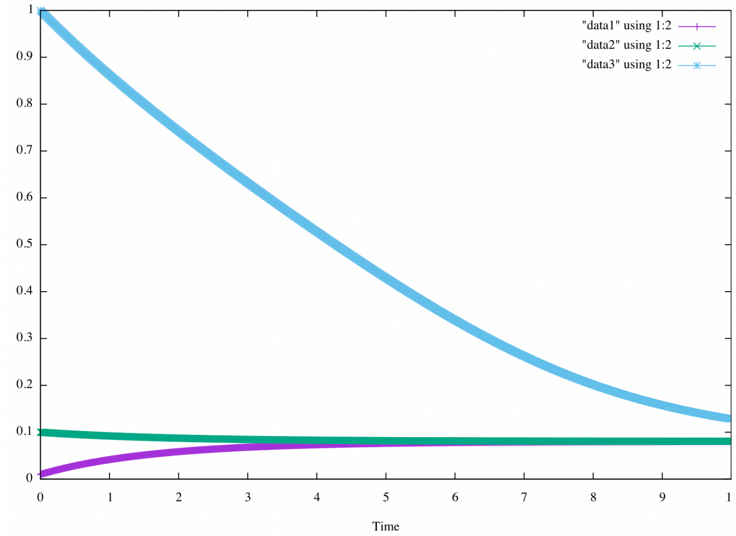
Background quiz

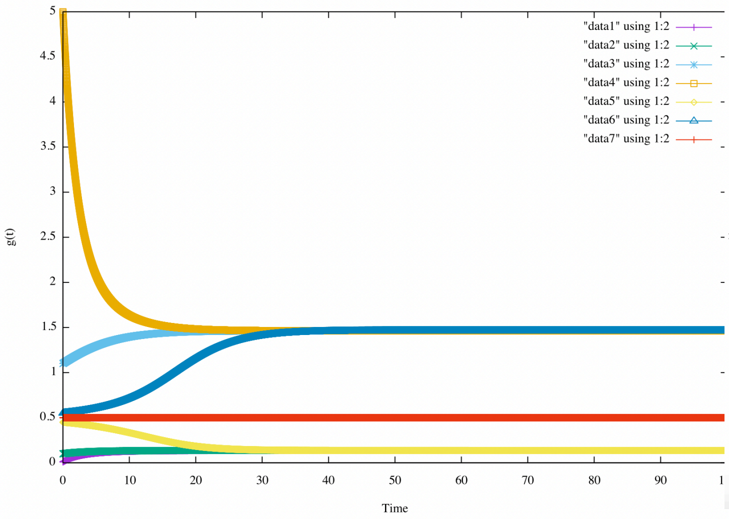
Background quiz: go.epfl.ch/turningpoint

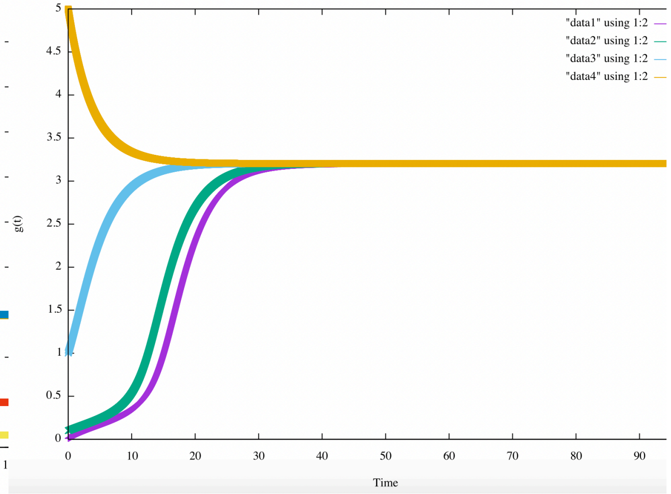
Session Id: julian23

All input is anonymous; data are stored outside CH

Break







High degradation

s = 0.05
r = 0.7

g* ~ 0.08

Initial points

g0 = 0.01, 0.1, 1.0

s = 0.05
r = 0.5

g* ~ 0.14, 0.5, 1.46

g0 = 0.01, 0.1, 0.49, 0.5, 0.51, 1.0, 5.0

Low degrad.

s = 0.05
r = 0.3

g* ~ 3.2

g0 = 0.01, 0.1, 1.0, 5.0

Tricky points

- Apply various steps to understand an equation: non-dimensionalise, combine parameters, set one to zero/explore the other, start from intuitive limits ($s = 0$, $r = 0$, etc)
- Know what defines hysteresis/bistability in the dg/dt versus g plot (Graph 1)

  bistability = two stable states separated by an unstable one

  hysteresis = the jump between low and high [G] depends on history, i.e., it occurs at different values of the parameters s, r when going from low to high or high to low
- Know how the time evolution of the fixed point g^* is related to the time variation of the parameters s, r
- Know how to plot $x/(1+x^2)$, $x^2/(1+x^2)$, $x^3/(1+x^3)$, $x^4/(1+x^4)$
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