Plasma Physics I

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Exercise 1

Consider the Lorentz distribution:

$$f(v) = \frac{A}{a^2 + v^2}$$

a.) Find the normalizing constant A of the function $f(v)$ such that:

$$\int_{-\infty}^{+\infty} f(v)dv = n$$

Suggestion: To solve the integral, use the method of residues. Choose a semicircle in the complex plane $z = v + iv_i$ with an infinite radius as integration path.

b.) Calculate $\langle v \rangle$, the first moment of $f(v)$.

c.) Can this function describe a real electron population of a plasma in a laboratory?

Exercise 2

Show that in a plasma described by the *Vlasov* equation:

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

the entropy, defined in a simple way (without numerical factors) as:

$$S(t) = -\int d\vec{v} \int d\vec{x} f(\vec{x}, \vec{v}, t) \ln\left(f(\vec{x}, \vec{v}, t)\right)$$

is conserved ($dS/dt = 0$).