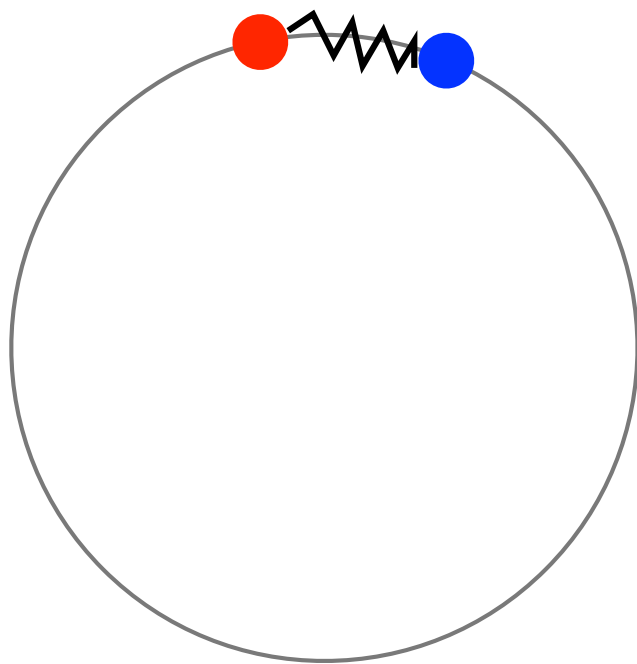


Phase oscillators & entrainment

Two coupling models

Model I (6.1)

- $\dot{\alpha} = \Omega$
- $\dot{\theta} = \omega + K \sin(\alpha - \theta)$



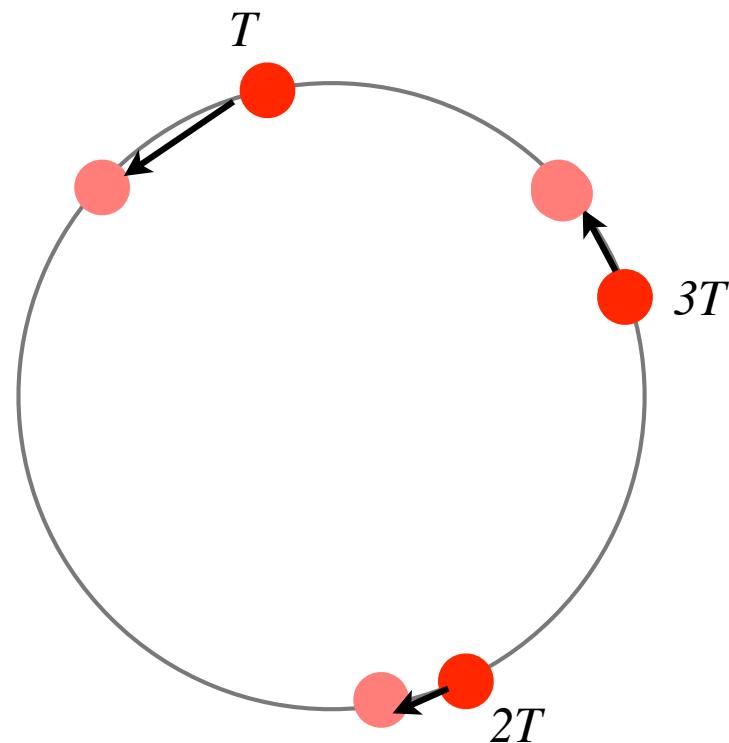
spring

Model I (6.2)

