November 5, 2024

Problem Set 7

Exercise 1. Let D_{2n} be the dihedral group, the group of symmetries of a regular n-gon. This group has $2n$ elements.

(a) Describe all irreducible complex representations of D_n. Start with the 1-dimensional representations, then consider the complexifications of the symmetries of a regular n-gon, and use the sum of the squares formula to complete the classification. Consider cases of odd and even n.

(b) Use the character table to find the decompositions of the tensor products $V_i \otimes V_j$ into a direct sum of irreducible representations. (It is enough to consider the case where $\mathrm{dimV_i} > 1, \mathrm{dimV_j} > 1$).

Exercise 2. Use results in representation theory of finite groups over $\mathbb{C}$ to show that every group of order p^2, where p is a prime, is abelian.

Exercise 3. Let G be a group of invertible upper triangular 2×2 matrices with coefficients in $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$.

(a) Find the conjugacy classes of G.

(b) Find a normal subgroup $H \subset G$ such that G/H is abelian.

(c) Use (a), (b) and the "sum of squares" formula to find the dimensions of the irreducible complex representations of G.

(d) Use the orthogonality relations to compute the table of characters of G.