

Glossary

Given the first-order, autonomous differential equation(s) in one dimension:

$$dx/dt = f(x)$$

or two dimensions:

$$dx/dt = f(x, y)$$

$$dy/dt = g(x, y)$$

or, generally, in N dimensions:

$d\mathbf{X}/dt = \mathbf{F}(\mathbf{X})$ where $\mathbf{X} = (x, y, \dots)$ and $\mathbf{F}(\mathbf{X}) = (f, g, \dots)$ is an N-component vector of functions, each of N variables.

Let $\mathbf{x}^*$ be a Fixed Point of the equation, so that $\mathbf{F}(\mathbf{x}^*) = 0$.

Here are some important terms related to fixed points and phase portraits.

Degenerate node = Noeud dégénéré

A type of fixed point similar to a node but for which the Jacobian has only a single eigenvector.

Fast direction = Direction rapide

The eigenvector of a node to which trajectories become tangent as time goes to minus infinity.

Fixed Point = Point Fixe

A point $\mathbf{x}^*$ (1 dimension) where $f(x) = 0$, or (x^*, y^*) (2 dimensions) where the nullclines $f(x, y) = g(x, y) = 0$ cross, i.e., both derivatives are zero, and the point is therefore a solution of the equation

$$d\mathbf{X}/dt = \mathbf{0}.$$

Node = Noeud

A type of fixed point at which multiple trajectories merge (stable node) or diverge (unstable node).

Non-isolated = Non-isolé

Nullcline = Isocline = Nullcline

Curve along which one of the derivatives of $\mathbf{F}(\mathbf{X})$ is zero, i.e., $f(x,y) = 0$ or $g(x,y) = 0$ in the 2D case.

Phase portrait = Phase Space = Potrait de phase

(2D) Graphical representation of the trajectories in the XY plane.

- trajectories are tangent to the vector field at each point
- trajectories cannot cross each other
- at all points except a fixed point, only a single trajectory passes through the point.

Saddle point = Point de Selle

A type of Fixed Point at which trajectories approach and recede but don't touch the point.

Slow direction = Direction lente

The eigenvector of a node for which trajectories approach the node tangent to the eigenvector as time goes to plus infinity,

Spiral = Spirale

Stable = Stable

Describes a Fixed Point (node, degenerate node, star, spiral, or non-isolated fixed points) for which small perturbations of a nearby trajectory decay away so the trajectory still approaches the Fixed point.

Stable (unstable) manifold = Collecteur stable (instable)

The set of initial points (x_0, y_0) whose trajectories approach a saddlepoint as time goes to plus (minus) infinity.

Star = Etoile

Unstable = Instable

Describes a Fixed Point (node, degenerate node, star, spiral, or non-isolated fixed points) for which small perturbations of a nearby trajectory grow so that the trajectory diverges from the fixed point.

Stable manifold of a saddlepoint = The set of initial points (x_0, y_0) such that the trajectory approaches the saddlepoint (x^*, y^*) as time goes to infinity (see Strogatz, p130).

Trajectory = Trajectoire

The path of a particle $\mathbf{x}(t)$ through its phase space as a function of time.

Unstable manifold of a saddlepoint = The set of initial points (x_0, y_0) such that a trajectory approaches the saddlepoint (x^*, y^*) as time goes to minus infinity (see Strogatz, p130).

Vector field = Champs de vecteur

(2D) Graphical representation of the velocity vector $(dx/dt, dy/dt)$ in the XY plane.

Two representations are common: either draw them all with the same length, and just vary their direction, or their magnitude can be proportional to the vector's length at each point.