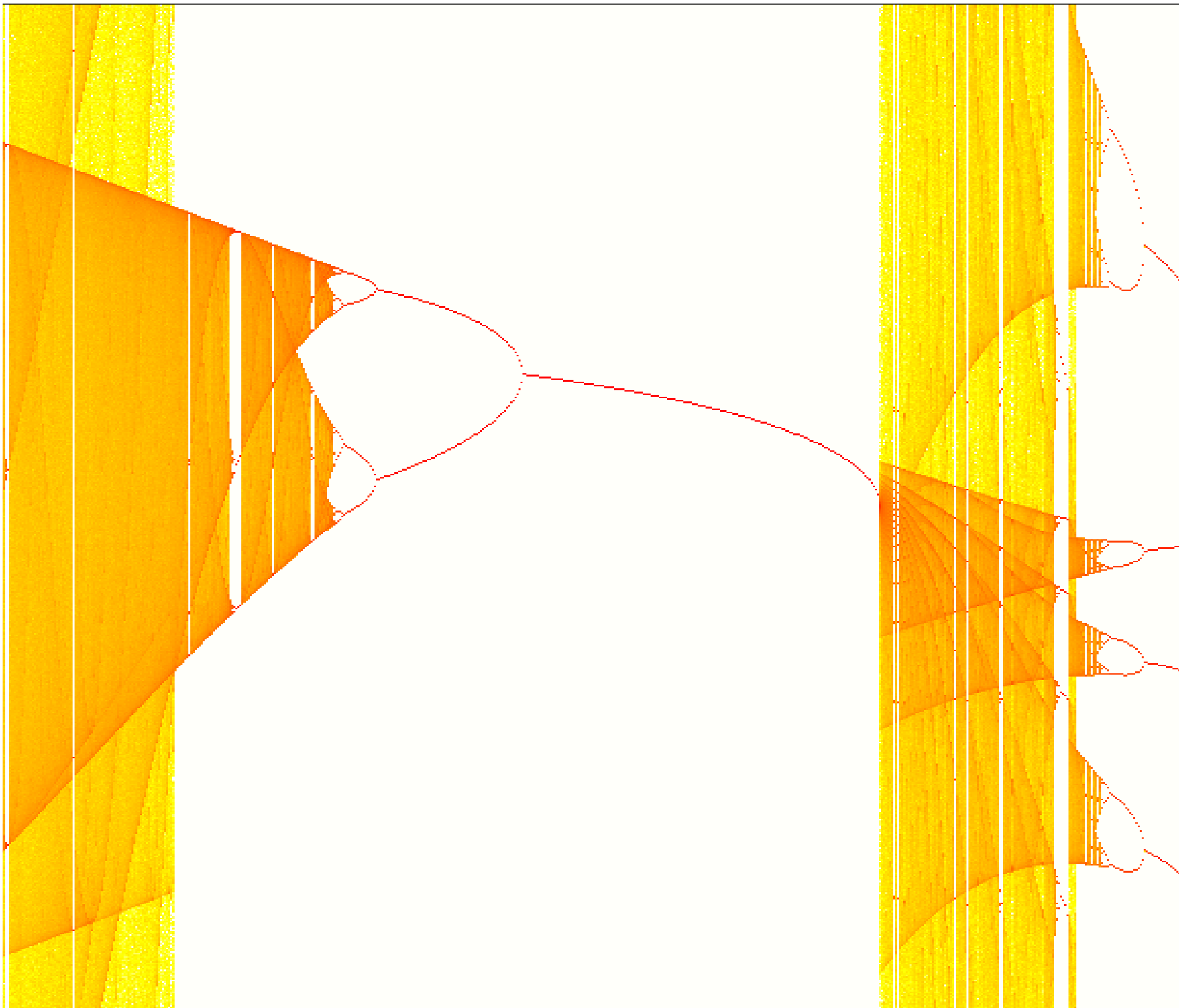
$$\dot{\theta} = \omega + \sum_n \delta(t - nT) g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

$$g(\theta) = e \sin(\theta)$$



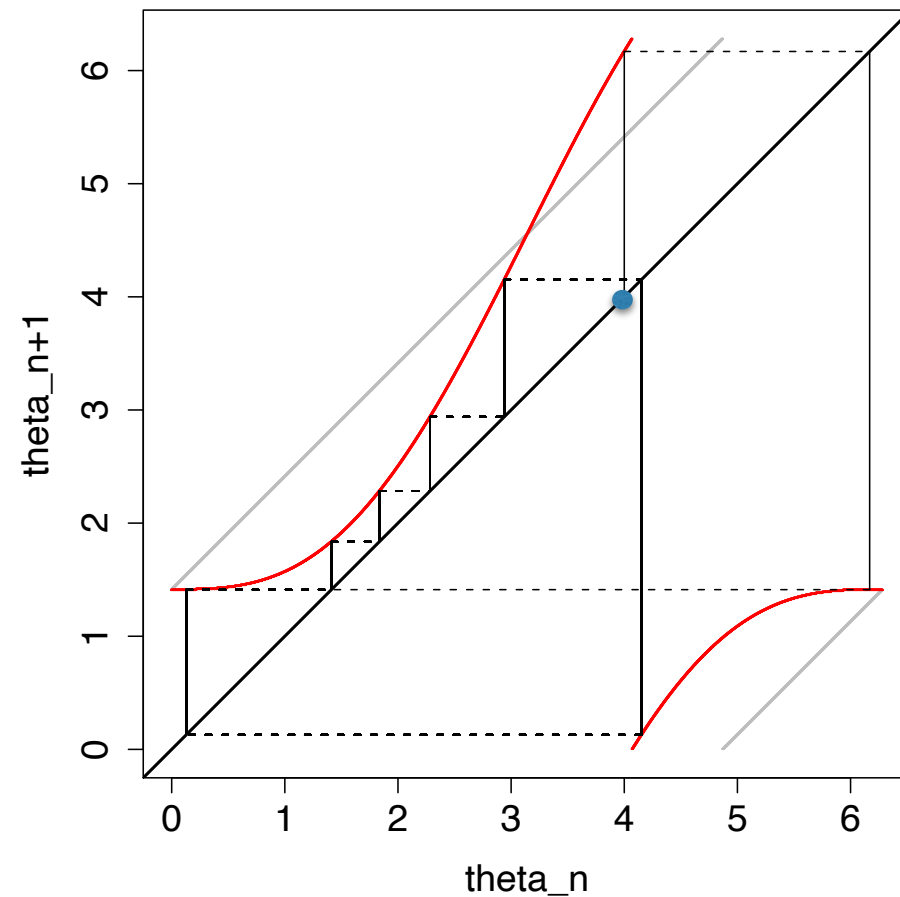
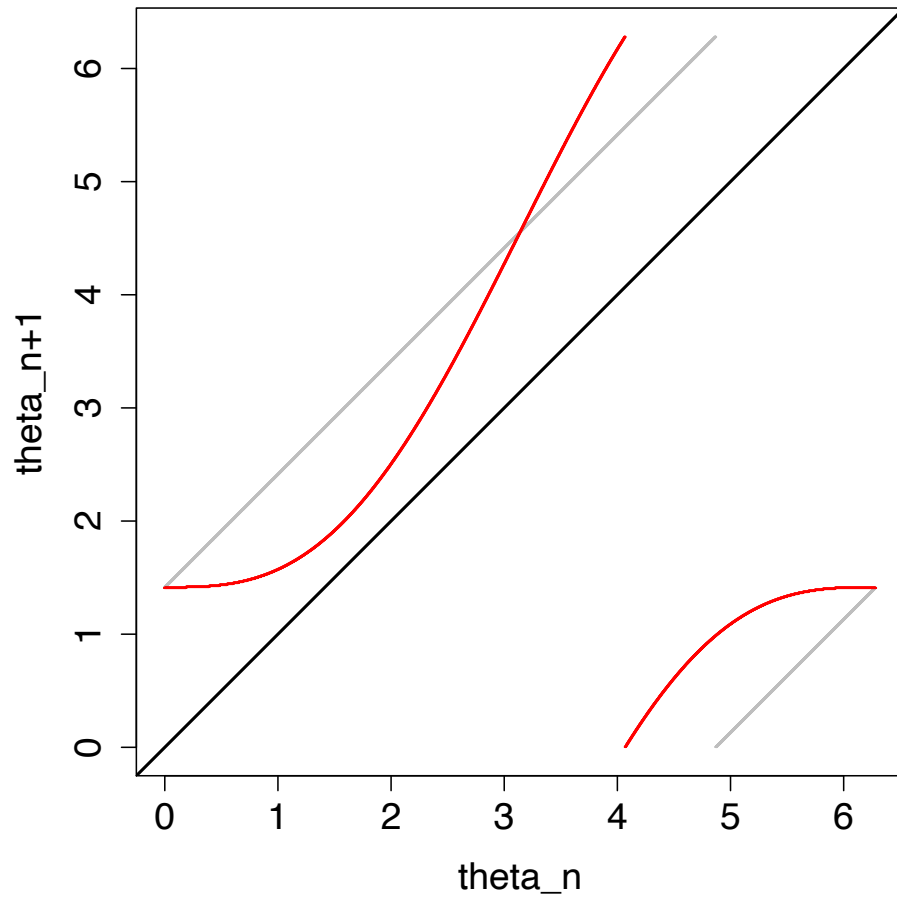
instantaneous kicks



Stability (no fixed point)

Here a period 6 orbit

$$|e| < T\omega \sim 1.4$$

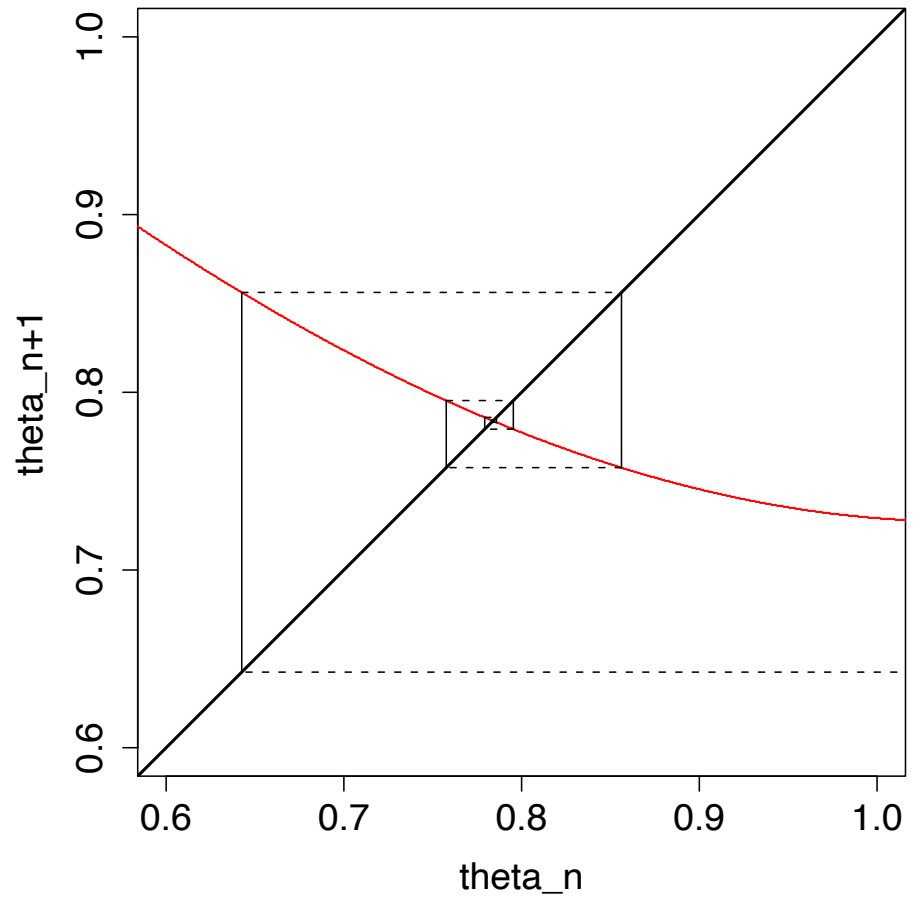
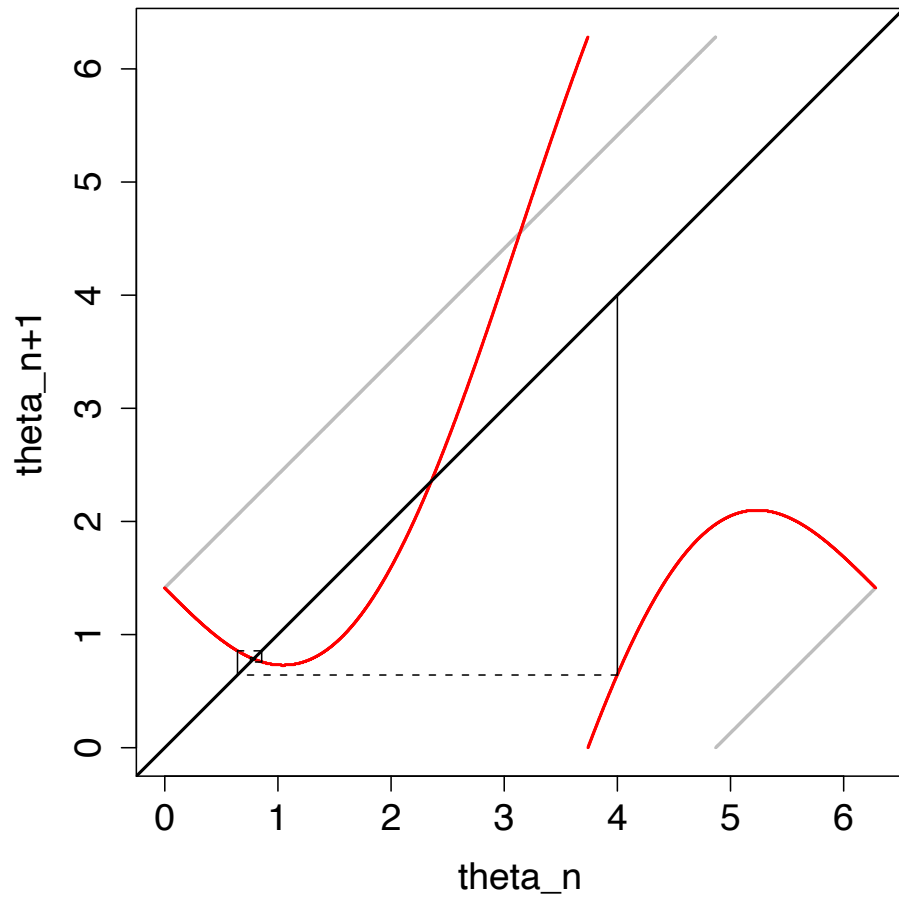


$$e = -1$$

$$\bullet : \theta_0$$

Stability (period 1 orbits)

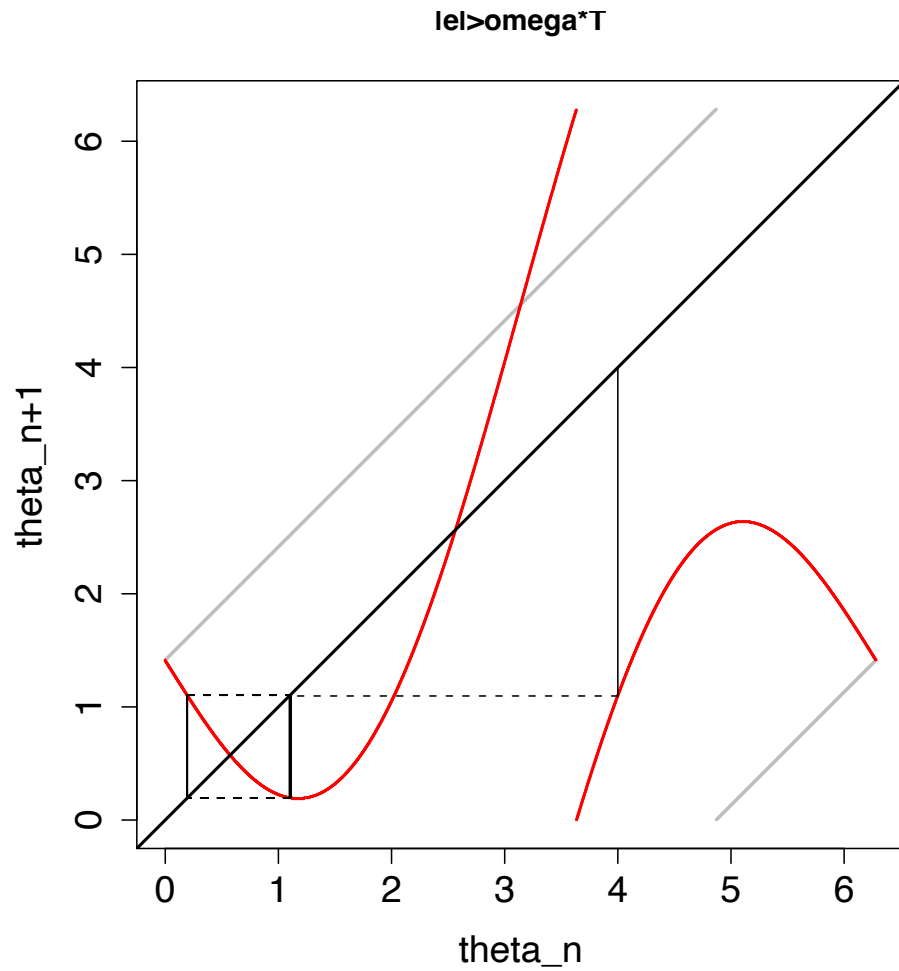
$$|e| > T\omega$$



$$e = -2$$

Stability (period 2 orbits)

$$|e| > T\omega$$

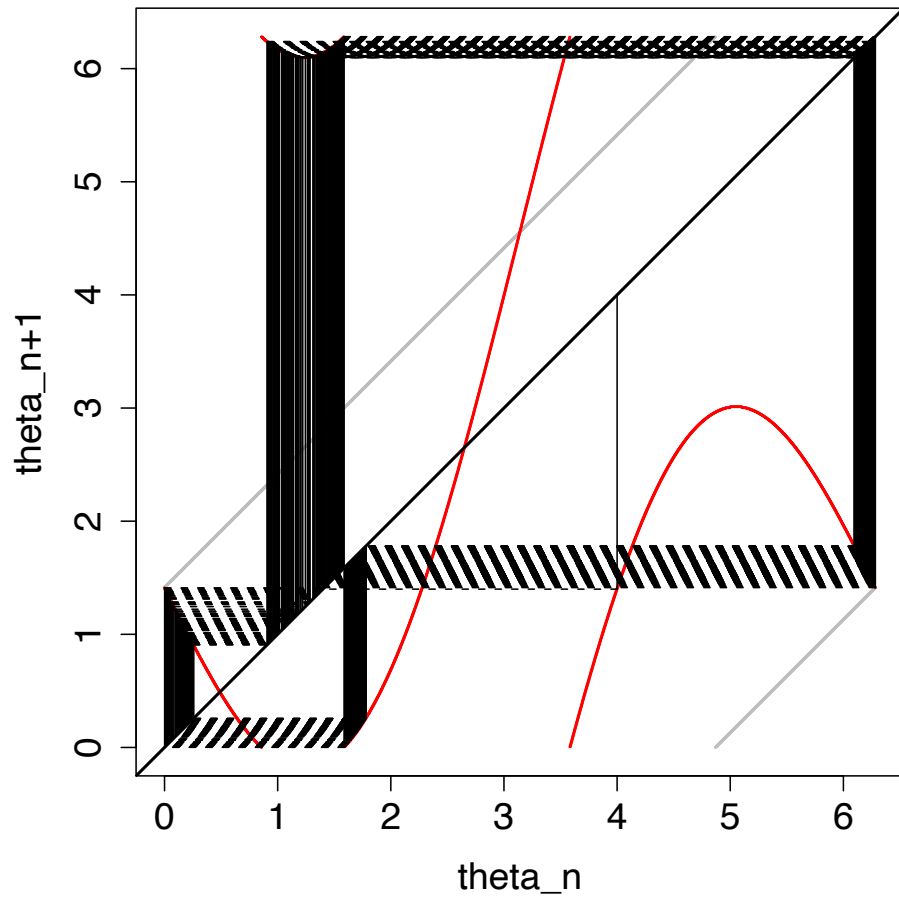


$$e = -2.6$$

Chaotic orbits

The trajectory fills a band (continuum)

$$|e| > T\omega$$



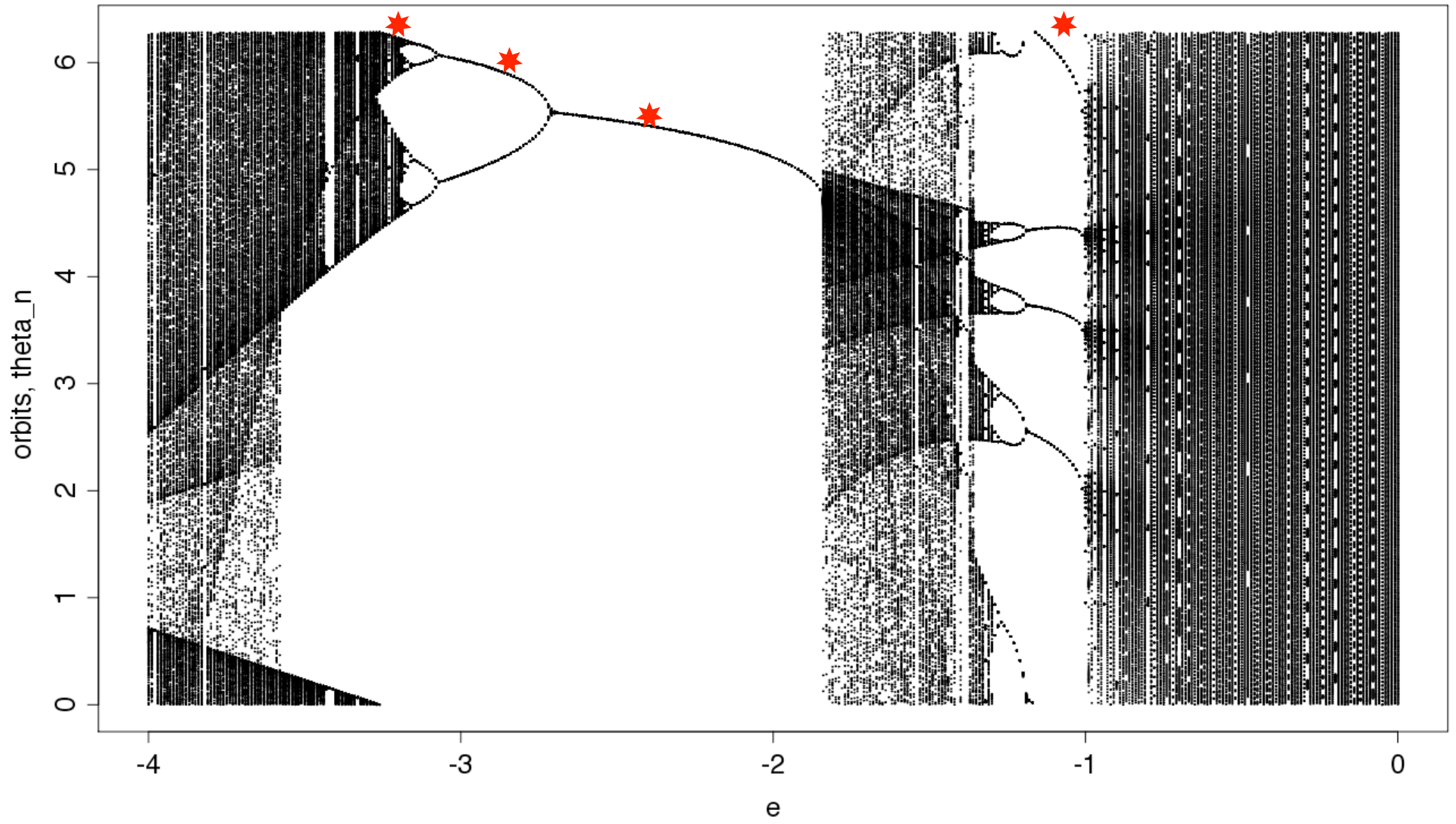
$$e = -3$$

$$\dot{\theta} = \omega + \sum_n \delta(t - nT) g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

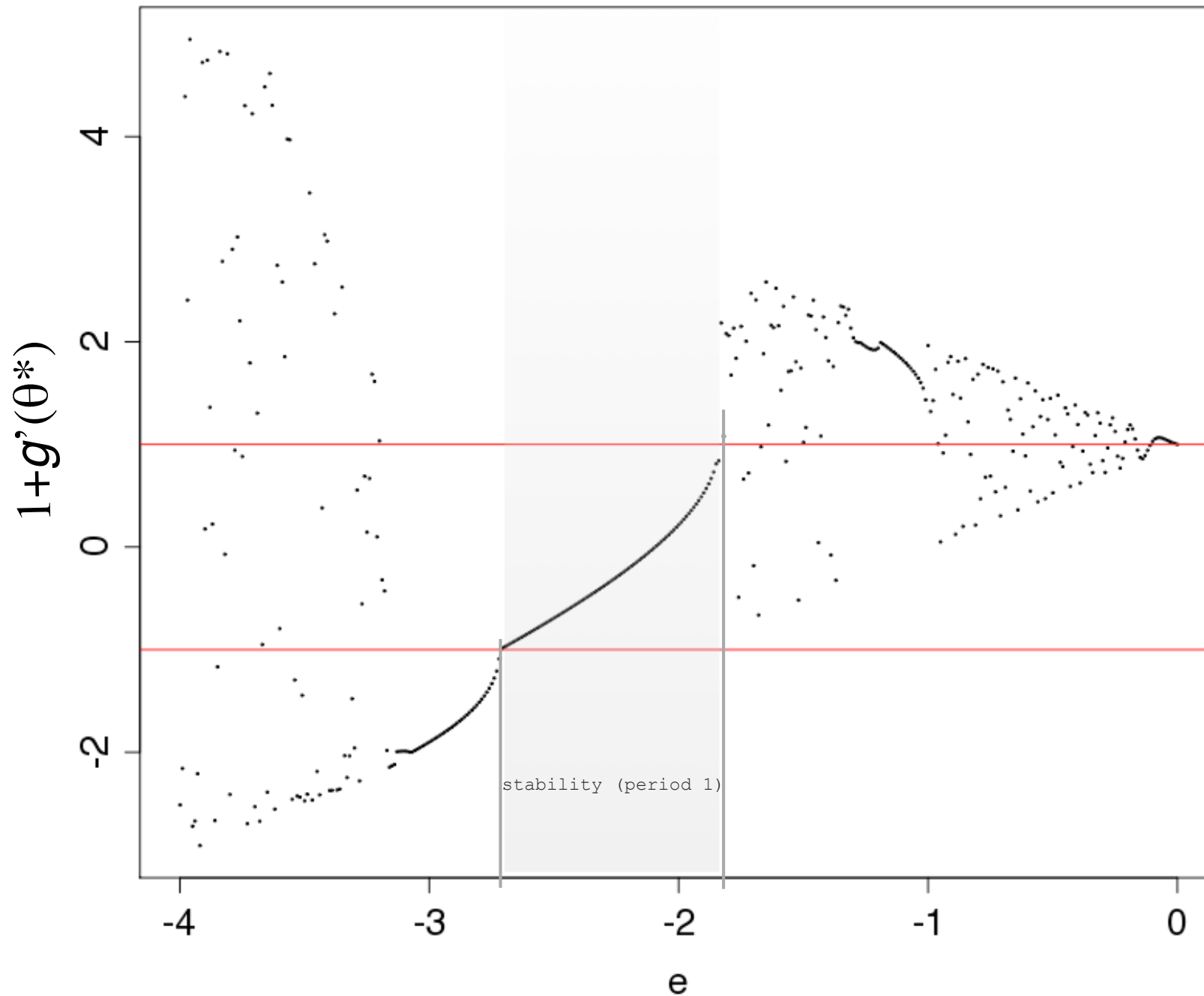
$$g(\theta) = e \sin(\theta)$$

$$e = -1$$



Stability region $|F'(\theta^*)| < 1$

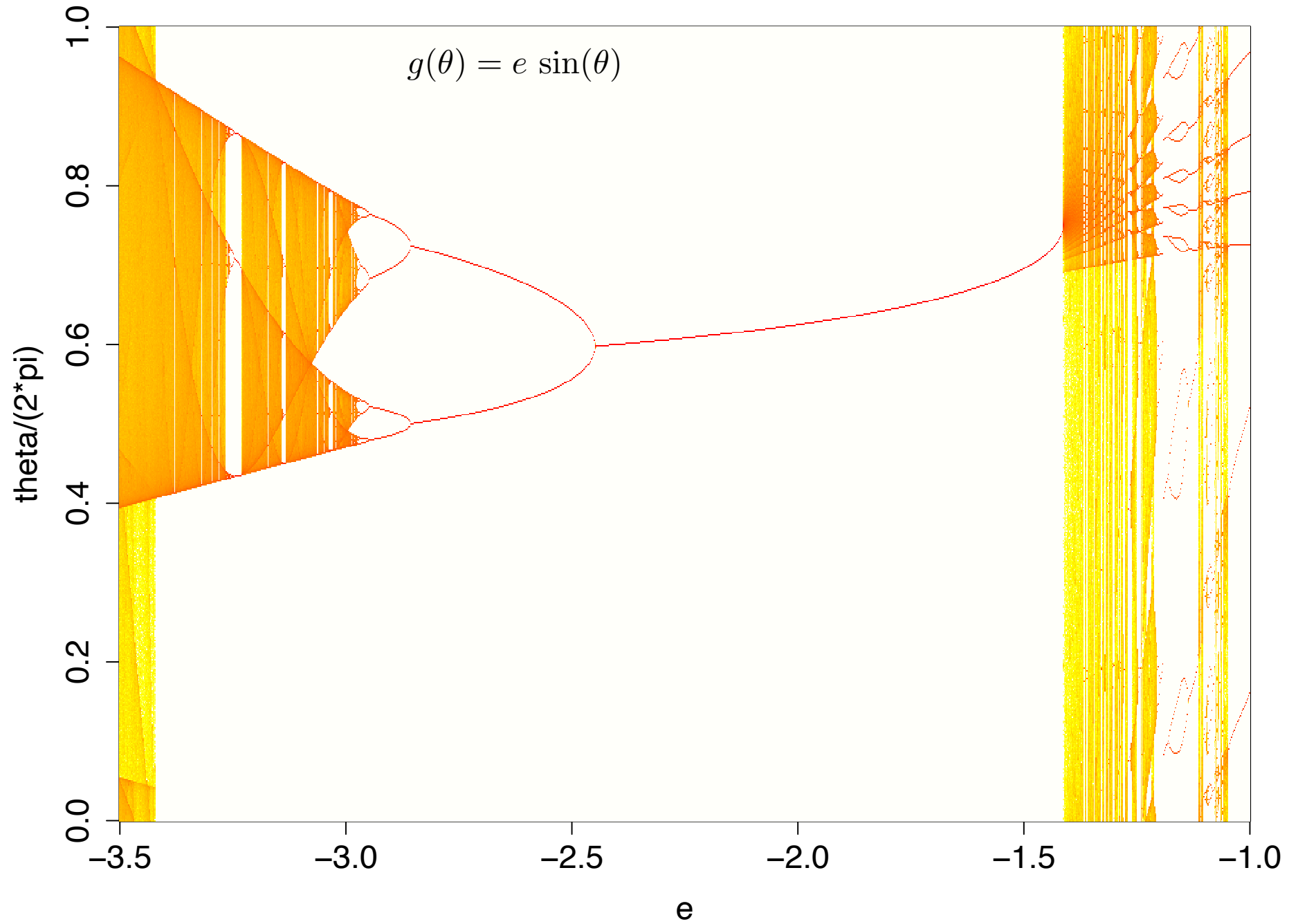
$$F'(\theta^*) = 1 + g'(\theta^*)$$



$$\dot{\theta} = \omega + \sum_n \delta(t - nT)g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

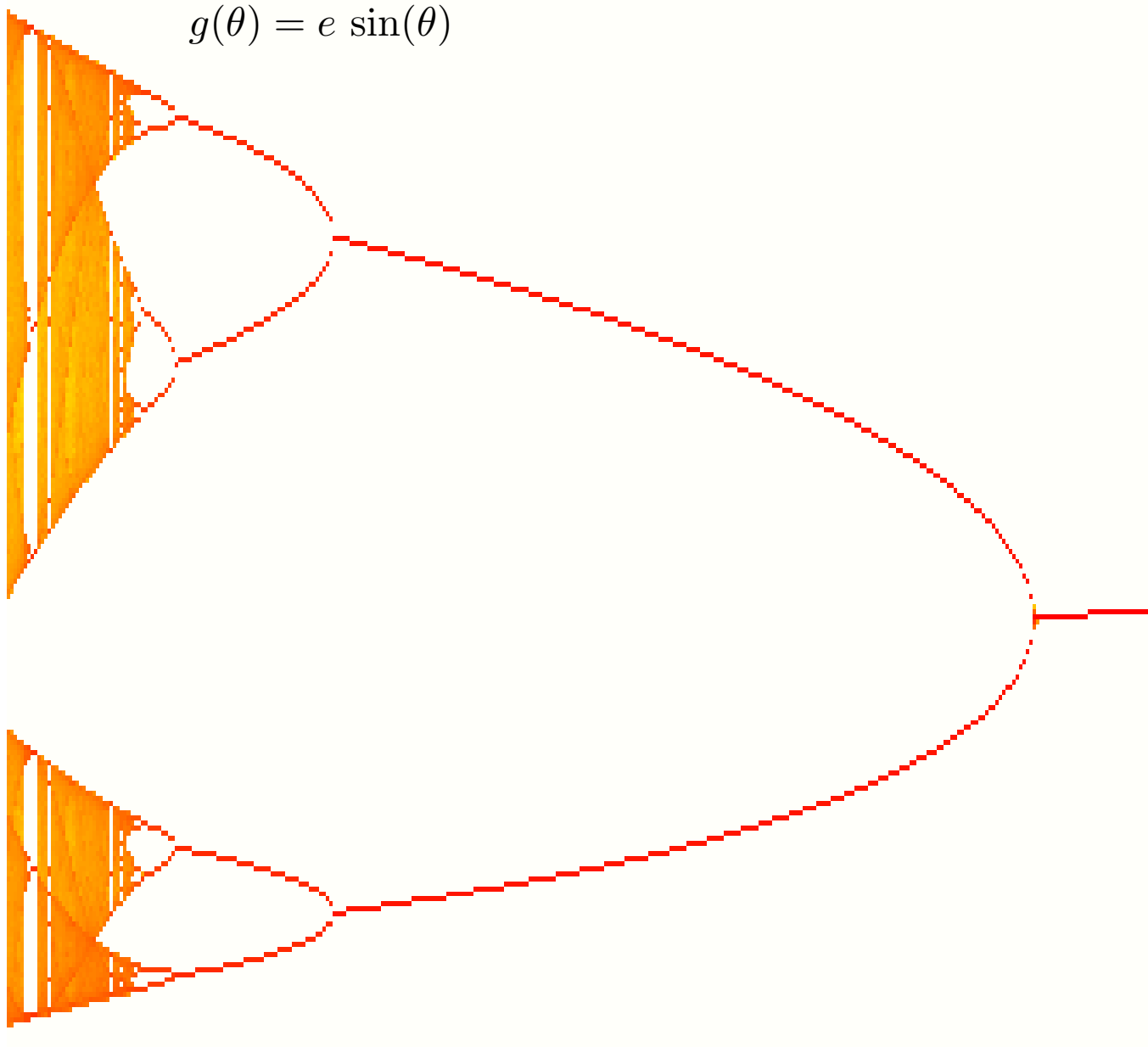
$$g(\theta) = e \sin(\theta)$$



$$\dot{\theta} = \omega + \sum_n \delta(t - nT) g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

$$g(\theta) = e \sin(\theta)$$



Remarks

- no chaos in 2D ODE system (but chaos in 3D)
- chaos in 1D maps (discrete dynamical systems)
- complex behavior in the kicked phase oscillator